INTERSECTION OF HYPERBOLAE ON THE EARTH

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PREFACE

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INTERSECTION OF HYPERBOLAE ON THE EARTH

by

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PREFACE

This report is an unaltered pointing of the author's senior undergraduate technical report, submitted to this department as part of the requirement for the course SE 4711.

ABSTRACT

Several methods are discussed of solving for the point of intersection of a pair of hyperbolic lines of position as generated by commonly used radionavigation systems e.g. Decca, Loran-C, Omega, Syledis, Raydist or HiFix.

Both the plane and the spherical problem are treated by the well-known iterative technique and by a direct trigonometrical solution. Numerous analogies are apparent between the plane and the spherical solutions.

For the direct method on the ellipsoid, a new and easier solution is presented. Notably, geodetic positions on the ellipsoid are calculated accurately for very long lines by spherical trigonometric formulae.

Numerical examples to test the algorithms and a set of Fortran routines are included. The results are verified by Vincenty's geodetic inverse formula.

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Chapter 1

INTRODUCTION

This report deals with the computational problem of determining the point of intersection of a pair of hyperbolic position lines on the earth's surface. These lines of position (LOP's) are generated by many of the electronic navigation systems operating today, for which a shipboard receiver measures the difference between arrival times of radio signals transmitted in synchronism from pairs of stations ashore.

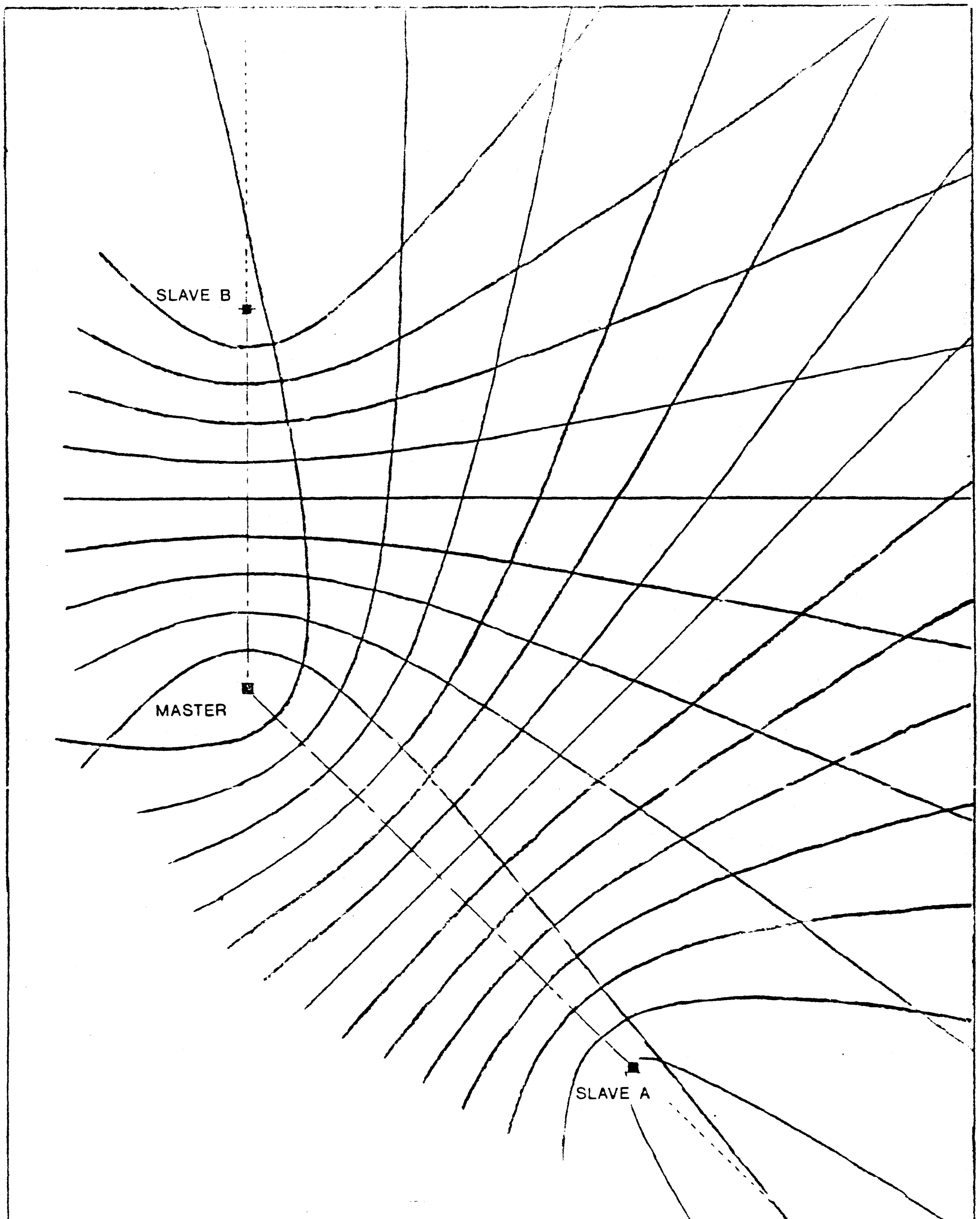
A number of algorithms exist to solve the hyperbolic positioning problem. The most commonly used methods are iterative which, with minor variations, operate by a trial-and error method. Beginning with an initial estimated point, the position is repeatedly improved by a correction vector, calculated using a local linear approximation of the pattern geometry, until the position found satisfies the observations. This technique is widely used for its simplicity, but has the drawback of sometimes converging on the wrong solution or diverging altogether. A version of the iterative method is described in this report and demonstrated in Fortran.

The 'direct methods' are rarely employed in practice. They are troublesome in a computer, with algorithms which are complicated and obscure. Few users seem to understand them. Typically they are based on a spherical approximation, with a mean sphere chosen to represent the local curvature of the ellipsoid in the working area. For the earlier, less accurate systems this approximation is quite adequate.

In this report a direct method is derived which does not make use of a mean best-fit sphere. Instead positions are found on the ellipsoid directly, by modifying the measured hyperbolae with ellipsoid correction terms and solving the problem on an auxiliary sphere. The modifying terms are such that the resulting latitude and longitude found on the auxiliary sphere are identical in value to the geodetic position of the point on the ellipsoid. In this way spherical trigonometric formulae can be used to calculate geodetic positions.

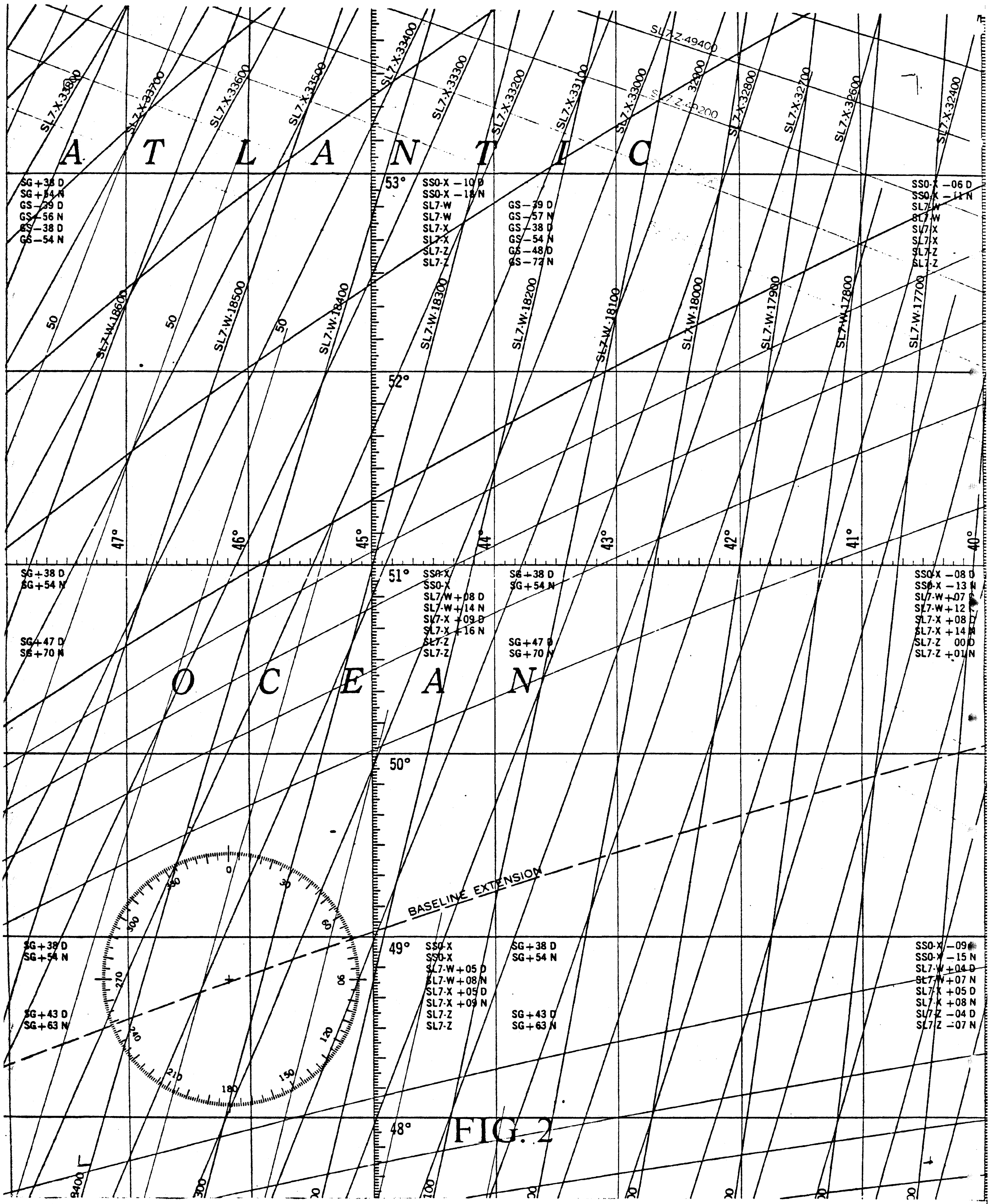
The Fortran program shown generates precise, fictitious observations for each test point, using Vincenty's geodetic inverse formula, and tests the direct solution by reproducing the latitude and longitude of the given test point.

In order to test the same idea in a simpler reference frame, the iterative and direct method are also developed for the plane solution and tested in Fortran. The observations are similarly modified, here to correspond to the auxiliary plane of the chosen map projection. This technique of computing on a plane of projection is widely used by surveyors, enabling them to use plane trigonometry for co-ordinate computations on a curved earth.



HYPERBOLIC POSITION LINES

FIGURE 1.


ATLANTIC
OCEAN
53°
52°
51°
50°
49°
48°
47°
46°
45°
44°
43°
42°
41°
40°
BASELINE EXTENSION
SL7-W-18600
SL7-W-18500
SL7-W-18400
SL7-W-18300
SL7-W-18200
SL7-W-18100
SL7-W-18000
SL7-W-17900
SL7-W-17800
SL7-W-17700
SL7-X-33700
SL7-X-33600
SL7-X-33500
SL7-X-33400
SL7-X-33300
SL7-X-33200
SL7-X-33100
SL7-X-33000
SL7-X-32800
SL7-X-32700
SL7-X-32600
SL7-X-32400
SL7-Z-49400
SL7-Z-49200
SG+38 D
SG+54 N
GS−39 D
GS−56 N
GS−38 D
GS−54 N
SS0-X −10 D
SS0-X −18 N
GS−39 D
GS−57 N
GS−38 D
GS−54 N
GS−48 D
GS−72 N
SS0-X −06 D
SS0-X −11 N
SG+47 D
SG+70 N
SL7-W+08 D
SL7-W+14 N
SL7-X +09 D
SL7-X +16 N
SS0-X −08 D
SS0-X −13 N
SL7-W+07
SL7-W+12
SL7-X +08 D
SL7-X +14 N
SL7-Z 00 D
SL7-Z +01 N
SG+43 D
SG+63 N
SL7-W+05 D
SL7-W+08 N
SL7-X +05 D
SL7-X +09 N
SS0-X −15 N
SL7-W+04 D
SL7-W+07 N
SL7-X +05 D
SL7-X +08 N
SL7-Z −04 D
SL7-Z −07 N
FIG. 2

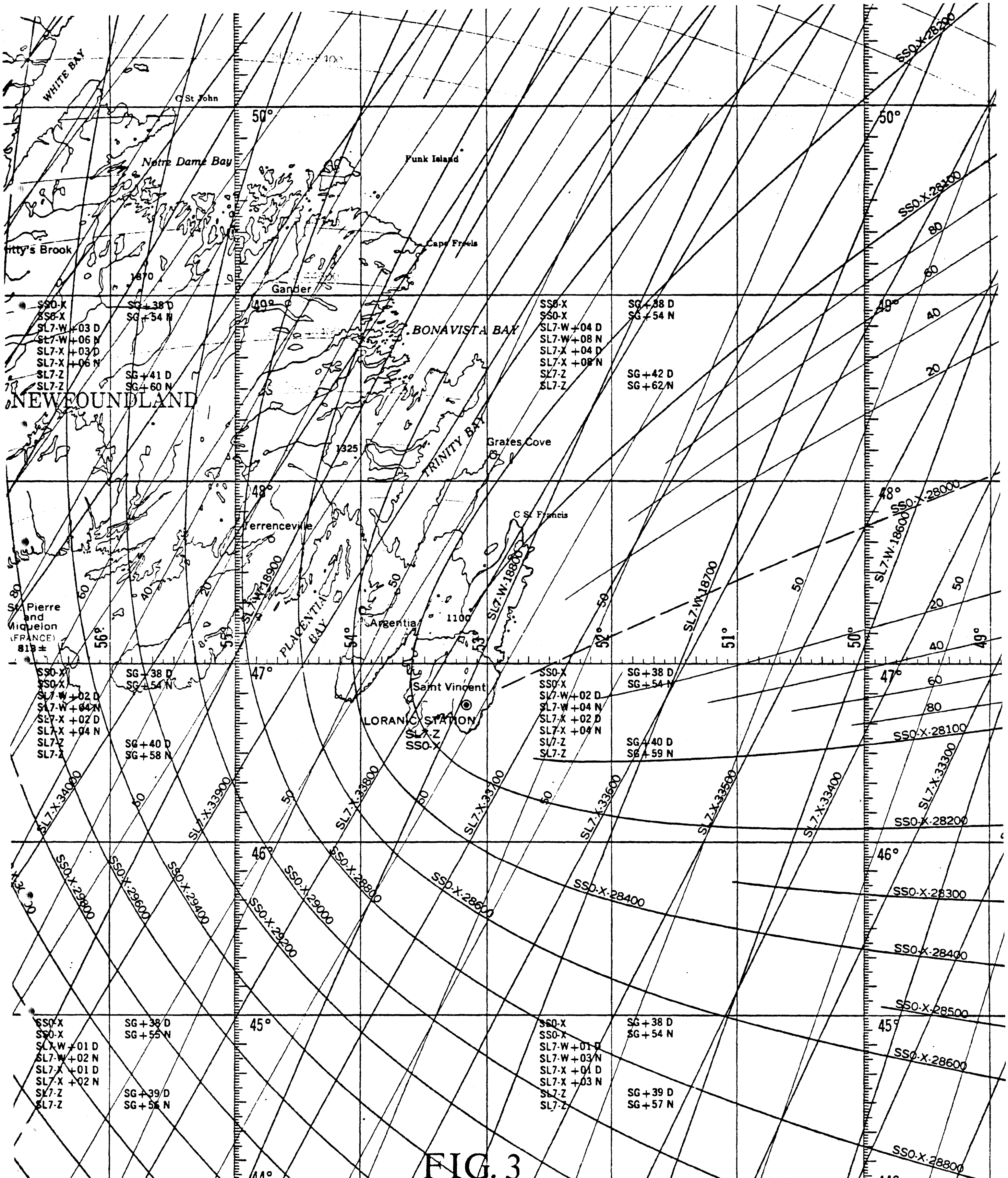

WHITE BAY
C St John
Notre Dame Bay
Funk Island
Kitty's Brook
Cape Freels
Gander
BONAVISTA BAY
NEWFOUNDLAND
TRINITY BAY
Grates Cove
Terrenceville
C St Francis
PLACENTIA BAY
Argentia
St Pierre and Miquelon (FRANCE)
Saint Vincent
LORAN-C STATION
SL7-Z
SS0-X
SL7-W-18900
SL7-W-18800
SL7-W-18700
SL7-W-18600
SL7-X-34000
SL7-X-33900
SL7-X-33800
SL7-X-33700
SL7-X-33600
SL7-X-33500
SL7-X-33400
SL7-X-33300
SS0-X-28000
SS0-X-28100
SS0-X-28200
SS0-X-28300
SS0-X-28400
SS0-X-28500
SS0-X-28600
SS0-X-28800
SS0-X-29000
SS0-X-29200
SS0-X-29400
SS0-X-29600
SS0-X-29800
SG+38 D
SG+54 N
SL7-W+03 D
SL7-W+06 N
SL7-X+03 D
SL7-X+06 N
SG+41 D
SG+60 N
SL7-W+04 D
SL7-W+08 N
SL7-X+04 D
SL7-X+08 N
SG+42 D
SG+62 N
SL7-W+02 D
SL7-W+04 N
SL7-X+02 D
SL7-X+04 N
SG+40 D
SG+58 N
SG+40 D
SG+59 N
SL7-W+01 D
SL7-W+02 N
SL7-X+01 D
SL7-X+02 N
SG+39 D
SG+56 N
SG+38 D
SG+55 N
SL7-W+01 D
SL7-W+03 N
SL7-X+01 D
SL7-X+03 N
SG+39 D
SG+57 N
50°
49°
48°
47°
46°
45°
44°
56°
55°
54°
53°
52°
51°
50°
49°

FIG. 3

Chapter 2

HYPERBOLIC POSITIONING SYSTEMS

A large number of positioning systems are based on the principle of location by intersecting hyperbolae. In the First World War this was the technique employed to find the location of distant artillery fire. The differences in time of arrival of the sound of a cannon burst taken between three separated listening posts were used to determine the position of enemy guns. The military still use this technique, known as "sound ranging by hyperbolae". In World War II a system known as Consol was operational; it cleverly generated a radial fan of position lines, observed by listening to an ordinary radio receiver. By counting the dots vs. dashes received from a closely spaced dipole of interfering transmitters, the navigator could locate himself on a particular radial LOP of the station. The radial pattern results from the degenerate form assumed by the family of hyperbolae when the focal points at the transmitter masts are very close together.

