Physical Interpretation of Ground Subsidence Surveys – A Case Study

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Abstract

The role of deformation surveys is much broader than just the conventional determination of the geometrical status of the deformable object. By comparing the observed deformation quantities (geometrical model) with the designed deformations obtained from the finite element analysis (deterministic model) one can verify the designed geomechanical parameters of the material and one may also determine the actual deformation mechanism. In this presentation, the geodetic monitoring surveys are used in identifying cause of irregular ground subsidence in a potash mining area in Canada. By iterative comparison of the observed subsidence results with the deterministic model, the irregular subsidence has been explained as an effect of hydrological changes due to the inflow of water to the mine. The subsidence surveys helped in identifying the location of a hypothetical aquifer.

1. Introduction

Analysis of deformations of any type of deformable body includes geometrical analysis and physical interpretation. Geometrical analysis describes the change in shape and dimensions of the monitored object, as well as its rigid body movements (translations and rotations). The ultimate goal of the geometrical analysis is to determine in the whole deformable object the displacement and strain fields in the space and time domains. In the generalised method of deformation analysis (Chen 1983; Chrzanowski et al.,1983; Chrzanowski et al., 1986), the displacement field is obtained by fitting an appropriate displacement function to the measured deformation quantities (e.g., absolute or relative displacements, strain measurements, tilts, changes in distances or angles).

Physical interpretation is to establish the relationship between the causative factors (loads) and the deformations (Chen and Chrzanowski, 1986). This can be determined either by

(a) a statistical method, which analyses the correlation between the observed deformations and loads, or
(b) a deterministic method, which utilises information on the loads, properties of the material, and physical laws governing the stress-strain relationship.

The deterministic modelling requires solving differential equations for which closed form solutions may be difficult or impossible to obtain. Therefore, numerical methods, such as the Finite Element Method (FEM) are used. In case of rock and soil materials, the in-situ geomechanical properties may significantly differ from the laboratory values. This must be taken under consideration when performing deterministic modelling of deformation.

By comparing the geometrical model of deformations, derived from the observed deformation quantities, with the designed deformations obtained from FEM, one can determine the actual deformation mechanism (Chrzanowski and Szostak-Chrzanowski, 1993; Chrzanowski and Szostak-

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Chrzanowski, 1995) and/or one can verify the designed geomechanical parameters (e.g., Szostak-Chrzanowski et al, 2000). Thus, the role of deformation monitoring surveys becomes much broader than just the conventional determination of the geometrical status of the deformable object.

In this presentation, after a short introduction to the behaviour and modelling of various types of rock material, an example is given on the use of geodetic monitoring surveys in explaining irregular ground subsidence in a potash mine in Canada.

2. Rock Behaviour

2.1 Simplified Classification of Rock types

In modelling rock behaviour, the authors consider two types of rocks:

(1) brittle rocks, which under tensional loading develop cracks and discontinuities and
(2) viscous or visco-elastic rocks, such as salt rock, which under loading exhibit continuous "flow" of the material

2.2 Behaviour of Brittle Rock

The rock mass behaviour in some range of compressive stresses may be assumed as linear elastic. The relation between stress and strain is expressed as:

$$\sigma = \mathbf{D}\varepsilon \tag{1}$$

where σ is the stress vector, **D** is a constitutive matrix , and ε is strain vector.

The constitutive matrix for isotropic material in case of plane strain is expressed as:

$$D = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1+2\nu}{2} \end{bmatrix} \tag{2}$$

where E is Young modulus, ν is Poisson ratio.

The behaviour of rock mass subjected to tensional stresses may be assumed as a behaviour of 'no-tension' material (Zienkiewicz et al. 1968). In case of the presence of tensional stress in only one direction, the constitutive matrix includes directional Young modulus E_a, and directional Poisson ratio ν_α and, for case of plane strain, has the form:

$$D_a = \frac{E_a}{(1+\nu)(1-\nu-2n\nu_a^2)} \begin{bmatrix} n(1-\nu_a^2) & n\nu_a(1+\nu) & 0 \\ n\nu_a(1+\nu) & 1-\nu^2 & 0 \\ 0 & 0 & m(1+\nu)(1-\nu-2n\nu_a^2) \end{bmatrix} \tag{3}$$

where $\nu = E / E_a$, $m = G_a / E_a$, $G_a = E_a / (2 (1+\nu_a))$, and $\nu_a = (3/2 - \nu) / (1+ E / E_a)$.

