# Investigation of the Aedes spread using a reaction-diffusion mathematical model

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## 1. Abstract

In this work, we developed a reaction-diffusion mathematical model to describe the spread of dengue infection in a two-dimensional computational domain to understand how the disease spreads from a specific location to another, considering the diffusion coefficients of both infected populations, mosquitoes, and humans. We aimed to understand how the disease spreads from a specific location to another, considering the diffusion coefficients of both infected populations, mosquitoes, and humans. Our contribution provides an in-depth analysis of the optimal control problem and it outlines a more explicit modeling framework based on real spatial-temporal data.

## 2. Objectives

To investigate the impact of human movement and the vector dispersal behavior on the spread of dengue disease, applying optimal control.

## 3. Model

The PDE system used in this work is given below, Herein, $H_s$ and $H_I$ are the density of human susceptible and infected populations. We denote by $M_s$ and $M_I$ the density of adult susceptible and infected mosquito populations, respectively, while $A$ presents the density of mosquitoes in the aquatic phase (eggs and larvae). $\nu$ is the control to combat mosquitoes. The biological parameters are showed in Tables (1) and (2).

$$\begin{cases} \partial_t M_I &= \epsilon_M \Delta M_I + f^{M_I}(M_I, H_I, A, M_s, H_s) - \alpha\nu M_I \quad \text{in } \Omega_T := (0,T)\times\Omega, \\ \partial_t H_I &= D_H \Delta H_I + f^{H_I}(M_I, H_I, A, M_s, H_s) \quad \text{in } \Omega_T, \\ \partial_t A &= f^{A}(M_I, H_I, A, M_s, H_s) - \alpha\nu A \quad \text{in } \Omega_T, \\ \partial_t M_s &= \epsilon_M \Delta M_s + f^{M_s}(M_I, H_I, A, M_s, H_s) - \alpha\nu M_s \quad \text{in } \Omega_T, \\ \partial_t H_s &= D_H \Delta H_s + f^{H_s}(M_I, H_I, A, M_s, H_s) \quad \text{in } \Omega_T, \\ \nabla M_I \cdot \eta &= 0,\ \nabla H_I \cdot \eta = 0,\ \nabla M_s \cdot \eta = 0,\ \nabla H_s \cdot \eta = 0 \quad \text{on } \Sigma_T = (0,T)\times\partial\Omega \\ M_I(0) &= M_{I,0},\ H_I(0) = H_{I,0},\ A(0) = A_0,\ M_s(0) = M_{s,0},\ H_s(0) = H_{s,0} \text{ in } \Omega, \end{cases} \tag{1}$$

where $\epsilon_M$ and $D_H$ are the diffusion coefficients of mosquitoes and humans, respectively. $\eta(x,y)$ is the normal vector on $\partial\Omega$. The interaction terms are written as follows:

$$\begin{aligned} f^{M_I}(M_I, H_I, A, M_s, H_s) &:= \alpha\rho A - \mu_m M_I + \frac{b\beta_m M_s H_I}{H} \\ f^{H_I}(M_I, H_I, A, M_s, H_s) &:= \frac{b\beta_H H_s M_I}{H} - \mu_H H_I - \sigma H_I \\ f^{A}(M_I, H_I, A, M_s, H_s) &:= k\delta\left(1 - \frac{A}{C}\right)(M_s + M_I) - (\mu_A + \alpha)A \\ f^{M_s}(M_I, H_I, A, M_s, H_s) &:= \alpha(1-\rho)A - \mu_m M_s - \frac{b\beta_m M_s H_I}{H} \\ f^{H_s}(M_I, H_I, A, M_s, H_s) &:= \mu_H(H - H_s) - \frac{b\beta_H H_s M_I}{H} \end{aligned}$$

The control function $\nu$ is governed by the following ODE:

$$\frac{d}{dt}\nu(t,x) = -\tau_1\nu(t,x) + \tau_2(t,x), \tag{2}$$

where $\tau_1$ and $\tau_2$ mean the forgetting rate to promove conditions unfavourable to Aedes breending and the government's investment in educational campaigns, respectively.