In 1945 the first modern hyperbolic system, Decca, became operational, followed shortly after by Loran-A. Since then a large number of systems have been developed. In the LF band we now have operating: Decca, Loran-C, Loran-D , AccuFix and Pulse-8. With the effectiveness of Loran-C proven, the Loran-A system in the HF band, is now in the process of being dismantled. Loran-B was a development which did not proceed past the experimental stage.

In the VLF band a world-wide system, Omega, operating at frequencies between 10.2-13.6 kHz provides coverage in all ocean areas from eight transmitter stations. An earlier VLF system, DECTRA (now dismantled), extended across the North Atlantic to guide airline traffic.

In the low HF band, around 2 mHz, a considerable number of short and medium range systems operate to provide radiolocation coverage near land for hydrographic survey, exploration work, dredging projects and harbour construction. Examples are: HiFix, MiniFix, RayDist in several versions, Toran, Lorac, Radan and Argo.

Recently, a very light-weight system, SYLEDIS (Systeme Legere de Distance) operating at 420 mHz (UHF) became available. Using a tropo-scatter propagation phenomenon, and correlation detection on faint signals, operating ranges of up to 400 km are attained with relatively low-power (200 Watt) transmitters. (Nard et al,1979)

An example of a typical pattern of position lines generated by a hyperbolic system is shown in diagram form in figure 1 . An excerpt from an ocean navigation chart for the Loran-C system used by ships and aircraft show (figure 2) the lineal pattern of position lines for an area in the mid-Atlantic; highly curved hyperbolae in the neighbourhood of the transmitter at Cape Race, Newfoundland, are shown in figure 3 .

An important related problem, not at all trivial, is that of an efficient algorithm to generate these position lines by computer in a form suitable for chart compilation.

For the purpose of developing a computational method an ideal hyperbolic navigation system is presumed, with a black box receiver which displays the time-of-arrival differences in units of metres. It is assumed to produce values which represent the difference in length between the geodetic lines from the ship to each shore station. With this presumption we avoid all of the peculiar technicalities, which differ from one system to the next, and leave aside issues such as the effective velocity of propagation, locking constants or emission delays, non-linear phase lag effects, calibration constants, skywave corrections ,overland signal path effects etc. All of these are assumed to be compensated for automatically in our hypothetical ideal receiver.

Chapter 3

GEODETIC INVERSE COMPUTATIONS

A basic building block of geodetic computations is the geodetic inverse algorithm, the calculation that yields the geodetic azimuth and distance between two points defined in latitude and longitude on the reference ellipsoid. Its counterpart, the geodetic forward or direct problem , arises less often for the long lines of navigational computations. The direct problem consists of finding the geodetic latitude and longitude of a point B, given the position of point A and the geodetic distance and azimuth from A to B.

For this pair of problems a considerable number of algorithms have been devised. Jank & Kivoija (1980) put the number as high as 50. To name a few, we have: Helmert's formula, Bessels formula, Gauss' formulae, Sodano's methods, the method of Levallois & Dupuy, Lilly's formula, Robbins' formula, Puissants formula (U.S. Coast & Geodetic Survey), Clarke's formulae, the Rainsford method, etc (Bomford,1973). The list is a long litany; each organisation seemingly tends to favour a particular algorithm. Some of the methods are variations of the same basic approach. The geodetic inverse formulae can be conveniently classified as either "short", "medium" or "long line" formulae, roughly according to the distance at which they begin to fail.

For this application two other formulae were selected: Vincenty's method, a precise, efficient and proven long-line formula for which the program was conveniently available (Vincenty,1975); the second, the Andoyer-Lambert formula is relatively simple, of lower accuracy (errors of up to 50 metres in long lines) with an an approach to the problem which makes it particularly suitable for this application.

A refinement due to Forsythe, has produced a version, now known as the Forsythe-Andoyer-Lambert formula with errors in the approximation of the distance amounting to about one metre for very long lines.

In this report the Andoyer-Lambert method is made use of for the hyperbolic intersection problem. The detailed explanation and derivation by (Thomas,1965) together with the algorithm by Razin (1967), further explained by Fell (1975), lead naturally to the approach taken here to solving the hyperbolic intersection problem on the ellipsoid. Vincenty's method is used as a standard to test the accuracy of the direct solution on the ellipsoid.

3.1 FORSYTHE-ANDOYER-LAMBERT FORMULAE

The Andoyer-Lambert method consists of calculating a spherical arc length on an auxiliary sphere of radius a, the ellipsoid major axis semi-diameter, and applying correction terms to find the distance corresponding to the ellipsoidal arc. The second correction is Forsythe's term. (Thomas,1965)

The formula for the distance s in metres:

$$s = a(d + f \Delta d)$$

$f = 1/\ 294.9787$ (Flattening)
$a = 6378\ 206.4$ m. (Clarke 1866 ellipsoid)

The spherical arc d is found by the cosine law:

$$\cos d = \sin\phi_A \sin\phi_B + \cos\phi_A \cos\phi_B \cos(\lambda_A - \lambda_B)$$

The flattening correction terms: $\Delta d = \Delta d_1 + \Delta d_2$

$$\Delta d_1 = -(Xd - 3Y\sin d)/4$$

$$X = P + Q \qquad Y = P - Q$$

$$P = \frac{(\sin\phi_A + \sin\phi_B)^2}{1 + \cos d} \qquad Q = \frac{(\sin\phi_A - \sin\phi_B)^2}{1 - \cos d}$$

$$\Delta d_2 = f\left(AX + BY + CX^2 + DXY + EY^2\right)/128$$

$$A = 64d + 16d^2/\tan d$$

$$D = 48\sin d + 8d^2/\sin d$$

$$B = -2D \qquad E = 30 \sin 2d$$

$$C = -(30d + 8d^2/\tan d + E/2)$$

Note that in the cosine law above the "spherical arc" length is obtained by entering geodetic latitudes into a spherical formula. The correction terms are matched to the data type (geodetic) to yield the proper ellipsoidal arc length.

A second version of the Andoyer-Lambert formulae exists which accepts parametric latitudes in the cosine law and uses a slightly different correction formula to match.

The parametric latitudes θ are related to geodetic latitudes ϕ by the formula:

$$\frac{\tan\theta}{\tan\phi} = \frac{b}{a}$$

The conversion to parametric latitudes, of the transmitter stations, and the conversion back to geodetic of the ship's position at each fix, as indicated by (Razin,1967) , and described by (Kayton & Fried,1971), can be avoided.

3.2 A NUMERICAL PITFALL OF THE COSINE LAW

The spherical cosine law breaks down at short distances, due to truncation effects of finite precision arithmetic in the computer.

In the table below, the effect of chopping all but the six most significant decimal digits at each step is shown for the calculation of :

$$d' = \arccos(\cos(d))$$

At 10 km from the transmitter station, the error due to this truncation amounts to 10%.

TABLE 1

Truncation Error in Short Lines

Dist. km.	d radians	Cos d	Dist. km.	Error m.	Rel. Err.
1000	0.156784	0.987735	999.982	18	18 ppm
100	0.0156784	0.999877	100.390	39	0.04 %
20	0.00313568	0.999995	20.170	170	0.85 %
10	0.00156784	0.999999	9.020	980	9.9 %

This type of problem buried in the code of a computer program could cause much trouble. Forsythe et al (1977,pp 20-23) show how even the lowly quadratic equation solver can break down in the computer. It is the type of problem that could easily be missed when testing with a variety of data values to verify an algorithm; only a careful analysis could uncover such flaws with any certainty.

3.3 ALTERNATIVES TO THE COSINE LAW

The arc length can be precisely found at any distance by a formulation in X,Y,Z differences:

$$\Delta X = \cos\phi_B \cos\Delta\lambda - \cos\phi_A$$

$$\Delta Y = \cos\phi_B \sin\Delta\lambda$$

$$\Delta Z = \sin\phi_B - \sin\phi_A$$

$$\text{Chord } c: \quad c^2 = (\Delta X)^2 + (\Delta Y)^2 + (\Delta Z)^2$$

$$\text{Arc } d: \quad \sin(d/2) = c/2$$

A second alternative, a half-angle formula, is useful:

$$\sin^2\frac{d}{2} = \left(\sin\frac{\Delta\phi}{2}\cos\frac{\Delta\lambda}{2}\right)^2 + \left(\sin\frac{\Delta\lambda}{2}\cos\phi_M\right)^2$$

$$\phi_M = (\phi_A + \phi_B)/2 \qquad \Delta\phi = \phi_A - \phi_B$$

$$\Delta\lambda = \lambda_A - \lambda_B$$

This form of the expression resembles Pythagoras' theorem. In a pocket programmable calculator one could use the "Rect-to-Polar" key to save steps.

Numerous simplifications in the correction terms also can be found by working out the expressions with half-angle substitutions.

3.4 APPLICATION OF THE ANDOYER-LAMBERT CORRECTIONS

The Andoyer-Lambert inverse formula is particularly suitable for electronic positioning problems, because of the simple form of the expression for the ellipsoidal arc distance which still yields adequate precision. The maximum error of 50 metres (only on very long lines)is well within the magnitude of the error that usually occurs in the measurement.

The observed quantities, distance differences in the case of a hyperbolic system, are measured on the surface of the ellipsoid. If the sphere-to-ellipsoid corrections of the Andoyer-Lambert method are applied in reverse to the measurements, converting them to corresponding spherical quantities, then spherical trigonometric formulae can be applied. With valid spherical formulae between geodetic positions, any spherical latitudes and longitudes calculated with the corrected measurements are in fact geodetic positions. Thus the basis of the Andoyer-Lambert method leads to a significant simplification in the determination of geodetic positions. The problem of mapping a solution on an auxiliary sphere back to the ellipsoid does not arise.

Initially an approximate position of the ship is needed to evaluate the ellipsoid corrections. But to find the position , one needs to have the corrections. Thus the method, here named a direct method, is not quite non-iterative. Because the ellipsoid corrections vary slowly with position their values at a nearby previous fix are close, so that each fix calculation practically amounts to a single cycle of iteration.

In practice an approximate position is anyway needed , to apply the necessary corrections for known or estimated non-linear propagation effects.

A set of test lines, ranging between 2500 and 7500 kilometres in length, are shown calculated in the table following to compare the Forsythe-Andoyer-Lambert method with Vincenty's geodetic inverse formula. The distances are shown for comparison in table 3 for the lines between test points shown in table 2 .

TABLE 3

Forsythe-Andoyer-Lambert Inverse Distance Checks

Comparison of the Forsythe-Andoyer-Lambert inverse formula with Vincenty's Formula

Major semi-axis: a=6378 206.4 Flattening f= 1/294.9787
Distances in metres

Test Line		Spherical Distance	Corr'ns Δd_1	Δd_2	Forsythe-And-Lamb.	Vincenty Geo-Inv.	Diff. (m.)
1	M-A	7397470.4	-35918.4	+52.2	7396234.28	234.40	-0.12
2	M-B	5509779.1	+2167.9	-3.0	5511943.95	943.97	-0.02
3	M-P	3104240.9	+57.9	-0.7	3104298.11	298.13	-0.02
4	P-M	3104240.9	+57.9	-0.7	3104298.11	298.13	-0.02
5	P-A	8349052.6	-44413.2	+20.9	8304660.33	660.40	-0.07
6	P-B	2591961.3	2766.0	-1.9	2594725.46	725.45	+0.01

TABLE 2

Test Point Locations

Geodetic Positions of Test Points

Clarke 1866 Ellipsoid

Point		Latitude	Longitude	Station
1	M	N 30-00	E 00-00	Master Transmitter
2	A	S 30-00	E 30-00	Slave A
3	B	N 60-00	E 60-00	Slave B
4	P	N 45-00	E 30-00	Ship position

An elaborate comparison of geodetic line formulae was carried out by Delorme(1978). Sodano's Fourth Method, Robbins' formula, Vincenty's formula, the Andoyer-Lambert and the Forsythe-Andoyer-Lambert methods were tested for speed and accuracy against benchmark data, a set of test lines published by ACIC(1959).

Vincenty's geodetic inverse is shown coded in Fortran in the appendix as subroutine VININ; the Andoyer-Lambert formula with Forsythe's second-order correction terms added is contained in subroutine FADLM.

Chapter 4

DIRECT SOLUTION IN THE PLANE

4.1 PROBLEM STATEMENT

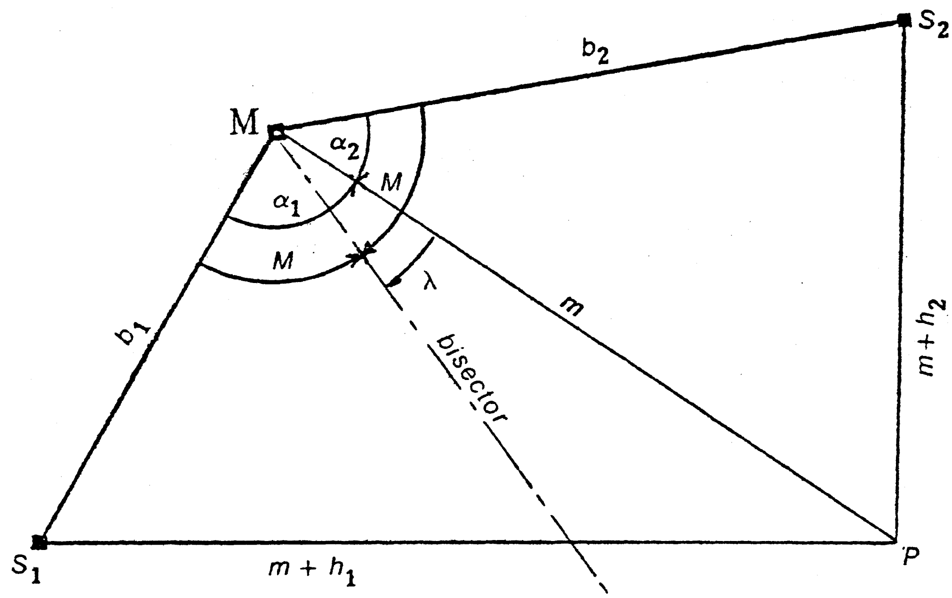


Figure 4: Intersection of Plane Hyperbolae

GIVEN:
The hyperbolic chain configuration defined by:
- the baseline lengths b_1 and b_2 (metres grid distance)
- the included grid angle 2M at Master
- the observed hyperbolae h_1 and h_2 (in units of metres)

The hyperbolae are taken as: $h = s - m$

("Slave-minus-Master" convention)

REQUIRED: The angle λ and distance m (=MP).

4.2 DERIVATION

The solution by plane geometry assumes an ideal flat earth model. In practice the effect of earth curvature must be taken into account, by means of the projection scale factor.
In this derivation the computation steps are shown interspersed as Fortran code segments.