After reaching the critical values of stress (compressive or tensional) the rock mass behaves as a non-linear material. The problem of non-linearity of the material is solved through sequential linear elastic solutions with iterative updating of the constitutive matrix through changes of the modulus of elasticity, E.

2.3 Behaviour of Salt Rock

The behaviour of salt is time dependent (visco-elastic) and the strain may be expressed as the sum of two components:

$$\varepsilon = \varepsilon_e + \varepsilon_c \tag{4}$$

where ε_e is the elastic strain and ε_c is the creep strain. It is difficult to obtain the long term characteristics of the creep model. Generally it requires several years of in-situ creep measurements to arrive at the proper model..

The salt rock may also be considered as a non-Newtonian liquid with high and not constant viscosity (Mraz et al., 1987). As a non-Newtonian liquid, the intact salt rock deposits are under isotropic lithostatic stress conditions. Therefore, the shear stress in intact salt rock is equal to zero. Development of shearing stresses due to mining activity causes the flow of the salt mass into the excavated areas until new equilibrium state of stresses is achieved.

3. Modelling of Rock Deformation Using FEM

3.1 Basic Definitions in Modelling of Deformations

Using the well known principles of the finite element displacement approach, the global equilibrium equation for the investigated object maybe written as:

$$\mathbf{Kd} = \mathbf{r} - \mathbf{f}^{b} + \mathbf{f}^{\sigma o} + \mathbf{f}^{\varepsilon o} \tag{5}$$

where $\mathbf{d}$ is the vector of nodal displacements, $\mathbf{K}$ is the total stiffness matrix, $\mathbf{r}$ is the vector of external forces concentrated at nodal points, $\mathbf{f}^{b}$ is the loading vector of body forces, $\mathbf{f}^{\sigma o}$ the loading vector from initial stresses, and $\mathbf{f}^{\varepsilon o}$ is the loading vector from initial strains. The global matrices and vectors are calculated through a superimposition of local (at each element or at each node of the FEM mesh) matrices $\mathbf{K}_e$ and vectors $\mathbf{r}_e$, $\mathbf{f}_e{}^{b}$, $\mathbf{f}_e{}^{\varepsilon o}$, and $\mathbf{f}_e{}^{\sigma o}$.

The stiffness matrix for an element in two dimensions is given as:

$$\mathbf{K}_e = \iint \mathbf{B}_e^T \mathbf{D} \mathbf{B}_e t dx dy \tag{6}$$

where $\mathbf{B}_e$ is the matrix relating strains in the element to its nodal displacements, $\mathbf{D}$ is the constitutive matrix of the material and t is the unit thickness of the elements. Since the total stiffness matrix $\mathbf{K}$ is singular, boundary conditions must be applied in order to solve equation (5) for the displacements.

3.2 Modelling of Deformation due to Mining in Brittle and Salt Rocks

A method of modelling the deformation and stress change in rock due to mining activity has been developed by (Szostak-Chrzanowski and Chrzanowski, 1991) based on FEM. The method is known as the S-C method. The most important concept of the S-C method is an introduction of the 'weak zone' in the qualitative model of ground subsidence. The 'weak zone' is delineated by the FEM elements in which the maximum shearing stresses develop at the boundary between the zone of rocks subjected to tensional stresses above the underground opening and the surrounding rocks subjected to compressive stresses. The boundary surface of the maximum shearing stresses had been identified by Kratzsch [1983] as a slippage surface of rocks reduces transferring of the tensional stresses beyond the boundary.