**Table 1:** Parameters of *Aedes aegypti* transmission.

| | | |
|---|---|---|
| $H$ | Human populations (susceptible, infected and recovered) | $indiv. \times km^{-2}$ |
| $M$ | Mosquito populations (susceptible and infected) | $indiv. \times m^{-2}$ |
| $\sigma$ | Recovery rate of humans | $day^{-1}$ |
| $\mu_H$ | Mortality of human population | $year^{-1}$ |
| $b$ | Proportion of the effective bite that transmits infection | $day^{-1}$ |
| $\beta_H$ | Probability of vector transmission to humans | $-$ |
| $\beta_m$ | Probability of human transmission to the vector | $-$ |

**Table 2:** Parameters of *Aedes aegypti* transmission.

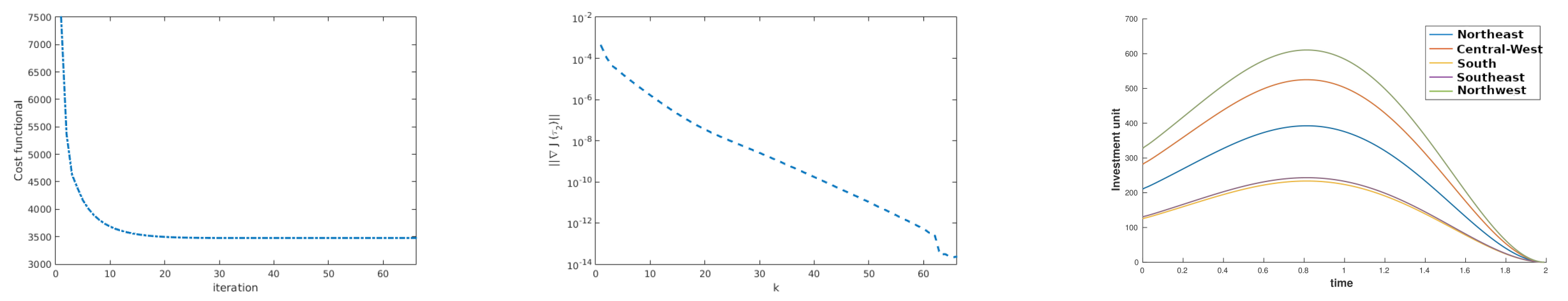
| | | |
|---|---|---|
| $C$ | Carrying capacity of aquatic phase of mosquitoes | $indiv. \times m^{-2}$ |
| $k$ | Ratio between male and female mosquitoes | $-$ |
| $\delta$ | Per-capita oviposition rate | $day^{-1}$ |
| $\mu_A$ | Mortality of aquatic stages of mosquitoes | $day^{-1}$ |
| $\mu_m$ | Mortality of adult mosquito populations | $day^{-1}$ |
| $\alpha$ | Transformation rate of water phase to the adult phase | $day^{-1}$ |
| $\epsilon_M$ | Diffusion coefficient of mosquitoes | $m^2 \times day^{-1}$ |
| $D_H$ | Diffusion coefficient of humans | $km^2 \times day^{-1}$ |

## 4. Cost function

The main idea is to compute the optimized control that minimizes the cost function $(P)$ that makes $H_I$ as small as possible over time.

$$(P) \quad \begin{cases} \min\limits_{\tau_2}\left[ J(\tau_2) = \frac{1}{2}\left(\iint_{\Omega_T}\left(\varepsilon_1|H_I - H_{I,d}|^2 + \varepsilon_2|\tau_2|^2\right)dx\,dt\right)\right], \\ \text{subject to the reaction-diffusion system (1).} \end{cases} \tag{3}$$

where $\varepsilon_1$ and $\varepsilon_2$ are the regularization parameters.