Applying the cosine law to each of the triangles MPS_1 and MPS_2 (Fig. 4):

$$(m+h)^2 = s^2 = m^2 + b^2 - 2mb\cos\alpha$$

$$m^2 + 2mh + h^2 = m^2 + b^2 - 2mb\cos\alpha$$

Cancelling and re-arranging,

$$2mh + 2mb\cos\alpha = b^2 - h^2$$

$$2m(h + b\cos\alpha) = b^2 - h^2$$

Divide by b and isolate $\cos\alpha$,

$$m(h/b + \cos\alpha) = (b^2-h^2)/2b$$

$$\cos\alpha = (1/m)(b^2-h^2)/2b - (h/b)$$

Substitute new variables,

$$M + \lambda = \alpha_1 \qquad M - \lambda = \alpha_2$$

$$X = 1/m$$

$$A_i = (b_i^2 - h_i^2)/2b_i \qquad B_i = -h_i/b_i \qquad \ldots i = 1,2$$

We obtain a pair of equations, one for each triangle,

$$\cos(M+\lambda) = A_1 X + B_1$$

$$\cos(M-\lambda) = A_2 X + B_2$$

in which the unknowns are X and λ

Fortran Code Segment

```
      A1 = (BASE1-HYP1)*(BASE1+HYP1)/(2.0*BASE1)
      A2 = (BASE2-HYP2)*(BASE2+HYP2)/(2.0*BASE2)

      B1 = -HYP1/BASE1
      B2 = -HYP2/BASE2
```

Expanding the $\cos(M+\lambda)$ and $\cos(M-\lambda)$ terms,

$$\cos M \cos\lambda - \sin M \sin\lambda = A_1 X + B_1$$

$$\cos M \cos\lambda + \sin M \sin\lambda = A_2 X + B_2$$

Adding and subtracting equations,

$$2 \cos M \cos\lambda = (A_1 + A_2) X + (B_1 + B_2)$$

$$-2 \sin M \sin\lambda = (A_1 - A_2) X + (B_1 - B_2)$$

Substituting new variables P and Q:

$$P_1 = (A_1 + A_2)/2 \qquad P_2 = (A_1 - A_2)/2$$

$$Q_1 = (B_1 + B_2)/2 \qquad Q_2 = (B_1 - B_2)/2$$

We have.

$$\cos M \cos\lambda = P_1 X + Q_1$$

$$-\sin M \sin\lambda = P_2 X + Q_2$$

Multiplying equations by P_2 and P_1 respectively, and subtracting to eliminate X,

$$P_2 \cos M \cos\lambda = P_2 P_1 X + P_2 Q_1$$

$$-P_1 \sin M \sin\lambda = P_1 P_2 X + P_1 Q_2$$

$$P_2 \cos M \cos\lambda + P_1 \sin M \sin\lambda = P_2 Q_1 - P_1 Q_2$$

Substitute new variables U,V and D:

$$U = P_2 \cos M \qquad V = P_1 \sin M$$

$$D = P_2 Q_1 - P_1 Q_2$$

Fortran Code Segment

```
P1 = (A1+A2)/2.0;        Q1 = (B1+B2)/2.0
P2 = (A1-A2)/2.0;        Q2 = (B1-B2)/2.0

U = P2*COS(AMS/2.0);      V = P1*SIN(AMS/2.0)
D = P2*Q1 - P1*Q2
```

The expression,

$$U \cos\lambda + V \sin\lambda = D$$

can be cast in the form,

$$R \sin\phi \cos\lambda + R \cos\phi \sin\lambda = D$$

in which R and ϕ are a polar representation of the rectangular elements U and V i.e.

$$U = R \sin\phi \qquad V = R \cos\phi$$

Solving,

$$\phi = \arctan(U/\!/V)$$

$$R = U \sin\phi + V \cos\phi$$

(The double division // is meant to denote the 4-quadrant resolution of the arc-tan function)

Divide by R,

$$\sin\phi \cos\lambda + \cos\phi \sin\lambda = D/R$$

$$\sin(\phi + \lambda) = D/R$$

For D/R > 1.00 no solution exists; the pair of hyperbolae do not intersect.

Let $\theta = \phi + \lambda$

$$D/R = \sin\theta = \sin(\pi - \theta)$$

$$\lambda_1 = \theta - \phi$$

$$\lambda_2 = \pi - \theta - \phi$$

Or:

$$\lambda_i = \arcsin(D/R)_i - \arctan(U/V)$$

Two possible values of λ are provided by the arc-sine, corresponding to two possible points of intersection. At least one is a valid solution.

Fortran Code Segment

```
     PHI = ATAN2(U,V)
      R  = U*SIN(PHI) + V*COS(PHI)
    THETA= ARSIN( D/R )

    ALAM1= THETA - PHI
    ALAM2= PI - THETA - PHI
```

The two values of λ are substituted in equation

$$P_2 X + Q_2 = -\sin M \sin\lambda$$

Solving for X

$$X = -(Q_2 + \sin M \sin\lambda)/P_2$$

yields the distance m (= MP)

$$m_i = -P_2/(Q_2 + \sin M \sin\lambda_i) \quad \ldots \; i = 1,2$$

For m negative, an invalid solution results, corresponding to an intersection with a conjugate hyperbola.

Fortran Code Segment

```
DM1 = -P2/( Q2 + SIN(AMS/2.0)*SIN(ALAM1))
DM2 = -P2/( Q2 + SIN(AMS/2.0)*SIN(ALAM2))
```

Having solved for the angle λ and distance m, the ship's position is found by a bearing and distance calculation from Master. The complete algorithm is shown as a Fortran subroutine HYPLAN in the appendix.

4.3 PLANE SOLUTION WITH PROJECTION SCALE FACTOR

To account for the curvature of the earth and to correctly position in the plane co-ordinate system of the survey projection, the observed hyperbolic values are to be converted to their corresponding grid values (denoted by primed quantities).

The grid distance d' is found by applying the line scale factor k to the true ground distance d.

$$d' = kd = d + (k-1)d = d + \Delta d$$

$$\Delta d = (k-1)d$$

To apply the grid corrections Δd to the hyperbolae,

$$h_i' = s_i' - m'$$

$$h_i + \Delta h_i = (s_i + \Delta s_i) - (m + \Delta m)$$

Separating the grid distance from the correction term,

$$h_i = s_i - m$$

$$\Delta h_i = (k_i - 1)s_i - (k_m - 1)m$$

$$\Delta h_i \doteq (k_i - 1)s_i' - (k_m - 1)m'$$

The Δh being very small quantities, the actual distances s and m may be replaced by their grid values.The grid corrections Δh are to be applied to the observed h before entering the direct solution algorithm.

To find an accurate line scale factor k, the as yet unknown position of P is needed, at least to some approximation. Thus in practice this direct solution cannot be non-iterative.

The values of the line scale factors k vary very gradually with position, so that in most cases the position of a previous fix is sufficiently close for an adequate scale factor determination.

The line scale factors calculation, in this example for the UTM projection, is done by a Fortran subroutine UTSFLN shown in the appendix. This routine calls a latitude function XVIII, which is tabulated as a function of Northing in the U.S. Army Map Service UTM tables (AMS,1958), and represented here by an approximation formula. The formulae for the UTM and other conformal projections e.g. the Stereographic and Lambert Conformal Conic projection may be found in (Krakiwsky ,1974).

Chapter 5

DIRECT SOLUTION ON THE ELLIPSOID

5.1 PROBLEM STATEMENT

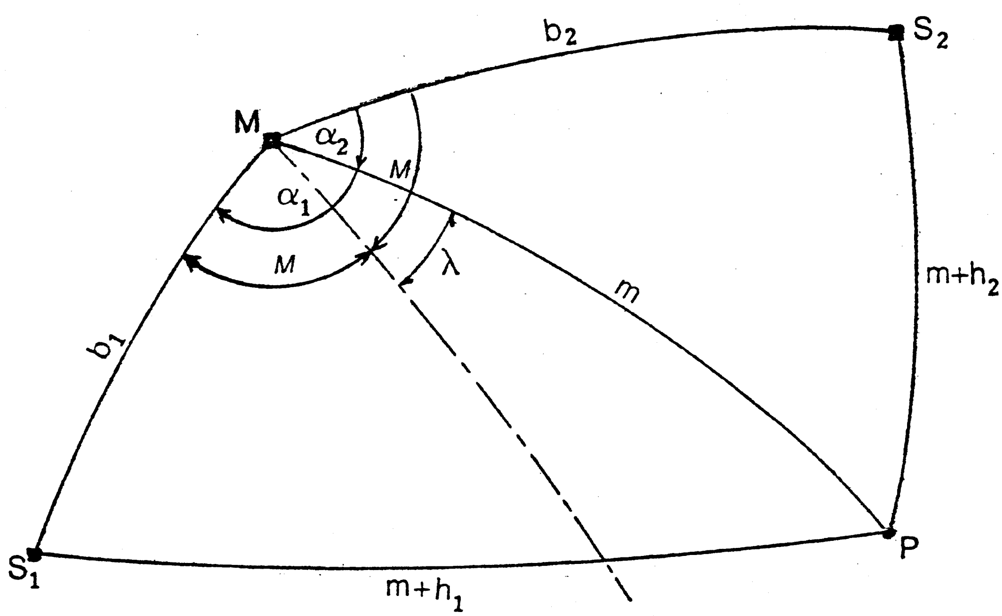


Figure 5: Intersection of Spherical Hyperbolae

GIVEN: The hyperbolic chain configuration defined by:
- the baseline lengths b_1 and b_2 in radians,
- the included spherical angle 2M between baselines,
- and the observed hyperbola h_1 and h_2 in radians.

REQUIRED: The angle λ and spherical distance m (= M-P).

5.2 DERIVATION OF THE SPHERICAL SOLUTION

Applying the cosine law of spherical trigonometry, to each of triangles MPS_1 and MPS_2 :

$$\cos(m+h) = \cos b \cos m + \sin b \sin m \cos\alpha$$

$$\cos m \cosh - \sin m \sinh = \cos b \cos m + \sin b \sin m \cos\alpha$$

Dividing by cos m and re-arranging:

$$\cos h - \tan m \sin h = \cos b + \tan m \sin b \cos\alpha$$

$$\sin b \cos\alpha + \sin h = -\cot m \,(\cos b - \cos h)$$

Divide by sin b, and isolate $\cos\alpha$:

$$\cos\alpha = -\cot m\left(\frac{\cos b - \cos h}{\sin b}\right) - \frac{\sin h}{\sin b}$$

Using a half-angle substitution: $\cos x = 1 - 2\sin^2\frac{x}{2}$

$$\cos\alpha = +\cot m\left(\frac{\sin^2\frac{b}{2} - \sin^2\frac{h}{2}}{\sin\frac{b}{2}\cos\frac{b}{2}}\right) - \frac{\sin h}{\sin b}$$

Substitute new variables

$$M+\lambda = \alpha_1 \qquad M-\lambda = \alpha_2 \qquad X = \cot m$$

$$A_i = \left(\sin^2\frac{b}{2} - \sin^2\frac{h}{2}\right) \Big/ \sin\frac{b}{2}\cos\frac{b}{2}$$

$$B_i = -\sin h_i / \sin b_i \qquad \ldots\ldots \quad i = 1,2$$

We now have a pair of equations:

$$\cos(M+\lambda) = A_1 X + B_1$$

$$\cos(M-\lambda) = A_2 X + B_2$$

with unknowns λ and X; the equations have the same form as the plane case and solving for λ is done in the same way. For the two values of λ_i , the corresponding values of the arc distance m are found by:

$$\tan m_i = -P_2 \big/ \left(Q_2 + \sin M \sin\lambda_i\right)$$

Using the azimuth of the bisector and the angle λ at Master, we have the azimuth from Master to the ship at point P. With the distance and azimuth, we can then calculate the latitude and longitude of P by spherical trigonometry. The resulting spherical answer is the geodetic position of the ship.

The parallels between the plane solution, using UTM scale factor corrections, and the ellipsoid solution with Andoyer-Lambert corrections, are striking.

The algorithm shown here is coded as subroutine HYSPH.

Chapter 6

ITERATIVE METHODS

6.1 ITERATIVE PLANE SOLUTION

In the iterative method , applied in a plane co-ordinate system, an estimated position in UTM coordinates is repeatedly updated with a correction vector, until the fix satisfies the observed hyperbolae.

The mathematical model:

(Computed Hyperbolae) - (Observed Hyperbolae) = 0

$$H_i(N,E) - h_i = 0$$

Expanding the vector function H by Taylor's series, and discarding second and higher-order terms, the model is linearized about a point of expansion near the solution point. H is a vector function of position in UTM Northing and Easting.

$$H_i(N_0,E_0) + \frac{\partial H_i}{\partial N}\Delta N + \frac{\partial H_i}{\partial E}\Delta E = h_i$$

Two linear equations with step corrections ΔN, ΔE as unknowns are extracted:

$$\begin{bmatrix} \frac{\partial H_1}{\partial N} & \frac{\partial H_1}{\partial E} \\ \frac{\partial H_2}{\partial N} & \frac{\partial H_2}{\partial E} \end{bmatrix} \times \begin{bmatrix} \Delta N \\ \Delta E \end{bmatrix} = \begin{bmatrix} h_1 - H_1 \\ h_2 - H_2 \end{bmatrix}$$

The corrections are found by solving with Cramers rule.

The expressions for the partial derivatives turn out to be simple functions of azimuth from P to Master and the respective Slave stations.

$$\frac{\partial H_i}{\partial N} = \cos\alpha_m - \cos\alpha_i$$

$$\frac{\partial H_i}{\partial E} = \sin\alpha_m - \sin\alpha_i$$

In each iteration cycle the estimate is improved by applying the correction step:

$$N_{n+1} = N_n + \Delta N$$

$$E_{n+1} = E_n + \Delta E$$

A subroutine HYPUTM, shown in the appendix, is based on this method of solution.

This iterative method, is essentially Newton's method applied to a two-dimensional problem. This type of solution can be usefully applied to any kind of positioning system. Actually the satellite fix computation, using Doppler measurements from the U.S. Navy Transit system, works in a very similar way.

6.2 ITERATIVE SOLUTION IN GEOGRAPHIC CO-ORDINATES

With a technique very similar to the plane iterative solution, an approximate position in latitude and longitude is refined by iteration until the position found satisfies the observed hyperbolic position lines. Except for the geodetic distance calculation , and the calculation of derivatives which are based on a spherical approximation, the method is quite similar to the plane iterative solution.

The mathematical model:

Computed H - Observed h = 0

$$H_i(\phi,\lambda) - h_i = 0$$

Linearized by Taylor's series about an initial point of expansion P:

$$H_i(\phi_P,\lambda_P) + \frac{\partial H_i}{\partial \phi}\Delta\phi + \frac{\partial H_i}{\partial \lambda}\Delta\lambda = h_i$$

In matrix form the equations are:

$$\begin{bmatrix} \frac{\partial H_1}{\partial \phi} & \frac{\partial H_1}{\partial \lambda} \\ \frac{\partial H_2}{\partial \phi} & \frac{\partial H_2}{\partial \lambda} \end{bmatrix} \times \begin{bmatrix} \Delta\phi \\ \Delta\lambda \end{bmatrix} = \begin{bmatrix} h_1 - H_1 \\ h_2 - H_2 \end{bmatrix}$$

$$\frac{\partial H_i}{\partial \phi} = \frac{\partial s_i}{\partial \phi} - \frac{\partial m}{\partial \phi} \qquad \frac{\partial H_i}{\partial \lambda} = \frac{\partial s_i}{\partial \lambda} - \frac{\partial m}{\partial \lambda}$$

The partial derivatives of the observed hyperbolae with respect to latitude and longitude are found by taking differences between the derivatives of the spherical distances; these in turn are found by differentiating the cosine law of spherical trigonometry:

$$\cos d_i = \sin\phi_i \sin\phi_P + \cos\phi_i \cos\phi_P \cos(\lambda_P - \lambda_i)$$

$$d_i = s_1, s_2 \text{ or } m$$

$$\frac{\partial d_i}{\partial \phi_P} = R(\sin\phi_P \cos\phi_i \cos\Delta\lambda - \cos\phi_P \sin\phi_i)/\sin d_i$$

$$\frac{\partial d_i}{\partial \lambda_P} = R\cos\phi_P \cos\phi_i \sin(\lambda_P - \lambda_i)/\sin d_i$$

R is an earth radius which converts arc lengths in radians to distances in metres.

The method indicated here is described in detail by Mackereth(1976). Instead of taking the tangent plane approximation by derivatives, he uses a secant plane, obtained by differencing computed hyperbolae between nearby points. A slightly better rate of convergence is obtained. A very similar approach, applied to a positioning system with ranging measurements, is described by Grant(1973).

A subroutine HYPGEO, listed in the appendix, uses this iterative solution with derivatives. The iterative methods were coded and run, mainly for comparison to judge the practical effectiveness of the direct solutions.

6.3 ANALOGIES BETWEEN PLANE AND SPHERICAL ITERATION

There is a recognizable parallel between the plane and the ellipsoidal solution, as shown below.

PLANE	ELLIPSOID
Plane distances by Pythagoras' theorem	Spherical distances by cosine law
Projection scale factor correction from grid to ground distance	Andoyer-Lambert corrections from spherical to ellipsoidal distance
Partial derivatives are functions of plane azimuth	Partial derivatives are functions of spherical azimuth.

The analogies show by the resemblance between the Fortran subroutines HYPUTM and HYPGEO, which demonstrate the iterative method on the UTM plane and the ellipsoid respectively.

6.4 REMARKS ON THE ITERATIVE METHOD

The following points apply to both the plane grid solution and to the solution on the ellipsoid.

1. For some pairs of hyperbolae there are two possible points of intersection. If an initial approximation of the position is very rough, it is possible for the iterative correction step to overshoot and cause the process to converge to the wrong solution.
   This is particularly likely to happen when positioning in the neighbourhood of the Master transmitter, where the two solutions are fairly close together and the hyperbolae are highly curved (see figure 3); here the linearized mathematical model could be stretched beyond its range of validity, i.e. the second and higher-order terms of the Taylor series expansion are not to be neglected for large increments. Because the correction steps would be quite inaccurate, but roughly in the right direction, they are usable if reduced in magnitude.