Once the 'weak zone' is introduced into the FEM model, the basic task is to perform a non-linear analysis. The non-linearity is modelled by changing the values of E according to criteria based on the critical tensional stresses. In cases of only one principal stress being tensional, the material is assumed to be anisotropic (transversely isotropic material with E = 0 in the direction of the critical principal stress). The FEM analysis is repeated until no more elements are identified as having developed critical tensional stresses.

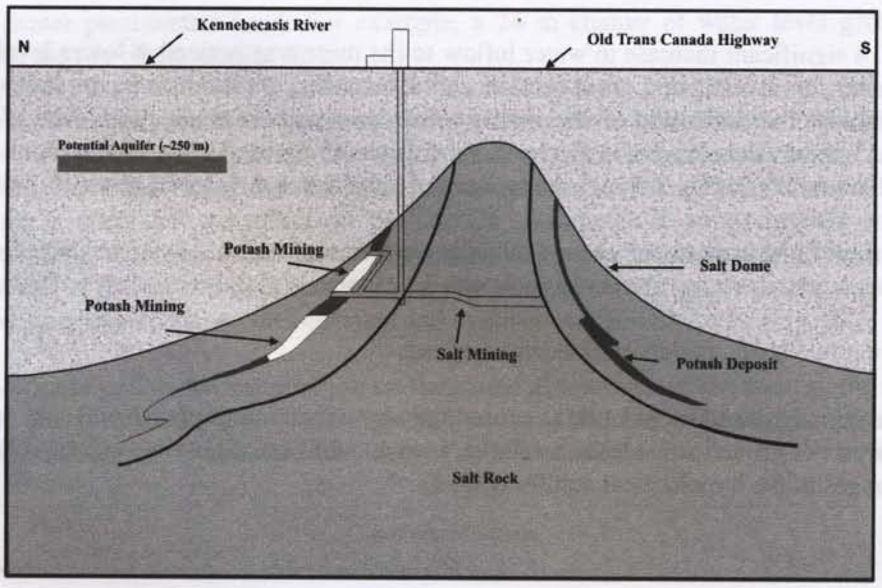
Modelling of salt rocks in salt and potash mines is performed in two phases:

(1) Modelling of the salt rock response to the mining extraction applying the condition that the volume of the subsidence basin (under the cap rock) must be the same as is the volume of mining openings (minus the volume of the compacted backfill). The 'flow-in' zone is delineated by the FEM elements in which maximum shearing stresses are developed.
(2) modelling of the response of the overburden rocks (above the cap rock) which are treated as brittle and non-tension material.

4. Monitoring and Modelling of Ground Subsidence in a Salt and Potash Mine in Canada

Mining of a large deposit of high grade sylvinite in New Brunswick has been carried out by Potash Corporation of Saskatchewan (PCS) since the mid 1980s. Potash and salt mining at PCS takes place at depths between 400 m to 700 m within a 25 km long dome-shaped salt pillow in which the potash is preserved in steeply dipping flanks (Figure 1). A strong, arch shaped, caprock provides an excellent natural support for the overlain brittle rocks. Potash is mined by using a mechanised cut-and-fill method with up to 100% extraction in the 1000 m long and about 150 m high stopes. Unsupported openings are up to 25 m wide. The potash deposit is structurally complex with a variable dip and width. Salt mining is by multi-level room-and-pillar method. Trans Canada Highway runs along the longitudinal axis of the mine and is affected by ground subsidence.

Annual monitoring of ground subsidence over the PCS mining operation near Sussex, N.B., has been carried out by Canadian Centre for Geodetic Engineering since 1989. Fig. 2 shows the layout of the mining workings and the distribution of monitored points, which have been re-observed annually using levelling surveys, traversing of high precision with robotic total stations, and GPS measurements. In 1995, a finite element analysis was performed to model the maximum expected subsidence along a selected cross-section (line C-C in Fig. 2). A summary of the results was presented in (Chrzanowski et al. 1998). The expected subsidence profile was to follow a regular shape with its maximum subsidence located above the room-and-pillar salt extraction (approximately above the centre of the salt dome)