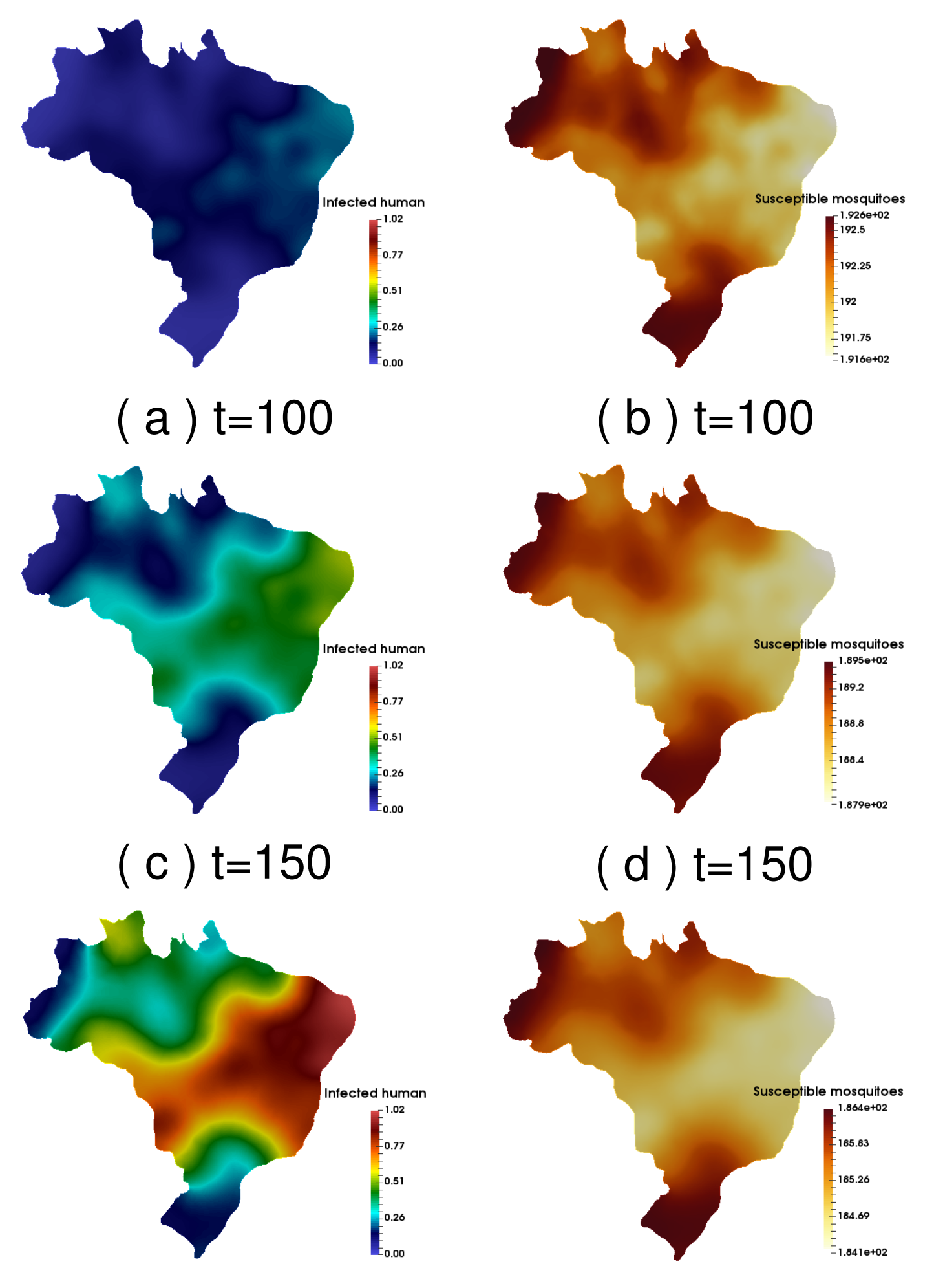


( a ) $J(\tau_2)$ ( b ) $\nabla J(\tau_2)$ ( c ) $\tau_2$

(a) the optimal solution for $\tau_2$;
(b) the gradient norm, $\nabla J(\tau_2)$, at each iteration;
(c) cost function, $J(\tau_2)$, evolution during the iterations.