At the cost of more iterations, the possibility of ambiguity may be minimized by limiting the size of the correction increment.

2. With an initial approximation taken sufficiently close, the problem does not arise. In practice, usually a sequence of fixes are taken at regular intervals a short time apart. The position of the previous fix, perhaps updated by dead-reckoning, provides a convenient initial value.

3. An incorrect choice of solution can be detected by a bearing and distance calculation between fixes; if they agree reasonably with the actual heading and speed of the vessel, the choice of solution is confirmed.

4. Certain pairs of hyperbolae do not intersect; the situation may arise where, due to bad data from the receiver or incorrect station co-ordinates having been entered, the computer will iterate endlessly in an attempt to compute an impossible fix. To detect this situation, a limit on the number of iterations is needed (e.g. maximum 20 iterations).
Endless looping could also occur if the break-out tolerance is set too finely i.e. smaller than the roundoff error of the finite precision arithmetic. Convergence might be had by chance, but often the computer simply thrashes around the exact solution, without leaving the iteration loop.

5. The linearized form of the mathematical model yields an expression which is identical to the observation equation of the least squares method. The partial derivatives are the elements of the design matrix A, the Jacobian matrix, relating the observations to the unknowns in linear form. Thus the iterative method has the advantage of being easily extended to include redundant position lines in a least squares solution. An overdetermined solution improves the fix quality and can provide an estimate of errors derived from the covariance matrix of the solution.

6. Including a third LOP makes the solution unique, eliminating the ambiguity between the two possible points of intersection.

In some systems e.g. Loran-C , a weak and noisy signal from a distant transmitter, or a skywave signal with a large systematic error (10-20 km) can be usefully included in the solution with a very low weight. The effect is to steer the process of convergence towards the desired solution.

7. The iterative algorithm is easily verified, an advantage particularly in micro-processor applications, in which the program code tends to become quite obscure.
If the distance calculations are correct and the process converges to satisfy the observed hyperbolae, one is assured of having found a solution.

The iterative method is widely applied. Recent advances in micro-circuitry have led to the development of the 'co-ordinate converter', a unit attached to the navigation receiver, which continuously runs the hyperbolic fix computation and displays the ship's position in rectangular grid or geographic co-ordinates. One such undertaking is described by (Culver & Danklefs,1977).

Chapter 7

TEST SOLUTIONS

7.1 RESULTS IN UTM

For the plane solution a cluster of four test points were evaluated by the iterative and the direct method, in the UTM coordinate system.

TABLE 4

Test Points on UTM

No.	Hyperbolae A	Hyperbolae B	Grid Corrections		U.T.M. Position Northing	U.T.M. Position Easting
1	58599.7	-41444.4	-21.0	23.1	N 4900000.0	E 600000.0
2	59307.8	-41739.1	-21.3	23.1	N 4901000.0	E 600000.0
3	58891.0	-42148.6	-21.1	23.3	N 4900000.0	E 601000.0
4	59595.5	-42446.6	-21.4	23.4	N 4901000.0	E 601000.0
	-metres-		-metres-		-metres-	

7.2 TEST POINTS ON THE ELLIPSOID

Three test points were evaluated in the neighbourhood of Lat 45 degrees North and 30 degrees East. The points are separated by about 60 nautical miles North-South and 42 n.m. East-West.

TABLE 5

Results on the Ellipsoid

No.	Hyperbolae A	Hyperbolae B	Ellipsoid Corrections		Position Lat.	Position Long.	Error (metres)	
1	5200362	-509572	44449	-2711	N 45-00	E 30-00	1.7	1.4
2	5268143	-638007	44362	-3196	N 46-00	E 30-00	0.7	0.7
3	5127620	-632566	44624	-2376	N 45-00	E 31-00	2.2	1.4
	-metres-		-metres-					

7.3 DISCUSSION OF RESULTS

For the UTM calculation the magnitude of the corrections indicate the difference, an earth curvature effect, between actually observed hyperbolae and their representation in the plane grid system. The ellipsoid-to-sphere corrections, in a similar way, indicate a component of ellipticity of the earth's surface contained in the observed hyperbolae. At the larger distance from the transmitter , we have a 45 kilometre shift.

For the direct method the errors in position (relative to Vincenty's formula) amount to a few metres. This is due to the approximation in the Forsythe-Andoyer-Lambert distance formula, magnified by lane expansion and the oblique angle of intersection of the hyperbolic position lines. Any shortcomings of the direct method would have shown up as a larger error here.

The test points are located so, that the geometry is favourable and no extreme values arise in the values at intermediate calculation steps. Thus the agreement indicates that here the round-off problem does not appear. The positions were chosen purposely far away, with distances to the transmitters ranging between 2500 to 7500 kilometres, to reveal any basic faults in the method. In this test, the round-off problem is not properly tested for.

The intermediate steps of the iterative calculation show that the convergence is rapid and stable. The maximum stepsizes of 50 km and 600 km, for the plane and ellipsoidal solutions respectively, have the effect of cutting the correction vector short, and here needlessly increase the number of iteration steps.

Chapter 8

TWO PROBLEMS REMAINING

In the development of the direct solution, two matters are unresolved. First, the effect of round-off error in the computer arithmetic is not analysed and second, the method of distinguishing between the two possible solutions should be more definite.

The effect of round-off is much less significant in the iterative procedure; each cycle begins the process anew and the accumulation of round-off is contained in the correction vector, a diminishing quantity approaching zero at convergence. If the distance calculations are correct and the process converges to satisfy the observations, then the answer will be correct. At worst, the effect of round-off will cause an extra step of iteration.

In the direct method, an open-ended process, any round-off error accumulated at intermediate steps will affect the final outcome. Testing the program with an assortment of data values will not do to provide a guarantee that the algorithm is everywhere numerically stable. In testing it is quite possible to miss a particular combination of values which cause the answer to blow up.

In Forsythe,Malcolm & Moler (1977) examples are given of quite straight-forward expressions in Fortran yielding answers which are significantly off and even quite wrong, also in double precision calculations.

A round-off error analysis is needed, for each intermediate step in the calculation.

In the derivation , we find that two possible solutions will satisfy the data. On the sphere a hyperbola is a closed curve. It can be shown that this curve is also an ellipse, having one of its focal points at a point diametrically opposite to a transmitter station. A pair of closed curves can only intersect at an even number of points. Thus by geometrical reasoning we also arrive at two possible points of intersection. Which would be the desired solution point? A way to resolve it is by defining a rectangular window, representing the working area, a circular window representing some radius of action or perhaps best a sector defined in polar co-ordinates centred on Master which spans the working area. Near the transmitter stations, where the two solutions are close together, both points might fall inside the window and the ambiguity remains.

In the plane solution it was found that one of the solutions actually is an intersection with a conjugate hyperbola. What is needed is a tightly defined convention for specifying the chain configuration and a rigourous derivation to find a characteristic which identifies each solution point mathematically. A similar idea, applied to spherical triangles for astronomic fixing at sea is described by Bennett (1980) for use with pocket programmable calculators.

Chapter 9

SUMMARY

1. A relatively simple , almost non-iterative, solution of the hyperbolic intersection problem on the ellipsoid is verified numerically and partly derived. The method is explained by way of analogy with the plane solution.

   The ellipsoidal solution, as a spherical solution with modified data, follows an idea similar to the technique, widely employed by surveyors, of modifying measured quantities (angles and distances) to correspond to the plane of a map projection. Coordinates on the earth's surface are then more easily calculated by plane trigonometry.

   The ellipsoid-to-sphere corrections of the Forsythe-Andoyer-Lambert method are analogous to the line scale factor corrections of the surveyor's plane rectangular grid system, based for example on a Transverse Mercator, Stereographic or Lambert Conformal Conic Projection.

2. Test runs were made to compare the direct solution with the currently widely used iterative technique. The direct solution is slightly faster. It can easily detect impossible fixes due to bad data, is less prone to failure e.g. no iterative process to diverge, and it can be improved further.

   On the other hand the iterative method has the advantage of being easily extended to form a least squares solution with redundant data. Also it can be applied to a configuration where the baselines are separated i.e. do not share a common master station. The direct solution shown here must have a central transmitter station common to both baselines.

3. The direct solution can be implemented in the more powerful types of pocket programmable calculators. Using the simpler (first-order terms only) Andoyer-Lambert corrections, a compact algorithm can be prepared to fit easily into a TI-59 calculator.

4. For batch processing on a larger machine, this algorithm can serve to provide a close initial position which is then refined and verified in a single iteration pass using a more precise geodetic inverse routine (e.g. Vincenty's formula).

   The first-order correction terms, for a maximum error of 50 metres, are quite adequate for currently operating extended-range radio-positioning systems. An error tolerance of 1 metre in the iterative computation generally exceeds by far the precision of existing navigation systems. The fine tolerance used in these test runs was chosen for the purpose of comparing algorithms with precise values for answers to check their validity numerically.

5. For real-time applications, e.g. co-ordinate conversion devices based on a micro-processor attached to the navigation receiver, this direct solution would not be suitable without further development.

   In particular the effects of round-off error in finite precision arithmetic need to be examined for a variety of test points and chain configurations. The blanket solution of using double precision variables throughout could be uneconomical in a micro-computer; the additional memory adds to the cost of the product and then one is not assured of avoiding instances of the algorithm failing due to numerically ill-conditioned situations. A failure of just the algorithm at some remote locality could be mistaken for a receiver fault, reflecting adversely on the manufacturer of the equipment.

6. A positive method of distinguishing between the two possible solutions is needed. This could be found by a tightly specified set of conventions to define the geometrical quantities and a rigourous derivation of the spherical solution.

7. The basis of the direct method, i.e. using ellisoidal quantities modified by the Forsythe-Andoyer-Lambert corrections to yield corresponding data for the sphere, could be applied more generally in the computation of very large geometric figures on the earth. It would appear that geodetic positions on the ellipsoid can be computed , to a precision of 1 metre at any distance on the globe, by simple spherical trigonometric formulae.

Chapter 10

CONCLUSION

1. A simpler direct method to find the intersection of hyperbolic position lines on the ellipsoid is obtained by derivation and by geometric reasoning with respect to the Andoyer-Lambert ellipsoid corrections. The validity of the solution is verified numerically by Vincenty's geodetic inverse formula.

2. Iterative procedures were written in Fortran , for the plane and the ellipsoidal case, for a comparison with the direct method. The direct method would be slightly faster and more compact if developed further.

3. A possible problem with round-off, due to truncation effects in computer arithmetic, was identified in the direct method.

4. A more positive , mathematical method needs to be found to distinguish between the two possible points of intersection.

5. The idea of modifying the observations to an equivalent spherical representation, offers an alternative method of geodetic position computation.