Figur 1: Cross-section of the PCS potash and salt mine in New Brunswick

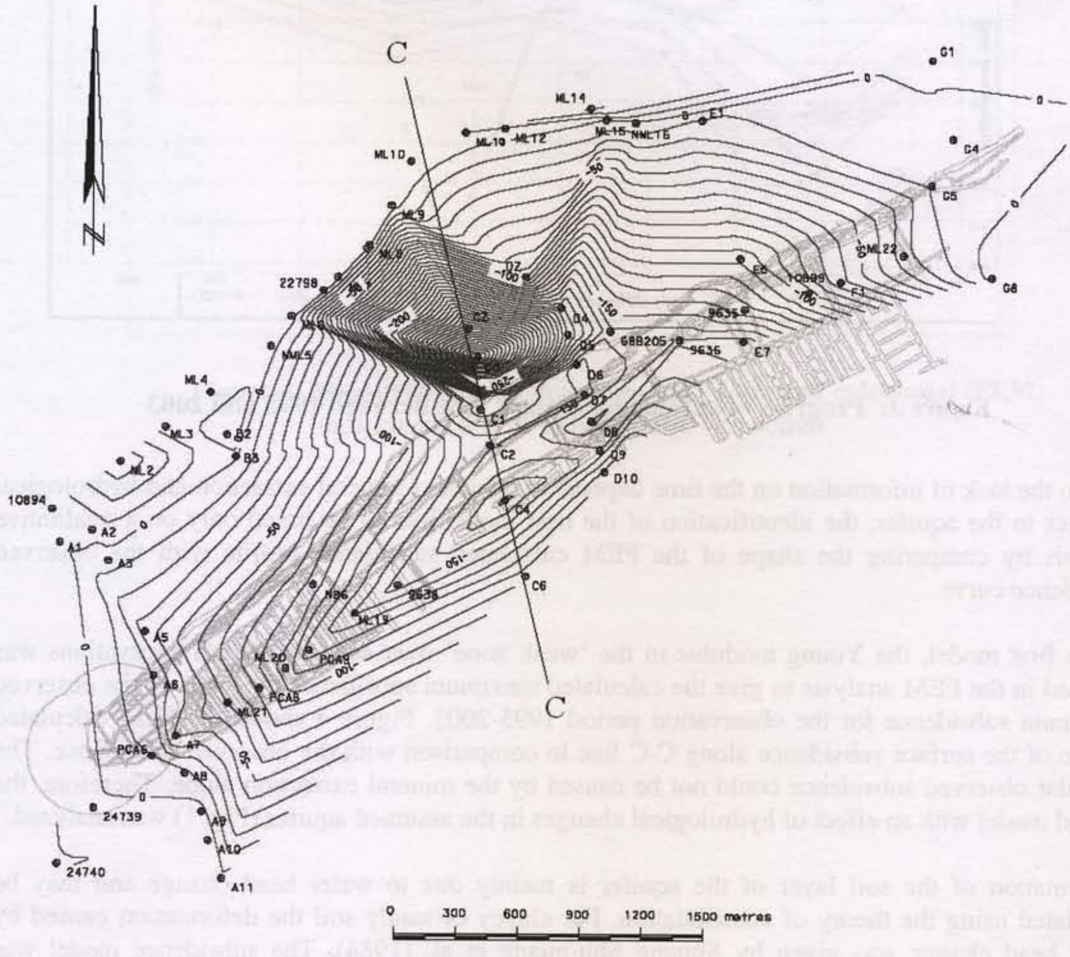


Figure 2: Mine layout , monitoring points, and isolines of subsidence 1996-2003

Since 1997, a significant increase in water inflow to the mine was noticed at lower levels of potash extraction near the investigated cross-section and a secondary subsidence basin started occurring on the surface at the north end of the investigated cross-section along the C line of monitored points. Fig. 3 shows development of ground subsidence along the C line of the monitored points between 1995 and 2003. Fig. 2 shows the isolines of subsidence developed between 1996 and 2003.

In 2003, a new FEM analysis of ground subsidence was undertaken to explain whether the water inflow from an unknown aquifer could cause the development of the secondary subsidence basin.

