Problem 1 Find the numbers that satisfy each inequality and sketch the solution on a numberline.

(a) $|x-5| \leq 3$

(b) $|50-x| < 3$

(c) $1 < |y-6|$

(d) $|x-4| + |x-3| < 1$

(e) $|x-4| + |x-3| > 0.75$

Problem 2 Use inequalities to describe the numbers *not* in the interval $[-4, 2]$ in two ways: (a) using an absolute-value inequality, and (b) using one or more simple inequalities

Problem 3 Our goal here is to visually prove an important inequality involving geometric and arithmetic means.

(a) Sketch a square with sides of length $(a+b)$.

(b) Sketch four rectangles with sides of length a and b inside the square. Your rectangles should all fit inside the square without overlapping.

(c) Explain why the sum of the areas of the four rectangles is no greater than the sum of the area of the square.

(d) Explain how this proves the inequality

$$\sqrt{ab} \leq \frac{1}{2}(a+b).$$

Problem 4

The arithmetic mean of two numbers, a and b, is $A(a,b) = \frac{1}{2}(a+b)$

The geometric mean of two positive numbers, a and b, is $G(a,b) = \sqrt{ab}$

The harmonic mean of two positive numbers, a and b, is $H(a,b) = \dfrac{2}{\frac{1}{a}+\frac{1}{b}}$

Starting from $G(\frac{1}{a}, \frac{1}{b}) \leq A(\frac{1}{a}, \frac{1}{b})$, show that $H(a,b) \leq G(a,b)$.

Problem 5 Suppose we drive half way to Maugerville at 40 km/h and the rest of the way at 60 km/hr. Show that the average speed is the harmonic mean of 40 and 60.

Problem 6 Verify that the geometric mean of the three numbers 3, 9 and 27 is less than there arithmetic mean.