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Appendix I

FORTRAN PROGRAMS FOR THE PLANE SOLUTION

```
*********************************************************************************************************************TABLE*
$JOB   WATFIV STU IFBERGEN/U,PAGES=25,T=3
C
C**************************************
C      SELF-TEST OF PLANE SOLUTIONS
C---------------------------------------
          IMPLICIT REAL*8 (A-H,O-Z)
          COMMON IBUG
C
          PI=4.0*DATAN(1.0D00);      PI2=PI*2.0 D00
          DR=PI/180.0 D00;           RD=180.0 D00/PI
          IBUG=1
          IBUG=0
C
C         /*** ENTER STATION POSITIONS IN UTM NORTHING AND EASTING
C         /*** FOR MASTER, SLAVES -A- AND -B-
          READ, UNM,UEM, UNA,UEA, UNB,UEB
C
C         /*** CALCULATE BASELINE GRID DISTANCES AND AZIMUTHS
          CALL UTINV(UNM,UEM, UNA,UEA, BASEA,AZMA)
          CALL UTINV(UNM,UEM, UNB,UEB, BASEB,AZMB)
C
C         /*** GRID ANGLE AT MASTER AND BISECTOR AZIMUTH
          AMS=DMOD(AZMA-AZMB, PI2)
          AZBIS=DMOD(AZMA-AMS/2.0 D00, PI2)
C
                   CALL TITLE(UNM,UEM, UNA,UEA, UNB,UEB,
     $                        BASEA,BASEB, AZMA,AZMB, AMS,AZBIS)
C
C         /*** MAIN OPERATING LOOP
          CORRA=0.0;    CORRB=0.0
          NTEST=0
      100 NTEST=NTEST+1
C         /*** ENTER THE TEST POINT IN UTM CO-ORDS
C         /*** AND GENERATE EXACT FICTITIOUS OBSERVED HYPERBOLAE AT -P-
          READ, UNP,UEP
          IF(UNP .LT. 0.0) STOP
          PRINT 701
      701 FORMAT(1H1/)
          PRINT 502, NTEST
      502 FORMAT(1H , 7X, 'DIRECT PLANE SOLUTION AT TEST POINT NO.', I2/
     $          1H ,7X, 41(1H*) ///)
C
C         /*** GRID DISTANCES TO TRANSMITTER STATIONS
          CALL UTINV(UNP,UEP, UNM,UEM, DM,AZM)
          CALL UTINV(UNP,UEP, UNA,UEA, DA,AZA)
          CALL UTINV(UNP,UEP, UNB,UEB, DB,AZB)
C
C         /*** UTM SCALE FACTORS APPLIED TO FIND TRUE DISTANCES
          CALL UTSFLN(UNP,UEP, UNM,UEM, SCFM)
          CALL UTSFLN(UNP,UEP, UNA,UEA, SCFA)
          CALL UTSFLN(UNP,UEP, UNB,UEB, SCFB)
C
          DM=DM/SCFM;    DA=DA/SCFA;    DB=DB/SCFB
```

```
C
C        /*** FICTITIOUS OBSERVED HYPERBOLAE
C        /*** FROM PRECISE DISTANCES TO TEST DIRECT PLANE SOLUTION
         HYPA=DA-DM;    HYPB=DB-DM
                 CALL CCLHED( HYPA,HYPB, UNP,UEP)
C
         ITER=0
   200   ITER=ITER+1
         CALL HYPLAN(ANS,BASEA,BASEB, HYPA+CORRA, HYPB+CORRB,
      $               ALAM1, ALAM2, DM1, DM2, ICODE)
C
C        /*** POSITION BY  DISTANCE & AZIMUTH
         AZMP=AZBIS-ALAM1
         CALL UTFOR(UNM,UEM, DM1, AZMP, UNPP, UEPP)
         CALL CYCDIR(CORRA,CORRB, HYPA,HYPB, UNPP,UEPP, ITER)
C
C        /*** SCALE FACTORS & GRID CORRECTIONS FOR -P-
         CALL UTSFLN(UNPP,UEPP, UNM,UEM, SCFM)
         CALL UTSFLN(UNPP,UEPP, UNA,UEA, SCFA)
         CALL UTSFLN(UNPP,UEPP, UNB,UEB,SCFB)
C
         DDM=(SCFM-1.0)*DM1
         DDA=(SCFA-1.0)*(DM1+HYPA)
         DDB=(SCFB-1.0)*(DM1+HYPB)
C
C        /*** GRID CORRECTIONS TO HYPERBOLAE
         CORR1=DDA-DDM;    CORR2=DDB-DDM
         CHECK=DABS(CORR1-CORRA) + DABS(CORR2-CORRB )
         CORRA=CORR1;     CORRB=CORR2
C
C        /*** RECYCLE IF THE CORRECTIONS ARE TOO DIFFERENT
         TOL=0.1
         IF ( CHECK .GT. TOL) GOTO 200
C
         PRINT 512
   512   FORMAT(1H ,45X, 9(1H=), 4X, 8(1H=) /)
C
C        /*** GENERATE FICTITIOUS INITIAL POSITION
C        /*** TO TEST ITERATIVE METHOD
         PRINT 701
         PRINT 602, NTEST
   602   FORMAT(1H ,7X,'ITERATIVE PLANE SOLUTION AT TEST POINT NO,',I2/
      $          1H ,7X, 44(1H*) //)
                 CALL CCLHED( HYPA,HYPB, UNP,UEP)
C
         UNPI=UNPP;    UEPI=UEPP
         UNPI=(UNPP+UNM)/2.0 D00;    UEPI=(UEPP+UEM)/2.0 D00
C
         PRINT 604,      UNPI, UEPI
   604   FORMAT(1H ,7X,'INITIAL POINT', 2('  -->' , 5X),
      $              'AT N', F10.1, '  E', F9.1 //)
C
         CALL HYPUTM(HYPA,HYPB, UNPI,UEPI, UNM,UEM, UNA,UEA, UNB,UEB,ITER)
C
         PRINT 612
   612   FORMAT(1H ,45X, 9(1H=), 4X, 8(1H=) /)
         GO TO 100
         END
```

```
C
C
      SUBROUTINE HYPLAN(AMS, BASE1,BASE2, HYP1,HYP2,
     $                  ALAM1,ALAM2, DM1,DM2, ICODE)
C*****************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON IBUG
C
      PI=4.0D00*DATAN(1.0D00);    RD=180.0D00/PI
C
      A1=(BASE1-HYP1)*(BASE1+HYP1)/( 2.0 D00*BASE1)
      A2=(BASE2-HYP2)*(BASE2+HYP2)/( 2.0 D00*BASE2)
      B1=-HYP1/BASE1;    B2=-HYP2/BASE2
C
      P1=(A1+A2)/2.0D00;    P2=(A1-A2)/2.0D00
      Q1=(B1+B2)/2.0D00;    Q2=(B1-B2)/2.0D00
C
      D=P2*Q1 - P1*Q2;    SINM=DSIN(AMS/2.0D00);    COSM=DCOS(AMS/2.0D00)
      U=P2*COSM;    V=P1*SINM
      PHI=DATAN2(U,V);    R=U*DSIN(PHI) + V*DCOS(PHI)
C
      IF(IBUG .NE. 0) PRINT 100,A1,A2,B1,B2, P1,P2, Q1,Q2,
     $                          D,U,V, R, PHI
  100 FORMAT(1H ,2E20.8/)
C
C     // CHECK FOR NO POSSIBLE INTERSECTION
      ICODE=0;    IF( R .LT. D) RETURN
C
C     // ONE OR TWO SOLUTIONS POSSIBLE
      THETA=DARSIN(D/R)
      ALAM1=THETA-PHI;    ALAM2=PI-THETA-PHI
      DM1=-P2/(Q2+SINM*DSIN(ALAM1))
      DM2=-P2/(Q2+SINM*DSIN(ALAM2))
C
C     // SET UP CODE TO IDENTIFY THE SOLUTIONS
      IF(DM1 .GT. 0.0D00) ICODE=1
      IF(DM2 .GT. 0.0D00) ICODE=2
      IF(DM1 .GT. 0.0D00 .AND. DM2 .GT. 0.0D00) ICODE=3
C
      IF(IBUG .EQ. 0) RETURN
      PRINT, ICODE
      PRINT 100, THETA,PHI, ALAM1,ALAM2, DM1,DM2
      RETURN
      END
```

```
C
C
      SUBROUTINE HYPUTM(OHA,OHB,UNP,UEP,UNM,UEM,UNA,UEA,UNB,UEB,ITER)
C*************************************************************************
C     ITERATIVE INTERSECTION OF HYPERBOLAE ON UTM PLANE
C------------------------------------------------------------------------
C
C     OHA,OHB        = OBSERVED HYPERBOLAE IN METRES
C     UNP,UEP        = INITIAL APPROXIMATE POSITION OF -P- IN UTM
C                           ALSO THE FINAL ITERATED POSITION
C     UNM,UEM        = UTM POSITION OF MASTER TRANSMITTER
C     UNA,UEA        = POSITION OF SLAVE -A- IN UTM NORTHING & EASTING
C     UNB,UEB        = POSITION OF SLAVE -B- IN UTM NORTHING & EASTING
C
C     UTINV          = SUBROUTINE FOR DISTANCE AND AZIMUTH BY UTM COORDS.
C     UTSFLN         = SUBROUTINE FOR UTM LINE SCALE FACTOR
C
C-------------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON IBUG
C
C     /*** ITERATION LOOP
      ITER=0 ;    ITMAX=20 ;    DMAX=25000.0
      DMAX=50000.0
      TOL=0.1;    TEST=TOL+TOL
      WHILE( ITER .LE. ITMAX  .AND. TEST .GE. TOL) DO
         ITER=ITER+1
C
C        /*** GRID AZIMUTH AND DISTANCE BY UTM CO-ORDINATES
         CALL UTINV(UNP,UEP, UNM,UEM, DM,AZM)
         CALL UTINV(UNP,UEP, UNA,UEA, DA,AZA)
         CALL UTINV(UNP,UEP, UNB,UEB, DB,AZB)
C
C        /*** UTM LINE SCALE FACTORS
         CALL UTSFLN(UNP,UEP, UNM,UEM, SCFM)
         CALL UTSFLN(UNP,UEP, UNA,UEA, SCFA)
         CALL UTSFLN(UNP,UEP, UNB,UEB, SCFB)
C
C     /*** GRID DISTANCES CONVERTED TO GROUND DISTANCES
      DM=DM/SCFM ;    DA=DA/SCFA;    DB=DB/SCFB
C
C     /*** COMPUTED HYPERBOLAE AT -P-
      CHA=DA-DM;    CHB=DB-DM
C
C    /** DIFFERENCE (OBSERVED) - (COMPUTED) HYPERBOLAE
      DHA=OHA-CHA;    DHB=OHB-CHB
C
C    /*** ELEMENTS OF A-MATRIX, THE HYPERBOLIC GRADIENTS
      DAN=DCOS(AZM)-DCOS(AZA);    DAE=DSIN(AZM)-DSIN(AZA)
      DBN=DCOS(AZM)-DCOS(AZB);    DBE=DSIN(AZM)-DSIN(AZB)
C
C     /*** SOLVE FOR CORRECTION STEP  DN,DE BY CRAMERS RULE
      DET=DAN*DBE - DBN*DAE
      DN=(DHA*DBE - DHB*DAE)/DET
      DE=(DAN*DHB - DBN*DHA)/DET
```

```
C
C
C      /*** LIMIT SIZE OF CORRECTION STEP
       DIST=DSQRT(DN*DN+DE*DE);    FRACTN=1.0
        IF (DIST .GT. DMAX) FRACTN=DMAX/DIST
C
       CALL CYCLIT(CHA,CHB,UNP,UEP,DHA,DHB,DN,DE,FRACTN,ITER)
       UNP=UNP + DN*FRACTN;    UEP=UEP+DE*FRACTN
C
C     /*** BREAK-OUT TEST FOR CONVERGENCE
       TEST=DABS(DHA)+DABS(DHB)
       ENDWHILE
C
       RETURN;     END

       SUBROUTINE UTINV(UNA,UEA, UNB,UEB, DIST,AZ)
C*****************************************************
C     GRID  DISTANCE AND AZIMUTH FROM UTM CO-ORDINATES
C     GRID  DISTANCE AND AZIMUTH FROM RECTANGULAR CO-ORDINATES
C     IN NORTHINGS (Y) AND EASTINGS (X)
C-----------------------------------------------------
       IMPLICIT REAL*8 (A-H,O-Z)
       COMMON IBUG
       PI2= 8.0 D00*DATAN(1.0 D00)
       DN=UNB-UNA;    DE=UEB-UEA;    DIST=DSQRT(DN*DN+DE*DE)
       AZ=DATAN2( DE,DN);    AZ=DMOD( AZ+PI2, PI2)
       RETURN;    END

       SUBROUTINE UTFOR(UNA,UEA, DIST, AZ, UNE,UEE)
C*******************************************************
C     POSITION OF -B- BY GRID DISTANCE AND AZIMUTH FROM -A-
C-------------------------------------------------------------------
       IMPLICIT REAL*8 (A-H,O-Z)
       COMMON IBUG
       UNB=UNA+DIST*DCOS(AZ)
       UEB=UEA+DIST*DSIN(AZ)
       RETURN
       END
```

```
C
C
      SUBROUTINE UTSFLN(YNA,XEA, YNB,XEB, SCFACT)
C*****************************************************
C     -- UTM LINE SCALE FACTOR  FOR LINE -A- TO -B-
C-----------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      CMSCF=0.9996 D00;    XCM= 500 000.0 D00
      YNM=(YNA+YNB)/ 2.0 D00;          XEM=(XEA+XEB)/2.0 D00
C
      QA=(XEA-XCM)/1.0D06;    QA2=QA*QA;    QA4=QA2*QA2
      QB=( XEB-XCM)/1.0D06;    QB2=QB*QB;    QB4=QB2*QB2
      QM=(XEM-XCM)/1.0D06;    QM2=QM*QM;    QM4=QM2*QM2
C
C     /*** POINT SCALE FACTORS AT A,B AND MID-POINT
      SCFA=1.0 D00 + XVIII(YNA)*QA2 + 3.0D-05*QA4
      SCFB=1.0 D00 + XVIII(YNB)*QB2 + 3.0D-05*QB4
      SCFM=1.0 D00 + XVIII(YNM)*QM2 + 3.0D-05*QM4
C
C     /*** LINE SCALE FACTOR BY SIMPSONHS RULE
      SCFACT= CMSCF*(SCFA+4.0D00*SCFM+SCFB)/6.0D00
      RETURN
      END

      DOUBLE PRECISION FUNCTION XVIII(YN)
C*******************************************************
C     -- LATITUDE FUNCTION XVIII
C     -- BY AN APPROXIMATION FORMULA IN UTM NORTHING
C-----------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      T=0.31113286 D-06*(YN-5.0D06)
      XVIII=0.0123 D00 - 83.927 D-06* DSIN(T)
      RETURN
      END

      SUBROUTINE RADMS(ANGLE, IDEGS, MINS, SECS)
C     ******************************************
C     ANGLES IN RADIANS TO DEGREES, MINUTES AND SECONDS
C     ------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      PI2= 8.0*DATAN(1.0 D00);    RD= 360.0 D00/PI2
      SD=DSIGN(1.0 D00, ANGLE)
      FUZZ= 1.0D-11;    ANGD=(SD*ANGLE*RD) +FUZZ
      IDEGS=ANGD;    ANGD=ANGD-DFLOAT( IDEGS)
      ANGM=ANGD*60.0 D00;    MINS=ANGM;    ANGM=ANGM-DFLOAT(MINS)
      SECS=ANGM*60.0 D00;    IDEGS= (( SD*IDEGS ))
      RETURN;    END
```

```
C
C
      SUBROUTINE TITLE(UNM,UEM, UNA,UEA, UNB,UEB,
     $                 BASEA,BASEB, AZMA,AZMB, AMS,AZBIS)
C     ****************************************************
C     PRINT TITLE PAGE FOR PLANE SOLUTION
C     -----------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON IBUG
      PI=4.0*DATAN(1.0 D00);    RD= 180.0 D00/PI
C
      PRINT 400
  400 FORMAT(1H1/// 1H ,15X, 50(1H*) /
     $       1H ,15X,'TEST OF PLANE HYPERBOLIC INTERSECTION COMPUTATIONS'
     A      /1H ,25X,'(DIRECT AND ITERATIVE METHODS)'/1H ,15X,50(1H*)///
     B       1H ,16X,'TRANSMITTER STATION CO-ORDINATES IN UTM (METRES)'/
     C       1H ,16X, 48(1H-) //
     D       1H ,31X,'NORTHING' , 10X, 'EASTING' //)
C
      PRINT 402, UNM,UEM, UNA,UEA, UNB,UEB
  402 FORMAT(1H ,15X,'MASTER ',7X,'N', F10.1 , 7X ,'E', F10.1//
     A        1H ,15X,'SLAVE A',7X,'N', F10.1 , 7X ,'E', F10.1//
     B        1H ,15X,'SLAVE B',7X,'N', F10.1 , 7X ,'E', F10.1///)
      PRINT,' '
C
      PRINT 404
  404 FORMAT(1H ,15X,'CHAIN CONFIGURATION '/ 1H ,15X,20(1H-) //
     $        1H ,45X,'PATTERN A',5X,'PATTERN B' //     )
C
      CALL RADMS(AZMA,IDA,MINA,SECA)
      CALL RADMS(AZMB,IDB,MINB,SECB)
      PRINT 406, BASEA,BASEB, IDA,MINA,SECA, IDB,MINB,SECB
  406 FORMAT(1H ,15X,'BASELINE GRID DISTANCE ',3X,2F14.1 //
     $        1H ,15X,'GRID AZIMUTH (MASTER=>SLAVE)',
     A        2(I4, '-', I2, '-', F4.1, 2X)    ///)
C
      CALL RADMS(AMS, IDM,MINM,SECM)
      CALL RADMS(AZBIS,IDBIS,MINBIS,SECBIS)
      PRINT 408, IDM,MINM,SECM, IDBIS,MINBIS, SECBIS
  408 FORMAT(1H ,15X,'GRID ANGLE BETWEEN BASELINES AT M',
     $        I4, '-', I2, '-', F4.1 /
     A        1H ,15X,'GRID AZIMUTH OF BISECTOR ',
     B        8X, I4, '-', I2, '-', F4.1 //)
C
      RETURN;    END
```

```
C
C
      SUBROUTINE COLHED( HYPA,HYPB, UNP,UEP)
C     *************************************
C     COLUMN HEADERS
C     --------------
      IMPLICIT REAL*8 (A-H,O-Z)
      PRINT 504
  504 FORMAT(1H ,23X, 'HYPERBOLAE', 13X, 'UTM GRID POSITION'/
     $          1H ,23X,'A',9X,'B',11X,'NORTHING',5X,'EASTING  / ITER/'//)
      PRINT,' '
C
      PRINT 506, HYPA,HYPB, UNP,UEP
  506 FORMAT(1H ,7X,'OBSERVED H ',2F10.1,'  AT N',F10.1,'   E',F9.1/
     $          1H ,7X, 59(1H-) //)
      RETURN;   END
C

      SUBROUTINE CYCDIR(CORRA,CORRB, HYPA,HYPB, UNPP,UEPP,ITER)
C     ***************************************************
C     PRINT DIRECT METHOD CYCLE VALUES
C     --------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      PRINT, ' '
      PRINT, ' '
      PRINT 508, CORRA, CORRB
  508 FORMAT(1H ,7X, 'GRID CORR-N', 2F10.1/)
C
      PRINT 510, HYPA+CORRA, HYPB+CORRB, UNPP,UEPP, ITER
  510 FORMAT(1H ,7X,'PLANE VALUE',2F10.1,' ==> N',F10.1,'   E',F9.1,
     $          ' /', I2,  '/' )
      RETURN;   END

      SUBROUTINE CYCLIT(CHA,CHB,UNP,UEP,DHA,DHB,DN,DE,FRACTN,ITER)
C     **************************************************************
C     PRINT ITERATIVE METHOD CYCLE DATA
C     ---------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      PRINT,' '
      PRINT, ' '
      PRINT 606, CHA,CHB, UNP,UEP
  606 FORMAT(1H ,7X,'COMPUTED HYP=', F8.1, F10.1,
     $'            '  AT N',F10.1,'   E',F9.1 )
C
      UNPN=UNP+DN*FRACTN;    UEPN=UEP+DE*FRACTN
      PRINT 608, DHA,DHB, DN*FRACTN, DE*FRACTN, UNPN,UEPN, ITER
  608 FORMAT(1H ,7X,'OBS-COMPUTED ',F8.1,F10.1,
     $          ' ==> DN=', F8.1, ' DE=', F8.1 /
     A          1H , 45X, 9(1H-), 4X, 8(1H-)  /
     $          1H ,43X,'N',F10.1,'   E', F9.1, ' /', I2, '/')
C
      RETURN;   END
C
$ENTRY
```

Appendix II

TEST RUN OF PLANE SOLUTIONS

```
********************************************************
TEST OF PLANE HYPERBOLIC INTERSECTION COMPUTATIONS
        (DIRECT AND ITERATIVE METHODS)
********************************************************

 TRANSMITTER STATION CO-ORDINATES IN UTM (METRES)
 ------------------------------------------------

                    NORTHING             EASTING

MASTER            N 5000000.0          E  500000.0

SLAVE A           N 4900000.0          E  400000.0

SLAVE B           N 5000000.0          E  600000.0


CHAIN CONFIGURATION
-------------------

                                    PATTERN A       PATTERN B

BASELINE GRID DISTANCE                141421.4        100000.0

GRID AZIMUTH (MASTER=>SLAVE) 225- 0- 0.0       90- 0- 0.0


GRID ANGLE BETWEEN BASELINES AT M 135- 0- 0.0
GRID AZIMUTH OF BISECTOR          157-30- 0.0
```

DIRECT PLANE SOLUTION AT TEST POINT NO. 1
****

```
                 HYPERBOLAE                  UTM GRID POSITION
                  A         B              NORTHING      EASTING   /ITER/

OBSERVED H     58599.7  -41444.4  AT N 4900000.0  E 600000.0
-------------------------------------------------------------

GRID CORR-N        0.0       0.0
PLANE VALUE    58599.7  -41444.4 ==> N 4900019.6  E 600024.3 /  1/

GRID CORR-N      -21.6      23.1
PLANE VALUE    58578.6  -41421.4 ==> N 4900000.0  E 600000.0 /  2/
                                       =========    ========
```

```
ITERATIVE PLANE SOLUTION AT TEST POINT NO. 1
********************************************

                  HYPERBOLAE                   UTM GRID POSITION
                  A           B                NORTHING       EASTING   /ITER/


OBSERVED H      58599.7   -41444.4   AT N 4900000.0  E 600000.0
--------------------------------------------------------------

INITIAL POINT  -->          -->      AT N 4950000.0  E 550000.0


COMPUTED HYP= 87434.0       -4.4   AT N 4950000.0  E 550000.0
OBS-COMPUTED -28834.4   -41440.1 ==> DN=-35094.3 DE= 29302.6
                                        ---------    --------
                                      N 4914905.7  E 579302.6 / 1/

COMPUTED HYP= 63625.1   -28760.2   AT N 4914905.7  E 579302.6
OBS-COMPUTED   -5025.4  -12684.2 ==> DN=-12805.1 DE= 17164.4
                                        ---------    --------
                                      N 4902100.6  E 596466.9 / 2/

COMPUTED HYP= 59057.8   -39500.5   AT N 4902100.6  E 596466.9
OBS-COMPUTED    -458.1   -1943.9 ==> DN= -2048.1 DE=   3430.9
                                        ---------    --------
                                      N 4900052.6  E 599897.8 / 3/

COMPUTED HYP= 58606.9   -41387.5   AT N 4900052.6  E 599897.8
OBS-COMPUTED      -7.2     -56.9 ==> DN=    -52.5 DE=    102.1
                                        ---------    --------
                                      N 4900000.0  E 600000.0 / 4/

COMPUTED HYP= 58599.7   -41444.4   AT N 4900000.0  E 600000.0
OBS-COMPUTED      -0.0      -0.0 ==> DN=     -0.0 DE=      0.0
                                        ---------    --------
                                      N 4900000.0  E 600000.0 / 5/
                                        =========    ========
```