The following two basic models have been analysed:

(1) analysis of ground subsidence as caused only by extraction of potash and salt, and
(2) analysis of ground subsidence as above, with an addition of possible effects of hydrological changes in the hypothetical aquifer (Fig. 1).

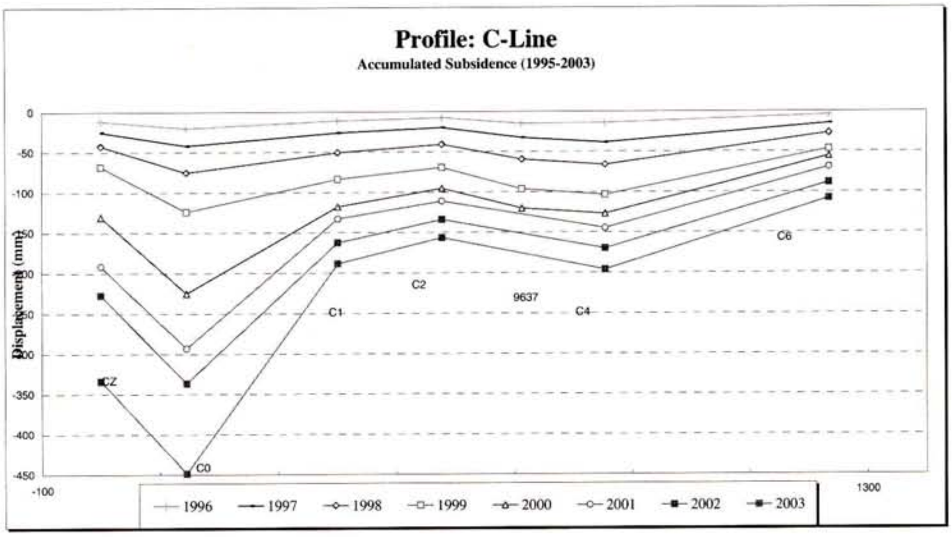


Figure 3: Progress of subsidence along C line between 1995 and 2003

Due to the lack of information on the time dependent effects of mineral extraction and hydrological changes in the aquifer, the identification of the best model had to be based only on a qualitative analysis by comparing the shape of the FEM calculated subsidence profile with the observed subsidence curve.

In the first model, the Young modulus in the 'weak zone' over salt and potash excavations was selected in the FEM analysis to give the calculated maximum subsidence the same as the observed maximum subsidence for the observation period 1995-2003. Figure 4 shows the FEM calculated profile of the surface subsidence along C-C line in comparison with the observed subsidence. The irregular observed subsidence could not be caused by the mineral extraction alone. Therefore, the second model with an effect of hydrological changes in the assumed aquifer (Fig. 1) was analyzed.

Deformation of the soil layer of the aquifer is mainly due to water head change and may be calculated using the theory of consolidation. For clayey or sandy soil the deformation caused by water head change was given by Shueng Shu-guang et al. (1984). The subsidence model was derived by Bravo et al. (1991) who used the principle of relationship of elastic compaction of soil and ground-water piezometric head. For example, a 24 m change of water level gives 0.076 m subsidence on the top of the aquifer (Riley, 1984).

Three analyses were performed for assumed depths of the aquifer to be 350 m, 250 m, and 150 m. In each analysis, it was assumed that the centre of the aquifer is located under the point of the maximum subsidence (pt. C_0) of the secondary subsidence basin and that the compressibility value in the aquifer is such that the effect on the surface subsidence is approximately equal to the observed maximum subsidence of 0.46 m. The dimensions of the aquifer were arbitrarily taken as having the width of 330 m and thickness of 40 m. The analysis of the aquifer at the depth of 150 m gave the best agreement between the observed and modeled subsidence curve (Fig. 5).

In 2003, in order to gain better information on the actual dimensions of the aquifer, the monitoring network was densified in the area of the expected aquifer. The analysis will be repeated in 2005 including information on the observed subsidence of the expanded network.