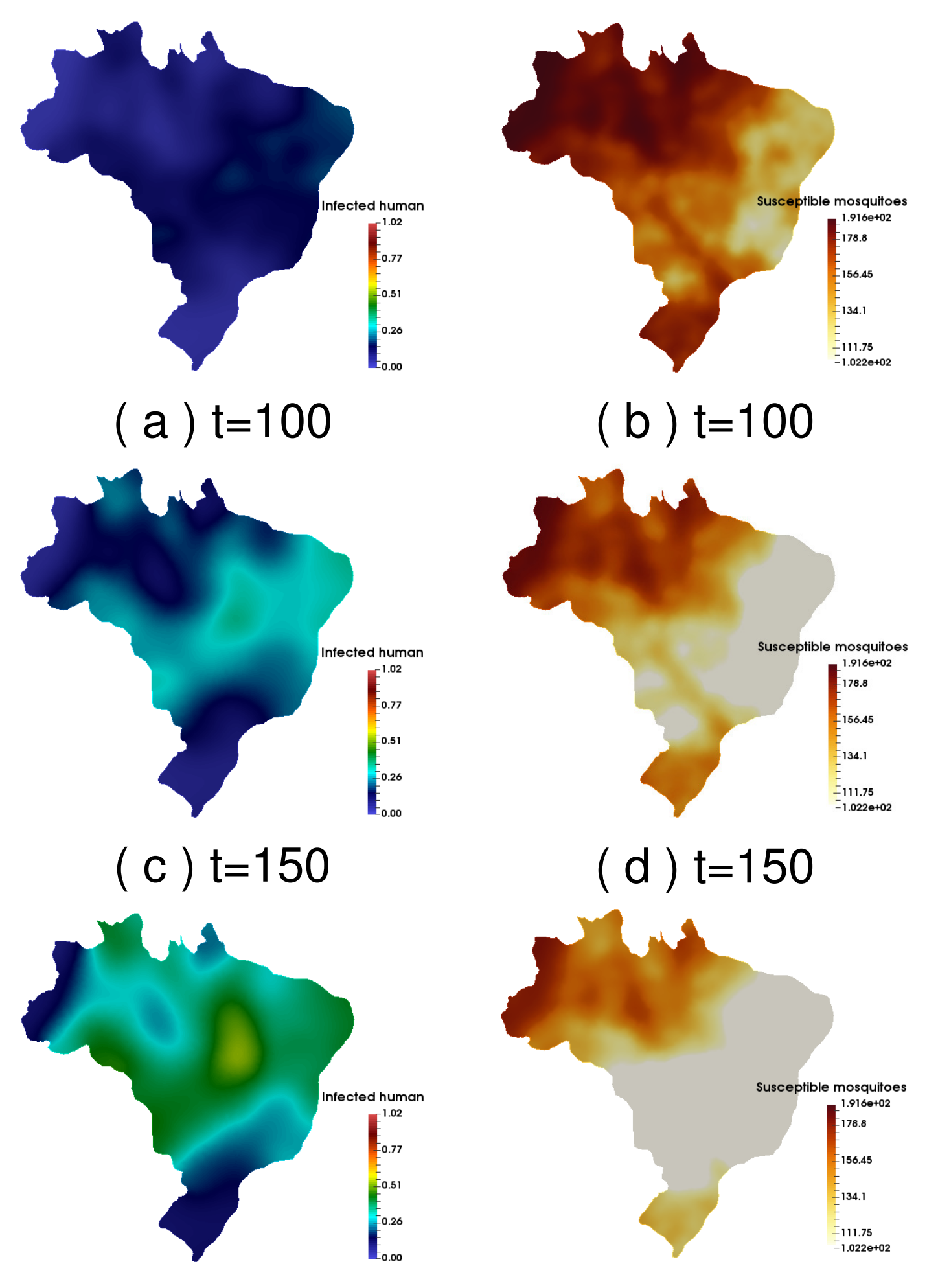
## 5. Results

We present here the results.



( a ) t=100 ( b ) t=100
( c ) t=150 ( d ) t=150
( e ) t=200 ( f ) t=200

**Figure 1:** Spatial distribution of densities of infected human and susceptible mosquito using the arbitrary control, $\tau_1 = 0.1$ and $\tau_2 = 0.0$..



( a ) t=100 ( b ) t=100
( c ) t=150 ( d ) t=150
( e ) t=200 ( f ) t=200

**Figure 2:** spatial distribution of densities of infected human and susceptible mosquito applying the optimal control solution. $\tau_1 = 0.1$.

## 6. Conclusions

The main conclusion is: we strongly suggest maintaining the control during the epidemic period in the Central-West, Northeast and Southeast regions, in order to optimize the spread of Dengue in Brazil.

## 7. Acknowledgments

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## 8. References

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