DIRECT PLANE SOLUTION AT TEST POINT NO. 2
****

```
                    HYPERBOLAE                      UTM GRID POSITION
                    A             B             NORTHING       EASTING   /ITER/

OBSERVED H      59307.8   -41739.1   AT N 4901000.0   E 600000.0
-------------------------------------------------------------------


GRID CORR-N       -21.0       23.1

PLANE VALUE     59286.7   -41716.1 ==> N 4901000.4   E 599999.9 /  1/

GRID CORR-N       -21.3       23.1

PLANE VALUE     59286.5   -41716.0 ==> N 4901000.0   E 600000.0 /  2/
                                         =========     ========
```

ITERATIVE PLANE SOLUTION AT TEST POINT NO. 2
****

```
                   HYPERBOLAE                   UTM GRID POSITION
                   A            B            NORTHING       EASTING  /ITER/

OBSERVED H      59307.8   -41739.1   AT N 4901000.0  E 600000.0
------------------------------------------------------------------

INITIAL POINT  -->          -->      AT N 4950500.0  E 550000.0


COMPUTED HYP= 87945.7       -4.3   AT N 4950500.0  E 550000.0
OBS-COMPUTED -28638.0  -41734.8 ==> DN=-34812.3 DE= 29363.8
                                       ---------    --------
                                     N 4915687.7  E 579363.8 /  1/

COMPUTED HYP= 64281.7  -29006.6   AT N 4915687.7  E 579363.8
OBS-COMPUTED   -4973.9  -12733.6 ==> DN=-12626.8 DE= 17119.6
                                       ---------    --------
                                     N 4903060.9  E 596483.4 /  2/

COMPUTED HYP= 59757.9  -39790.1   AT N 4903060.9  E 596483.4
OBS-COMPUTED    -450.1   -1949.0 ==> DN= -2009.6 DE=  3415.1
                                       ---------    --------
                                     N 4901051.3  E 599898.5 /  3/

COMPUTED HYP= 59314.8  -41682.2   AT N 4901051.3  E 599898.5
OBS-COMPUTED      -7.0     -57.0 ==> DN=    -51.3 DE=   101.4
                                       ---------    --------
                                     N 4901000.0  E 600000.0 /  4/

COMPUTED HYP= 59307.8  -41739.1   AT N 4901000.0  E 600000.0
OBS-COMPUTED      -0.0      -0.0 ==> DN=     -0.0 DE=     0.0
                                       ---------    --------
                                     N 4901000.0  E 600000.0 /  5/
                                       =========    ========
```

DIRECT PLANE SOLUTION AT TEST POINT NO. 3
****

```
                    HYPERBOLAE                UTM GRID POSITION
                    A           B           NORTHING       EASTING   /ITER/

OBSERVED H     58891.0   -42148.6   AT N 4900000.0   E 601000.0
--------------------------------------------------------------------------


GRID CORR-N      -21.3       23.1

PLANE VALUE    58869.7   -42125.5 ==> N 4899999.6   E 601000.5 /  1/

GRID CORR-N      -21.2       23.3

PLANE VALUE    58869.8   -42125.2 ==> N 4900000.0   E 601000.0 /  2/
                                        =========     ========
```

```
ITERATIVE PLANE SOLUTION AT TEST POINT NO. 3
********************************************

                    HYPERBOLAE                 UTM GRID POSITION
                    A          B              NORTHING      EASTING   /ITER/


OBSERVED H       58891.0   -42148.6   AT N 4900000.0  E 601000.0
--------------------------------------------------------------------


INITIAL POINT   -->          -->      AT N 4950000.0  E 550500.0


COMPUTED HYP= 37554.1     -711.7   AT N 4950000.0  E 550500.0
OBS-COMPUTED -28663.1   -41436.8 ==> DN=-35029.1 DE= 29476.5
                                         ---------    --------
                                       N 4914970.9  E 579976.5 / 1/

COMPUTED HYP= 63889.2   -29393.8   AT N 4914970.9  E 579976.5
OBS-COMPUTED   -4998.3  -12754.8 ==> DN=-12835.2 DE= 17388.1
                                         ---------    --------
                                       N 4902135.7  E 597364.6 / 2/

COMPUTED HYP= 59349.3   -40170.8   AT N 4902135.7  E 597364.6
OBS-COMPUTED    -458.3   -1977.8 ==> DN= -2080.8 DE=  3527.2
                                         ---------    --------
                                       N 4900055.0  E 600891.8 / 3/

COMPUTED HYP= 58898.3   -42089.0   AT N 4900055.0  E 600891.8
OBS-COMPUTED      -7.4     -59.6 ==> DN=    -54.9 DE=    108.1
                                         ---------    --------
                                       N 4900000.0  E 601000.0 / 4/

COMPUTED HYP= 58891.0   -42148.5   AT N 4900000.0  E 601000.0
OBS-COMPUTED      -0.0      -0.0 ==> DN=     -0.0 DE=      0.0
                                         ---------    --------
                                       N 4900000.0  E 601000.0 / 5/
                                         =========    ========
```

```
DIRECT PLANE SOLUTION AT TEST POINT NO. 4
*****************************************


                 HYPERBOLAE                    UTM GRID POSITION
                 A           B               NORTHING       EASTING   /ITER/


OBSERVED H     59595.5   -42446.7   AT N 4901000.0   E 601000.0
---------------------------------------------------------------


GRID CORR-N      -21.2       23.3

PLANE VALUE    59574.3   -42423.4 ==> N 4901000.4   E 600999.9 / 1/

GRID CORR-N      -21.4       23.4

PLANE VALUE    59574.1   -42423.4 ==> N 4901000.0   E 601000.0 / 2/
                                        =========     ========
```

```
ITERATIVE PLANE SOLUTION AT TEST POINT NO. 4
********************************************

                  HYPERBOLAE                   UTM GRID POSITION
                  A          B                 NORTHING      EASTING   /ITER/


OBSERVED H     59595.5   -42446.7   AT N 4901000.0  E 601000.0
-----------------------------------------------------------------


INITIAL POINT  -->          -->      AT N 4950500.0  E 550500.0


COMPUTED HYP= 88063.5     -715.2   AT N 4950500.0  E 550500.0
OBS-COMPUTED -28468.0   -41731.5 ==> DN=-34747.3 DE= 29536.3
                                        ---------     --------
                                     N 4915752.7  E 580036.3 /  1/

COMPUTED HYP= 64542.6   -29641.3   AT N 4915752.7  E 580036.3
OBS-COMPUTED   -4947.1   -12805.4 ==> DN=-12656.8 DE= 17344.1
                                        ---------     --------
                                     N 4903095.9  E 597380.4 /  2/

COMPUTED HYP= 60045.8   -40463.3   AT N 4903095.9  E 597380.4
OBS-COMPUTED    -450.3    -1983.5 ==> DN= -2042.1 DE=  3512.1
                                        ---------     --------
                                     N 4901053.7  E 600892.5 /  3/

COMPUTED HYP= 59602.6   -42387.1   AT N 4901053.7  E 600892.5
OBS-COMPUTED      -7.1      -59.6 ==> DN=   -53.7 DE=   107.4
                                        ---------     --------
                                     N 4901000.0  E 601000.0 /  4/

COMPUTED HYP= 59595.5   -42446.7   AT N 4901000.0  E 601000.0
OBS-COMPUTED      -0.0       -0.0 ==> DN=    -0.0 DE=     0.0
                                        ---------     --------
                                     N 4901000.0  E 601000.0 /  5/
                                       =========     ========
**ERROR***  END OF FILE ENCOUNTERED ON UNIT    5 (IBM CODE IHC217)

     PROGRAM WAS EXECUTING LINE    19 IN ROUTINE M/PROG WHEN TERMINATION OCCURRED

TATEMENTS EXECUTED=     4536

ORE USAGE       OBJECT CODE=    15712 BYTES,ARRAY AREA=      16 BYTES,TOTAL AREA AVAILABLE=  102400  BYTES

IAGNOSTICS        NUMBER OF ERRORS=       1, NUMBER OF WARNINGS=      0, NUMBER OF EXTENSIONS=
```