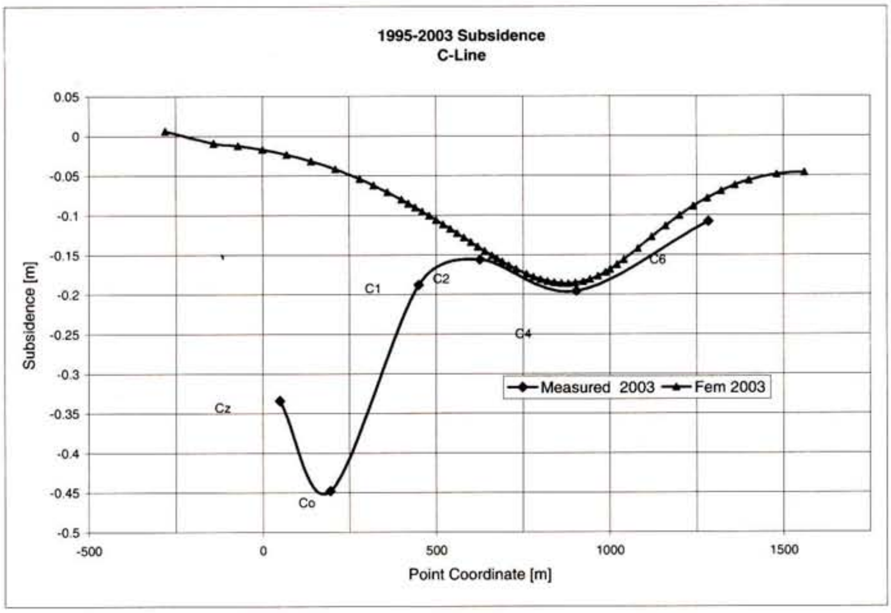


Figure 4: Measured subsidence along C-C Line and calculated (FEM) subsidence due to only mineral extraction

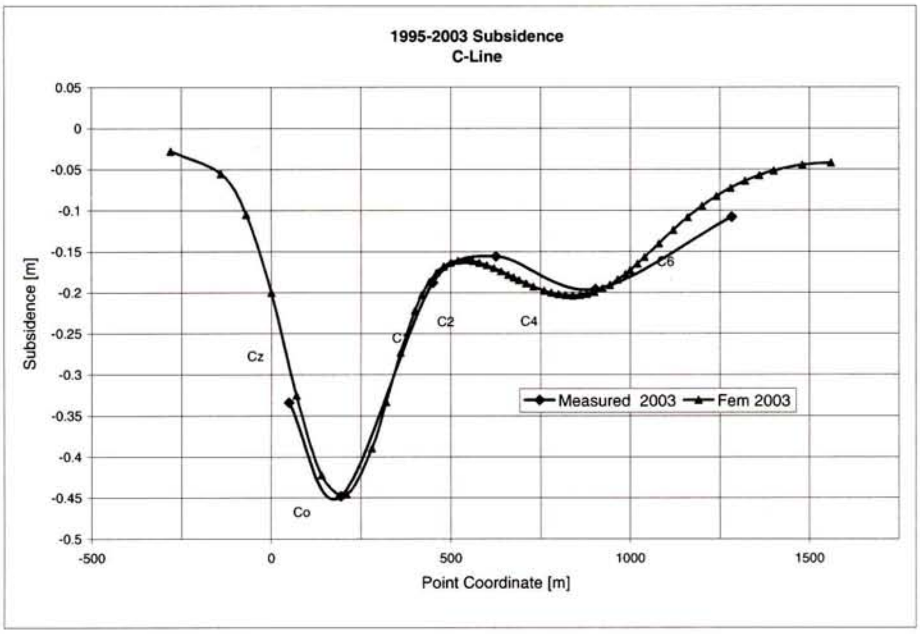


Figure 5: Measured subsidence along C-C Line and calculated (FEM) subsidence with aquifer at 150 m depth

5. Conclusions

The presented case study demonstrates the usefulness of the monitoring surveys in solving geomechanical problems. On the other hand, the comparison between the observed and modelled deformation helps in redesigning the deformation monitoring scheme to make it more useful for the physical interpretation of the monitoring results.

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