Appendix III

FORTRAN PROGRAMS FOR THE ELLIPSOIDAL SOLUTION

```
*****************************************************************************************************************FABLE**
      $JOB    WATFIV STUIFBERGEN/U,PAGES=20,T=3
      C       ********************************
      C       SELF-TEST OF ELLIPSOIDAL SOLUTIONS
      C       ----------------------------------
      C
                IMPLICIT REAL*8 (A-H,O-Z)
              COMMON IBUG
      C
              PI=4.0*DATAN(1.0 D00);      PI2=PI+PI
              DR=PI/180.0 D00;            RD= 180.0 D00/PI
              IBUG=1
              IBUG=0
              NTEST=0
      C
      C       /*** CLARKE 1866 ELLIPSOID
              AE=6378 206.4 D00;       FI= 294.9786986 D00
      C
      C       /*** ENTER STATION CO-ORDINATES OF MASTER, SLAVES A & B
      C       /*** GEODETIC LATITUDES AND LONGITUDES IN DEGREES
      C       /*** LONGITUDES POSITIVE EAST
      C
              READ, PHM,GLM,  PHA,GLA,  PHB,GLB
              PRINT 30, PHM,GLM, PHA,GLA, PHB,GLB
           30 FORMAT('  MASTER LAT=', F13.8, 5X, '  LONG=', F13.8//
             $        ' SLAVE A LAT=', F13.8, 5X, '  LONG=', F13.8//
             A        ' SLAVE B LAT=', F13.8, 5X, '  LONG=', F13.8//)
      C
              PHM=PHM*DR;    GLM=GLM*DR
              PHA=PHA*DR;    GLA=GLA*DR
              PHB=PHB*DR;    GLB=GLB*DR
      C
      C       /*** SPHERICAL DISTANCES AND AZIMUTHS OF BASELINES
              CALL SPHINV(PHM,GLM, PHA,GLA, BASE1,AZMA,AZ)
              CALL SPHINV(PHM,GLM, PHB,GLB, BASE2,AZMB, AZ)
      C
      C       /** INCLUDED ANGLE AT MASTER AND BISECTOR AZIMUTH
              AMS=DMOD(AZMA-AZMB, PI2)
              AZBIS=DMOD(AZMA-AMS/2.0D00, PI2)
      C
              PRINT, BASE1, BASE2
              PRINT, AZMA*RD, AZMB*RD
              PRINT, AMS*RD, AZBIS*RD
                   CALL TITLE(PHM,GLM,PHA,GLA,PHB,GLB,
             $                 BASE1,BASE2, AZMA,AZMB, AMS,AZBIS)
```

```
C
C
C
C        /***  MAIN OPERATING LOOP
         CORRA=0.0;    CORRB=0.0
     100 NTEST=NTEST+1
         READ, PHP,GLP
         IF(PHP+GLP .EQ. 0.0D00) STOP
         PHP=PHP*DR;    GLP=GLP*DR
C
C        /*** GEODETIC DISTANCES POINT -P- TO TRANSMITTERS
C        /*** PRECISE GEODETIC DISTANCES ON THE ELLIPSOID
C        /*** BY VINCENTY'S FORMULA
         CALL VININ(AE,FI, PHP,GLP, PHM,GLM, DM, AZPM)
         CALL VININ(AE,FI, PHP,GLP, PHA,GLA, DA, AZPA)
         CALL VININ(AE,FI, PHP,GLP, PHB,GLB, DB,AZPB)
C
C        /*** FICTITIOUS OBSERVED HYPERBOLAE
C        /*** DERIVED FROM EXACT DISTANCES BY VINCENTY'S FORMULA
         HYPA=DA-DM;    HYPB=DB-DM
C
C        /*** TEST OF DIRECT SOLUTION
         PRINT 398
     398 FORMAT(1H1//)
         PRINT 102, NTEST
     102 FORMAT(1H ,7X,'DIRECT SOLUTION - TEST PT. NO.', I2/
        $       1H ,7X,31(1H*) /)
               CALL COLHED( HYPA,HYPB, PHP,GLP)
```

```
C
C
      ITER=0
  200 ITER=ITER+1
C
C     /*** SPHERICAL SOLUTION WITH MODIFIED HYPERBOLAE
      CALL HYSPH( AMS,BASE1,BASE2,HYPA+CORRA, HYPB+CORRB,
     $            ALAM1,ALAM2, DM1,DM2, ICODE)
C
      CALL SPHFOR(PHM,GLM, DM1, AZBIS-ALAM1, PHP1,GLP1)
C
      CALL CYCDIR(HYPA,HYPB, CORRA,CORRB, PHP1,GLP1, PHP,GLP)
C     /*** COMPUTE ELLIPSOID CORRECTIONS
      CALL FADLM(PHP1,GLP1, PHM,GLM, SM,DSM,DDSM,COSA,SINA)
      CALL FADLM(PHP1,GLP1, PHA,GLA, SA,DSA,DDSA,COSA,SINA)
      CALL FADLM(PHP1,GLP1, PHB,GLB, SB,DSB,DDSB,COSA,SINA)
C
C     /*** USE THE CORRECTION TERMS ONLY
      CORR1=(DSM+DDSM)-(DSA+DDSA)
      CORR2=(DSM+DDSM)-(DSB-DDSB)
C
      CHECK=DABS(CORRA-CORR1)+DABS(CORRB-CORR2)
      CORRA=CORR1;    CORRB=CORR2
      TOL=1.0
      IF( CHECK .GT. TOL) GOTO 200
C
C     /*** TEST OF ITERATIVE METHOD
C     /*** ROUGH INITIAL VALUE
      PRINT 398
      PRINT 104, NTEST
  104 FORMAT(1H ,7X, 'ITERATIVE SOLUTION - TEST PT. NO.', I2/
     $       1H ,7X,33(1H*) /)
      CALL COLHED(HYPA,HYPB, PHP,GLP)
      PHPI=(PHP1+PHM)/2.0
      GLPI=(GLP1+GLM)/2.0
C
C     /*** ITERATIVE ELLIPSOIDAL SOLUTION
      CALL HYPGEO(HYPA, HYPB, PHPI,GLPI, PHM,GLM, PHA,GLA,PHB,GLB,ITER)
C
      GO TO 100
      END
```

```
C
C
      SUBROUTINE HYSPH(AMS,BASE1,BASE2,HYPA,HYPB,
     $                 ALAM1, ALAM2, DM1,DM2, ICODE)
C***************************************************
C     -- DIRECT SOLUTION OF INTERSECTING HYPERBOLAE ON THE SPHERE
C-------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON IBUG
      PI=4.0D00*DATAN(1.0D00);    RD=180.0 D00/PI
      AE=6378 206.4;    HYP1=HYPA/AE;    HYP2=HYPB/AE
C
      SB1=DSIN(BASE1/2.0D00);      SB2=DSIN(BASE2/2.0D00)
      CB1=DCOS(BASE1/2.0D00);      CB2=DCOS(BASE2/2.0D00)
      SH1=DSIN(HYP1/2.0 D00);      SH2=DSIN(HYP2/2.0D00)
C
      A1=(SB1-SH1)*(SB1+SH1)/(SB1*CB1)
      A2=(SB2-SH2)*(SB2+SH2)/(SB2*CB2)
      E1=-DSIN(HYP1)/DSIN(BASE1)
      E2=-DSIN(HYP2)/DSIN(BASE2)
C
      P1=(A1+A2)/2.0D00;    P2=(A1-A2)/2.0 D00
      Q1=(E1+B2)/2.0D00;       Q2=(B1-B2)/2.0 D00
C
      D=P2*Q1-P1*Q2;    SINM=DSIN(AMS/2.0D00);    COSM=DCOS(AMS/2.0D00)
      U=P2*COSM;    V=P1*SINM
      PHI=DATAN2(U,V);    R=U*DSIN(PHI) + V*DCOS(PHI)
C
      IF(IBUG .NE. 0) PRINT 100, A1,A2, B1,B2, P1,P2, Q1,Q2,
     $                      D,U,V, R, PHI
  100 FORMAT(1H ,4E20.8/1H ,4E20.8/1H ,3E20.8/1H ,2E20.8//)
C
C     // CHECK FOR NO POSSIBLE SOLUTION
      ICODE=0;    IF( R .LT. D)  RETURN
C
C     // ONE OR TWO POSSIBLE SOLUTIONS
      THETA=DARSIN(D/R);    ALAM1=THETA-PHI;    ALAM2=PI-THETA-PI
C
C     // DISTANCES M I.E. MASTER TO POINT -P-
      DM1=DATAN( -P2/(Q2 + SINM*DSIN(ALAM1)))
      DM2=DATAN( -P2/(Q2 + SINM*DSIN(ALAM2)))
C
C     // CODE TO IDENTIFY THE SOLUTIONS FOUND
      IF(DM1 .GT. 0.0 D00)  ICODE=1
      IF( DM2 .GT. 0.0 D00) ICODE=2
      IF( DM1 .GT. 0.0 D00 .AND. DM2 .GT. 0.0D00) ICODE=3
      IF ( IBUG .EQ. 0) RETURN
C
      PRINT, '  HYSPH'
      PRINT, ICODE
      PRINT 110, THETA, PHI, ALAM1,ALAM2, DM1,DM2
  110 FORMAT(1H ,2F20.8/)
      RETURN
      END
```

```
C
C
      SUBROUTINE HYPGEO(OHA,OHB,PHP,GLP,PHM,GLM,PHA,GLA,PHB,GLB,ITER)
C***********************************************************************
C     ITERATIVE  INTERSECTION OF HYPERBOLAE ON THE ELLIPSOID
C-----------------------------------------------------------------------
C      OHA,OHB       = OBSERVED HYPERBOLAE, PATTERNS A & B (METRES)
C      PHP,GLP       = INITIAL APPROX. POSITION IN LAT & LONG (RADIANS)
C                    - ALSO REFINED LAT AND LONG BY ITERATION
C      PHM,GLM       = GEODETIC LAT & LONG OF MASTER TRANSMITTER
C      PHA,GLA       = GEODETIC LAT & LONG OF SLAVE A
C      PHB,GLB       = GEODETIC LAT & LONG OF SLAVE B
C      ITER          = NUMBER OF ITERATIONS
C
C      EXTERNALS
C      FADLM         = SUBROUTINE FOR GEODETIC INVERSE BY ANDOYER-LAMBERT
C                    = FORMULA FOR ELLIPSOIDAL DISTANCE  A TO B
C-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON IBUG
C
      PI=4.0*DATAN(1.0D00);    RD=180.0 D00/PI;    AE=6378206.4 D00
C
C     /*** ITERATION LOOP
      TOL=1.0;    TEST=TOL+TOL;    ITER=0;    ITMAX=20
      DMAX=600.0D+03
      WHILE (ITER .LE. ITMAX .AND. TEST .GE. TOL) DO
         ITER=ITER+1
         CALL FADLM(PHP,GLP,PHM,GLM,SM,DSM,DDSM,COSM,SINM)
         CALL FADLM(PHP,GLP,PHA,GLA,SA,DSA,DDSA,COSA,SINA)
         CALL FADLM(PHP,GLP,PHB,GLB,SB,DSB,DDSB,COSB,SINB)
C
C        /*** SPHERICAL DISTANCES CORRECTED TO ELLIPSOID
         DM=SM+DSM+DDSM
         DA=SA+DSA+DDSA
         DB=SB+DSB+DDSB
C
C        /*** COMPUTED HYPERBOLICS AT -P-
         CHA=DA-DM;    CHB=DB-DM
C
C        /*** (OBSERVED) - (COMPUTED)  HYPERBOLIC DIFFERENCES
         DHA=OHA-CHA;    DHB=OHB-CHB
C
C        /*** A-MATRIX ELEMENTS(DH/D.PHI),(DH/D.LAMBDA)
         DAP=COSA-COSM;    DAL=SINA-SINM
         DBP=COSB-COSM;    DBL=SINB-SINM
C
C        /*** SOLVE FOR THE CORRECTIONS  DP, DL BY CRAMERS RULE
         DET=(DAP*DBL - DBP*DAL)*AE
          DP=(DHA*DBL - DHB*DAL)/DET
          DL=(DAP*DHB - DBP*DHA)/DET
```

```
C
C
C          /*** LIMIT THE SIZE OF CORRECTION IF IT IS LARGE
           DLC=DL*DCOS(PHP);    DIST=AE*DSQRT( DP*DP+DLC*DLC)
           FRACTN=1.0
           IF ( DIST .GT. DMAX) FRACTN=DMAX/DIST
              CALL CYCLIT(CHA,CHB,PHP,GLP,DHA,DHB,DP,DL,FRACTN,ITER)
           PHP=PHP+DP*FRACTN;      GLP=GLP+DL*FRACTN
           TEST=DABS(DHA) + DABS(DHB)
           ENDWHILE
        IF (IBUG .EQ. 0) RETURN
        PRINT, '   HYPGEO'
        PRINT, CHA, CHB, PHP*RD, GLP*RD
           PRINT, ' ITER=', ITER
           PRINT, PHP*RD, GLP*RD
           PRINT, DIST, TEST
        RETURN
        END
```

```
C
C
      SUBROUTINE VININ(AE,F,ALAT1,ALON1, ALAT2,ALON2, DIST,AZ)
C***********************************************************************
C    GEODETIC INVERSE BY VINCENTY'S METHOD
C-----------------------------------------------------------------
      IMPLICIT REAL*8  (A-H,O-Z)
      COMMON IBUG
C
C-- STATEMENT FUNCTIONS
      SIN(A)=DSIN(A)
      COS(A)=DCOS(A)
      TAN(A)= DSIN(A)/DCOS(A)
      ATAN(A)=DATAN(A)
      ATAN2(A,B)=DATAN2(A,B)
      SQRT(A)=DSQRT(A)
      ABS(A)=DABS(A)
      SICO(A)=DSQRT(1.0 D00 - A*A)
C
C     /*** CONSTANTS
      PI=4.0*DATAN(1.0 D00)
      FUZZ=1.0D-12
      FL=1.0/F
      BE= AE*(1.0-FL)
C
C-- REDUCED LATITUDES AND THEIR TRIG FUNCTIONS
      TU1=(1.0-FL)* TAN(ALAT1)
      TU2=(1.0-FL)* TAN(ALAT2)
      U1=ATAN(TU1)
      U2=ATAN(TU2)
C
      SU1=SIN(U1)
      SU2=SIN(U2)
      SU12=SU1*SU2
C
      CU1=COS(U1)
      CU2=COS(U2)
      CU12=CU1*CU2
C
C -- FIRST APPROX OF DIFFERENCE IN LONGITUDE = D.LONG ON SPHERE
      DL=ALON2-ALON1
      ?DL=DL
```

```
C
C
C -- ITERATION LOOP
  100 CONTINUE
      SDL=SIN(DL)
      CDL=COS(DL)
      CS=SU12 + CU12*CDL
      SS=SICO(CS)
      SIG=ATAN(SS/CS)
      IF ( ABS (SS)  .LT. FUZZ) SS=FUZZ
      SA=CU12*SDL/SS
      CA=SICO(SA)
      CA2=CA*CA
      C2SM=CS - ( 2.0*SU12)/CA2
C
C     -- DIFFERENCE OF LONG. ON AUXILIARY SPHERE
C
      C=(FL/16.0)*CA2*(4.0+(FL*(4.0-3.0*CA2)))
      DL1=XDL+(1.0-C)*FL*SA*(SIG+C*SS*(C2SM+C*CS*(-1.0+2.0*(C2SM**2))))
      IF ( ABS(DL1-DL) .LE. 1.0D-10) GOTO 45
      DL=DL1
      GOTO 100
   45 CONTINUE
C
      U=(CA**2) * ((AE**2) - (BE**2)) / ( BE**2)
      A=1.0+((U/256.0)*(64.0+U*(-12.0+5.0*U)))
      E = (U / 512.0) * (128.0 + U * ( -64.0 + ( 37.0 * U )))
      DSIG = B * SS * (C2SM + 0.25 * E * CS * (-1.0 + 2.0*(C2SM**2)))
C
C-- GEODESIC DISTANCE
      DIST= BE * A * ( SIG - DSIG)
C
C-- CALCULATE THE FORWARD AZIMUTH
      SDL1= SIN(DL1)
      CDL1= COS(DL1)
      AZ1= ATAN2((CU2*SDL1),( CU1*SU2-SU1*CU2*CDL1))
   70 CONTINUE
C
C-- BACK AZIMUTH
      AZ2= ATAN2((-1.0*CU1*SDL1),(SU1*CU2-CU1*SU2*CDL1))
      AZF=AZ1
      IF(IBUG .EQ. 0) RETURN
      PRINT,'  VININ'
      PRINT, DIST, AZ1,AZ2
      RETURN
      END
```

```
C
C
      SUBROUTINE FADLM(PHA,GLA,PHB,CLE, S,DS,DDS, COSA,SINA)
C**************************************************************
C     -- GEODETIC DISTANCE BY FORSYTHE-ANDOYER-LAMBERT METHOD
C--------------------------------------------------------------
      IMPLICIT REAL*8 (A-Z)
      INTEGER IBUG
      COMMON IBUG
C
      AE=6378 206.4 D00;        FL=1.0D00/294.9787 D00
      PHM=(PHB+PHA)/2.0D00;          DPM=(PHB-PHA)/2.0 D00
      DLON=GLB-GLA;             DLM=DLON/2.0 D00
C
      SPM=DSIN(PHM);            CPM=DCOS(PHM)
      SDP=DSIN(DPM);            CDP=DCOS(DPM)
      SDL=DSIN(DLM);            CDL=DCOS(DLM)
C
      K=SPM*CDP;                KK=SDP*CPM
      H=(CPM+SDP)*(CPM-SDP);  L=SDP*SDP + H*SDL*SDL
      U=2.0D00*K*K/(1.0D00-L);     V=2.0D00*KK*KK/L
      X=U+V;    Y=U-V
C
C     /*** SPHERICAL ARC DISTANCE, D (RADIANS) & S (METRES)
      D=2.0D00*DARSIN(DSQRT(L));    SIND=DSIN(D);    COSD=DCOS(D)
      S=AE*D
      T=D/SIND;    E=30.0 D00*COSD;    A=4.0D00*T*(8.0D00+T*E/15.0D00)
      D=4.0D00*(6.0D00 + T*T);    B=-(D+D);    C=T-(A+E)/2.0D00
C
      DS=-AE*SIND*(FL/4.0D00) * ( T*X - 3.0D00*Y)
      DDS=AE*SIND*(FL*FL/64.0 D00)*( X*(A+C*X)+Y*(B+E*Y)+D*X*Y)
C
C     /*** GRADIENTS PARTIAL DERIVATIVES
      SA=DSIN(PHA);    CA=DCOS(PHA)
      SB=DSIN(PHB);    CB=DCOS(PHB)
      SL=DSIN(DLON);    CL=DCOS(DLON)
      COSA=(SA*CB*CL-CA*SB)/SIND
      SINA=-(CA*CB*SL)/SIND
      IF(IBUG .EG. 0) RETURN
C
      DIST= S+DS+DDS
      PRINT,'  FADLM';   PRINT, S,DS,DDS, DIST
      PRINT, COSA, SINA
      RETURN
      END
```

```
C
C
      SUBROUTINE SPHFOR(PHA,GLA,D,AZ,PHE,GLB)
C*********************************************
C     -- POSITION OF -B- IN SPHERICAL LAT AND LONG
C     -- BY SPHERICAL DISTANCE AND AZIMUTH FROM -A-
C---------------------------------------------
      IMPLICIT REAL* 8  ( A-H,O-Z)
      COMMON IBUG
C
      SA=DSIN(PHA);      CA=DCOS(PHA)
      SD=DSIN(D);        CD=DCOS(D)
      SZ=DSIN(AZ);       CZ=DCOS(AZ)
C
      PHB=DARSIN(CD*SA+SD*CA*CZ)
      SE=DSIN(PHB);      CE=DCOS(PHB)
C
      SL=SD*SZ/CB;       CL=(CD-SA*SE)/(CA*CB)
      GLB=GLA + DATAN2( SL,CL)
C
      IF ( IBUG .EQ. 0) RETURN
      PRINT,' SPHFOR'
      PRINT , PHA,GLA,D,AZ,PHE,GLB
      RETURN
      END
```

```
C
C
      SUBROUTINE SPHINV(PHA,GLA,PHB,GLB,D,AZAB,AZBA)
C*************************************************************
C     SPHERICAL DISTANCE & AZIMUTH
C------------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON IBUG
      PI= 4.0 D00*DATAN(1.0 D00);   RD=180.0 D00/PI
      AE=6378206.4
C
      CA=DCOS(PHA);   SA=DSIN(PHA);   TA=SA/CA
      CB=DCOS(PHB);   SB=DSIN(PHB);   TB=SB/CB
      DLON=GLB-GLA;   SL=DSIN(DLON);   CL=DCOS(DLON)
C
      COSD=SA*SB + CA*CB*CL;   D=DARCOS(COSD)
      DIV=TB*CA-SA*CL;   AZAB=DATAN2(SL,DIV)
      DIV=TA*CB-SB*CL;   AZBA=DATAN2(-SL,DIV)
      IF(IBUG .LE. 1) RETURN
C
      PRINT,' SPHINV'
      PRINT,' '
      PRINT, PHA,GLA, PHB,GLB, D,AZAB,AZBA
      PRINT,D*AE,AZAB*RD, AZBA*RD
      RETURN
      END
```

```
C
C
      DOUBLE PRECISION FUNCTION RADN(ID,IM,AS)
C--- DEGRESS, MINUTES AND SECONDS TO RADIANS IN DOUBLE PRECISION
C-------------------------------------------------------------------
      IMPLICIT REAL*8  (A-H,O-Z)
      PI=4.0*DATAN(1.0 D00)
      SG=1.0
      IF (AS .NE. 0.0) SG=AS/DABS(AS)
      IF ( IM .NE. 0) SG= IM/IABS(IM)
      IF ( ID .NE. 0)     SG=ID/IABS(ID)
      ANGLE=AS + 60.0*DFLOAT( IM+60*IABS(ID))
      RADN= SG*ANGLE* (PI/180.0)/(60.0*60.0)
      PRINT 100, ID,IM,AS
  100 FORMAT(1H ,2I4,F9.2)
      RETURN
      END

      SUBROUTINE RADMIN( RADS, IDEGS, FMINS)
C     ******************************** ****
      IMPLICIT REAL*8( A-H,O-Z)
      PI=4.0 * DATAN(1.0 D00);   SGN=DSIGN( 1.0 D00, RADS)
      ANGLE=SGN*RADS*180.0 D00 / PI
      IDEGS=ANGLE;   FMINS=( ANGLE-DFLOAT (IDEGS))*60.0 D00
      ANGLE=ANGLE + 1.0 D-09
      RETURN;   END

      SUBROUTINE RADMS(ANGLE, IDEGS, MINS, SECS)
C     ****************************************
C     ANGLES IN RADIANS TO DEGREES, MINUTES AND SECONDS
C   ------------------------------------------------
      IMPLICIT REAL*8 (A-H, O-Z)
      PI2=8.0*DATAN(1.0 D00);   RD=360.0 D00/PI2
      SD=DSIGN( 1.0 D00, ANGLE)
        FUZZ=1.0D-11;   ANGD=(SD*ANGLE*RD) + FUZZ
      IDEGS=ANGD;   ANGD=ANGD-DFLOAT(IDEGS)
      ANGM=ANGD*60.0 D00 ;   MINS=ANGM;   ANGM=ANGM-DFLOAT(MINS)
      SECS=ANGM*60.0 D00;   IDEGS= (( SD * IDEGS ))
      RETURN;   END
```

```
C
C
      SUBROUTINE TITLE(PHM,GLM,PHA,GLA,PHB,GLB,
     $                 BASE1,BASE2, AZMA,AZMB, AMS,AZBIS)
C     ************************************************
C     TITLE PAGE FOR ELLIPSOIDAL PROBLEM
C     ----------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON IBUG
      PI=4.0*DATAN(1.0 D00);    RD=180.0D00/PI;    DR=PI/180.0 D00
      PRINT 398
  398 FORMAT(1H1)
C
      PRINT 400
  400 FORMAT(1H ///1H , 15X, 54(1H*) /
     $        1H ,15X,'TEST OF ELLIPSOIDAL HYPERBOLIC INTERSECTION ',
     A              'ALGORITHMS' /
     B        1H ,27X,'DIRECT AND ITERATIVE METHOD'/1H ,15X,54(1H*)///
C
      PRINT 402
  402 FORMAT(1H ,15X,'TRANSMITTER STATION POSITIONS',
     $              ' IN GEODETIC CO-ORDINATES' /
     1        1H ,15X, 54(1H*) /
     A        1H ,15X,'CLARKE 1866 ELLIPSOID  ',
     B              ' A=6378206.4    F= 1/294.9787'///
     C        1H ,26X,'LATITUDE', 7X,'LONGITUDE'//)
C
      CALL RADMS(PHM,IDA,MA,SECA);    CALL RADMS(GLM,IDO,MO,SECO)
      PRINT 404, IDA,MA,SECA,  IDO,MO,SECO
C
      CALL RADMS(PHA,IDA,MA,SECA);    CALL RADMS(GLA,IDO,MO,SECO)
      PRINT 406, IDA,MA,SECA,  IDO,MO,SECO
C
      CALL RADMS(PHB,IDA,MA,SECA);    CALL RADMS(GLB,IDO,MO,SECO)
      PRINT 408, IDA,MA,SECA,  IDO,MO,SECO
C
  404 FORMAT(1H ,15X,'MASTER  ',   2(I6, '-', I2, '-', F6.3 ) / )
  406 FORMAT(1H ,15X,'SLAVE A',    2(I6, '-', I2, '-', F6.3 ) / )
  408 FORMAT(1H ,15X,'SLAVE B',    2(I6, '-', I2, '-', F6.3 ) / )
C
      PRINT 396
  396 FORMAT(1H ///)
      PRINT 410
  410 FORMAT(1H ,15X,'CHAIN CONFIGURATION - SPHERICAL ANGLES & ARCS
     $        1H ,15X,  46(1H-) //
     A        1H ,15X,'ON SPHERE OF RADIUS  A=6378206.4 METRES' //
     B         1H ,43X,'RADIANS', 18X, 'DEGREES' /
     C         1H ,36X,2( 'SLAVE -A-',4X,'SLAVE -B-',5X) //)
C
      PRINT 412, BASE1,BASE2, BASE1*RD,BASE2*RD,
     $           AZMA,AZMB, AZMA*RD, AZMB*RD
  412 FORMAT(1H ,15X,'BASELINE LENGTH',2X,2F13.9,2F13.6 /
     $        1H ,15X,'BASELINE AZIMUTH ', 2F13.9, 2F13.6 //)
C
      PRINT 416, AMS,AMS*RD, AZBIS, AZBIS*RD
  416 FORMAT(1H ,15X,'ANGLE BETWEEN BASELINES', F17.9,F24.6 /
     $        1H ,15X,'BISECTOR SPHERICAL AZIMUTH',F14.9, F24.6 //)
      PRINT 398
      RETURN;   END;
```

```
      C
      C
            SUBROUTINE COLHED( HYPA,HYPB, PHP,GLP)
      C     **********************************
            IMPLICIT REAL*8 (A-H,O-Z)
            PRINT, ' '
            PRINT 636
        636 FORMAT(1H ,29X,'HYPERBOLAE',12X,'GEODETIC POSITION (DEG/MIN)'/
           $       1H ,27X,'-A-', 9X,'- -',12X,'LATITUDE', 6X,'LONGITUDE'//)
            CALL RADMIN(PHP,IDLAT, FMLAT)
            CALL RADMIN(GLP,IDLON,FMLON)
            PRINT 634, HYPA,HYPB, IDLAT,FMLAT, IDLON,FMLON
        634 FORMAT(1H ,7X,'OBSERVED HYPB.   ', F10.1,F12.1,'    AT
           $       2( I4,'-', F7.4,4X) / 1H ,7X, 72(1H-) //)
            RETURN;  END

            SUBROUTINE CYCDIR(HYPA,HYPB, CORRA,CORRB, PHPP,GLPP, PHP,GLP)
      C     -------------------------------------------------------------
      C     DIRECT METHOD CYCLE PRINT-OUT
      C     --------------------------------
            IMPLICIT REAL*8 (A-H,O-Z)
            PRINT, ' '
            PRINT 660, CORRA,CORRB
        660 FORMAT(1H ,7X,'ELLIPSOID CORR.', F11.1, F12.1)
      C
            CALL RADMIN(PHPP,IDLAT,FMLAT);   CALL RADMIN(GLPP, IDLON, FMLON)
            PRINT 670, HYPA+CORRA, HYPB+CORRB, IDLAT,FMLAT, IDLON,FMLON
        670 FORMAT(1H ,7X,'SPHERICAL VALUE', F11.1,F12.1, ' ==> ',
           $       2( I5,'-',F7.3,2X) / 1H , 48X, 2( I3(1H-), 2X ) )
      C
            DP=PHPP-PHP;  DL=GLPP-GLP;  CC=DCOS(PHP);   CONV=6378206.4
            CALL RADMIN(DP, IDLAT, FMLAT);   CALL RADMIN(DL, IDLON, FMLON)
            PRINT 650, IDLAT, FMLAT, IDLON,FMLON, DP*CONV, DL*CONV*CC
        650 FORMAT(1H ,46X,2(5X,11(1H-) ), 1X /
           $       1H ,41X, 'ERROR =', 2( I7,'-',F7.3,1X) /
           $       1H ,41X,' IN METRES', 2(F11.1,2X) )
            PRINT, ' '
            RETURN;   END
```

```
C
      SUBROUTINE CYCLIT(CHA,CHB,PHP,GLP,DHA,DHB,DP,DL,FRACTN,ITER)
C     *****************************************************************
C      ITERATION CYCLE PRINT-OUT
C     --------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      AE=6378206.4;    PI=4.0*DATAN(1.0 D00)
C
      CALL RADMIN(PHP,IDLAT,FMLAT)
      CALL RADMIN(GLP,IDLON,FMLON)
      PRINT 606, CHA,CHB, IDLAT,FMLAT, IDLON,FMLON
  606 FORMAT(1H ,7X,'COMPUTED HYPB.=', F11.1,F12.1,' AT ',
     $              2X, 2(I4,'-',F7.3, 4X) )
C
      CALL RADMIN(DP*FRACTN, IDLAT,FMLAT)
      CALL RADMIN(DL*FRACTN, IDLON,FMLON)
      PRINT 608, DHA,DHB, IDLAT,FMLAT, IDLON,FMLON
  608 FORMAT(1H ,7X,'OBS - COMPUTED=', F11.1,F12.1,
     $          6X, 2( I4,'-', F7.3, 4X) /
     A          1H ,51X, 12(1H-), 4X,12(1H-) )
C
      CALL RADMIN( PHP+DP*FRACTN, IDLAT, FMLAT)
      CALL RADMIN( GLP+DL*FRACTN, IDLON, FMLON)
      PRINT 610, ITER, IDLAT, FMLAT, IDLON, FMLON
  610 FORMAT(1H ,33X,'ITER NO.', I2,8X,2(I4,'-',F7.3,4X) //)
C
      RETURN
      END

    $ENTRY
IASTER LAT=  30.00000000        LONG=   0.00000000

.AVE A LAT= -30.00000000        LONG=  30.00000000

.AVE B LAT=  60.00000000        LONG=  60.00000000

     0.1159804177049415D 01       0.8638445989076787D 00
     0.1518132145679865D 03       0.3471500395394823D 02
     0.1170982106140382D 03       0.9326410926096736D 02
```

Appendix IV

TEST RUN OF ELLIPSOIDAL SOLUTIONS

```
*****************************************************
TEST OF ELLIPSOIDAL HYPERBOLIC INTERSECTION ALGORITHMS
            DIRECT AND ITERATIVE METHOD
*****************************************************
```

TRANSMITTER STATION POSITIONS IN GEODETIC CO-ORDINATES

****

CLARKE 1866 ELLIPSOID   A=6378206.4   F= 1/294.9787

	LATITUDE	LONGITUDE
MASTER	30- 0- 0.000	0- 0- 0.000
SLAVE A	-30- 0- 0.000	30- 0- 0.000
SLAVE B	60- 0- 0.000	60- 0- 0.000

CHAIN CONFIGURATION - SPHERICAL ANGLES & ARCS

ON SPHERE OF RADIUS A=6378206.4 METRES

	RADIANS SLAVE -A-	RADIANS SLAVE -B-	DEGREES SLAVE -A-	DEGREES SLAVE -B-
BASELINE LENGTH	1.159804177	0.863844599	66.451884	49.494550
BASELINE AZIMUTH	2.649640442	0.605891119	151.813215	34.715004

	RADIANS	DEGREES
ANGLE BETWEEN BASELINES	2.043749323	117.098211
BISECTOR SPHERICAL AZIMUTH	1.627765781	93.264109

DIRECT SOLUTION - TEST PT. NO. 1
****

	HYPERBOLAE -A-	-B-		GEODETIC POSITION (DEG/MIN) LATITUDE	LONGITUDE
OBSERVED HYPE.	5200362.3	-509572.7	AT	45- 0.0000	30- 0.0000
ELLIPSOID CORR.	0.0	0.0			
SPHERICAL VALUE	5200362.3	-509572.7	==>	44- 41.091	30- 18.352
			ERROR =	0- 18.909	0- 18.352
			IN METRES	-35082.2	24076.1
ELLIPSOID CORR.	44527.4	-2451.1			
SPHERICAL VALUE	5244889.7	-512023.8	==>	44- 59.969	29- 59.907
			ERROR =	0- 0.031	0- 0.093
			IN METRES	-56.6	-122.2
ELLIPSOID CORR.	44449.3	-2711.0			
SPHERICAL VALUE	5244811.6	-512283.6	==>	45- 0.001	30- 0.001
			ERROR =	0- 0.001	0- 0.001
			IN METRES	1.7	1.4

ITERATIVE SOLUTION - TEST PT. NO. 1
****

```
                             HYPERBOLAE                  GEODETIC POSITION (DEG/MIN)
                         -A-                 - -          LATITUDE       LONGITUDE

OBSERVED HYPB.      5200362.3     -509572.7  AT     45- 0.0000       30- 0.0000
------------------------------------------------------------------------------

COMPUTED HYPB.=     6016816.7     2408072.7  AT     37- 30.000       15-  0.001
OBS - COMPUTED=     -816454.4    -2917645.4          3- 25.285        5- 14.965
                                                   -----------     -----------
                         ITER NO. 1                 40- 55.285       20- 14.965

COMPUTED HYPB.=     5722665.5     1254122.4  AT     40- 55.285       20- 14.965
OBS - COMPUTED=     -522303.3    -1763695.1          2- 58.937        5- 56.500
                                                   -----------     -----------
                         ITER NO. 2                 43- 54.222       26- 11.466

COMPUTED HYPB.=     5414309.9      103898.5  AT     43- 54.222       26- 11.466
OBS - COMPUTED=     -213947.6     -613471.2          1-  9.268        3- 42.932
                                                   -----------     -----------
                         ITER NO. 3                 45-  3.490       29- 54.398

COMPUTED HYPB.=     5211201.7     -505533.8  AT     45-  3.490       29- 54.398
OBS - COMPUTED=      -10839.4       -4038.9          0-  3.488        0-  5.634
                                                   -----------     -----------
                         ITER NO. 4                 45-  0.003       30-  0.032

COMPUTED HYPB.=     5200326.1     -509643.8  AT     45-  0.003       30-  0.032
OBS - COMPUTED=          36.2          71.2          0-  0.003        0-  0.032
                                                   -----------     -----------
                         ITER NO. 5                 45-  0.000       29- 60.000

COMPUTED HYPB.=     5200362.4     -509572.5  AT     45-  0.000       29- 60.000
OBS - COMPUTED=          -0.1          -0.2          0-  0.000        0-  0.000
                                                   -----------     -----------
                         ITER NO. 6                 45-  0.000       29- 60.000
```

DIRECT SOLUTION - TEST PT. NO. 2
****

	HYPERBOLAE -A-	- -		GEODETIC POSITION (DEG/MIN) LATITUDE	LONGITUDE
OBSERVED HYPE.	5268142.6	-638006.7	AT	46- 0.0000	30- 0.0000
ELLIPSOID CORR.	44449.6	-2710.7			
SPHERICAL VALUE	5312592.1	-640717.4	==>	45- 59.917	29- 59.849
			ERROR =	0- 0.083	0- 0.151
			IN METRES	-153.5	-195.0
ELLIPSOID CORR.	44362.3	-3196.7			
SPHERICAL VALUE	5312504.9	-641203.4	==>	46- 0.000	30- 0.001
			ERROR =	0- 0.000	0- 0.001
			IN METRES	0.7	0.7

ITERATIVE SOLUTION - TEST PT. NO. 2
****

	HYPERBOLAE A			GEODETIC POSITION (DEG/MIN) LATITUDE	LONGITUDE
OBSERVED HYPB.	5268142.6	-638006.7	AT	46- 0.0000	30- 0.0000
COMPUTED HYPB.=	6045139.9	2338290.8	AT	38- 0.000	15- 0.000
OBS - COMPUTED=	-776997.3	-2976297.5		3- 34.175	5- 7.486
	ITER NO. 1			41- 34.175	20- 7.486
COMPUTED HYPB.=	5770790.2	1180045.2	AT	41- 34.175	20- 7.486
OBS - COMPUTED=	-502647.6	-1818051.9		3- 9.228	5- 50.528
	ITER NO. 2			44- 43.403	25- 58.014
COMPUTED HYPB.=	5482664.1	22346.2	AT	44- 43.403	25- 58.014
OBS - COMPUTED=	-214521.5	-660352.9		1- 20.663	3- 55.231
	ITER NO. 3			46- 4.066	29- 53.295
COMPUTED HYPB.=	5280531.6	-633186.2	AT	46- 4.066	29- 53.295
OBS - COMPUTED=	-12389.0	-4820.5		0- 4.065	0- 6.746
	ITER NO. 4			46- 0.001	30- 0.041
COMPUTED HYPB.=	5268094.7	-638092.0	AT	46- 0.001	30- 0.041
OBS - COMPUTED=	47.9	85.3		0- 0.001	0- 0.041
	ITER NO. 5			46- 0.000	29- 60.000
COMPUTED HYPB.=	5268142.7	-638006.5	AT	46- 0.000	29- 60.000
OBS - COMPUTED=	-0.1	-0.2		0- 0.000	0- 0.000
	ITER NO. 6			46- 0.000	29- 60.000

DIRECT SOLUTION - TEST PT. NO. 3
****

	HYPERBOLAE -A-	-B-		GEODETIC POSITION (DEG/MIN) LATITUDE	LONGITUDE
OBSERVED HYPE.	5127620.5	-632566.4	AT	45- 0.0000	31- 0.0000
ELLIPSOID CORR.	44362.6	-3196.5			
SPHERICAL VALUE	5171983.1	-635762.9	==>	45- 0.092	31- 0.309
		ERROR =		0- 0.092	0- 0.309
		IN METRES		170.8	404.8
ELLIPSOID CORR.	44625.5	-2374.9			
SPHERICAL VALUE	5172246.0	-634941.3	==>	45- 0.001	31- 0.001
		ERROR =		0- 0.001	0- 0.001
		IN METRES		2.3	0.7
ELLIPSOID CORR.	44624.7	-2375.8			
SPHERICAL VALUE	5172245.2	-634942.3	==>	45- 0.001	31- 0.001
		ERROR =		0- 0.001	0- 0.001
		IN METRES		2.2	1.4

```
ITERATIVE SOLUTION - TEST PT. NO. 3
*********************************

                         HYPERBOLAE               GEODETIC POSITION (DEG/MIN)
                  A                                   LATITUDE      LONGITUDE

OBSERVED HYPB.    5127620.5     -632566.4  AT     45- 0.0000        31- 0.0000
--------------------------------------------------------------------------------

COMPUTED HYPB.=   5966597.5     2341835.7  AT     37- 30.001        15- 30.001
OBS - COMPUTED=   -838977.0    -2974402.1          3- 22.171         5- 18.149
                                                 ------------   ------------
                             ITER NO. 1           40- 52.171        20- 48.150

COMPUTED HYPB.=   5670846.5     1190604.1  AT     40- 52.171        20- 48.150
OBS - COMPUTED=   -543226.1    -1823170.5          2- 55.829         5- 58.917
                                                 ------------   ------------
                             ITER NO. 2           43- 48.000        26- 47.066

COMPUTED HYPB.=   5361491.1       43774.5  AT     43- 48.000        26- 47.066
OBS - COMPUTED=   -233870.6     -676340.9          1- 16.230         4-  6.000
                                                 ------------   ------------
                             ITER NO. 3           45-  4.230        30- 53.066

COMPUTED HYPB.=   5141006.4     -627278.7  AT     45-  4.230        30- 53.066
OBS - COMPUTED=    -13385.9       -5287.7          0-  4.227         0-  6.974
                                                 ------------   ------------
                             ITER NO. 4           45-  0.003        31-  0.040

COMPUTED HYPB.=   5127574.8     -632654.4  AT     45-  0.003        31-  0.040
OBS - COMPUTED=        45.7          88.0          0-  0.003         0-  0.040
                                                 ------------   ------------
                             ITER NO. 5           45-  0.000        30- 60.000

COMPUTED HYPB.=   5127620.6     -632566.2  AT     45-  0.000        30- 60.000
OBS - COMPUTED=        -0.1          -0.2          0-  0.000         0-  0.000
                                                 ------------   ------------
                             ITER NO. 6           45-  0.000        30- 60.000

ATEMENTS EXECUTED=     6687
RE USAGE        OBJECT CODE=   27464 BYTES,ARRAY AREA=       16 BYTES,TOTAL AREA AVAILABLE=  102400  BYTES
AGNOSTICS          NUMBER OF ERRORS=        0, NUMBER OF WARNINGS=        0, NUMBER OF EXTENSIONS=      22
MPILE TIME=     0.40 SEC,EXECUTION TIME=     0.30 SEC,     10.44.57     TUESDAY      17 FEB 81     WATFIV - M
```