Representing spatially-coupled ecological oscillators by dynamical Ising model with memory

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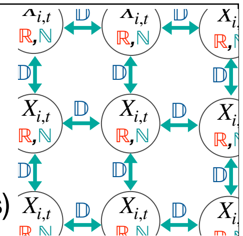
Motivation

- Spatially-coupled ecological oscillators can be modeled by coupled lattice maps
- Coupled lattice maps with noise in two-cycle regime undergo a transition to global synchrony in Ising universality class [1]
- Universal properties of coupled lattice maps can be studied using the standard Ising model

Questions

- How can we study the non-universal dynamical properties of ecological oscillators?
- Is there a dynamical Ising model that represents even the non-universal properties?
- How can we study the ecological oscillators when the local dynamics of the system is unknown?

Models

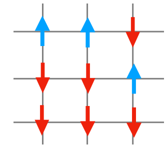


Noisy coupled lattice maps

- Two-dimensional lattice on a torus
- Each $X_{i,t}$ undergoes three processes

1. $\mathbb{R}$: Local dynamics (Ricker, Logistic maps)
2. $\mathbb{D}$: Dispersal (Nearest neighbor interactions)
3. $\mathbb{N}$: Multiplicative noise

- Four models studied at criticality, each model undergoes transition from incoherence to synchrony in the Ising universality class



Dynamical Ising model with memory

- Magnetic spins, $S_{i,t} = \pm 1$ ↑↓
- Spins can align with neighbors and lead to magnetization
- P_f: Flip probability of a spin
- J: Nearest neighbor coupling
- K: memory

$$P_f = \frac{\exp[-JS_{i,t}\sum S_{j,t} - K]}{2\cosh[JS_{i,t}\sum S_{j,t} + K]}$$

Ising representation

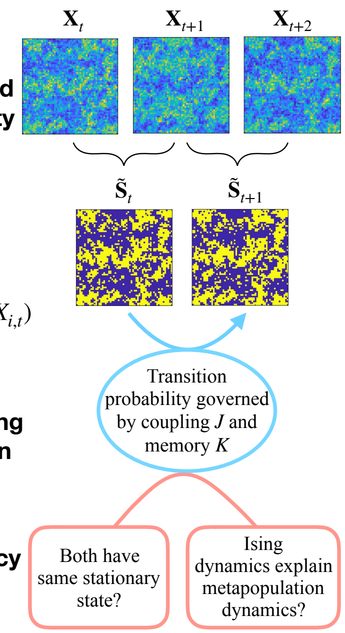
Consecutive lattice snapshots of a coupled lattice map at criticality

Lattice variables are reduced to discrete values $\{\pm 1\}$

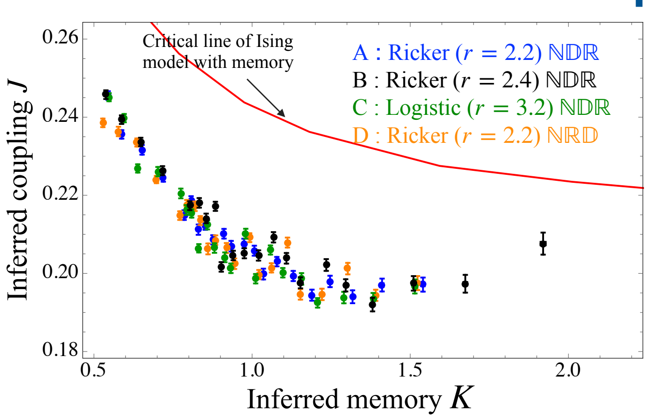
$$\tilde{S}_{i,t} = (-1)^t \mathrm{sgn}(X_{i,t+1} - X_{i,t})$$

Ising parameters are inferred by maximizing the likelihood function

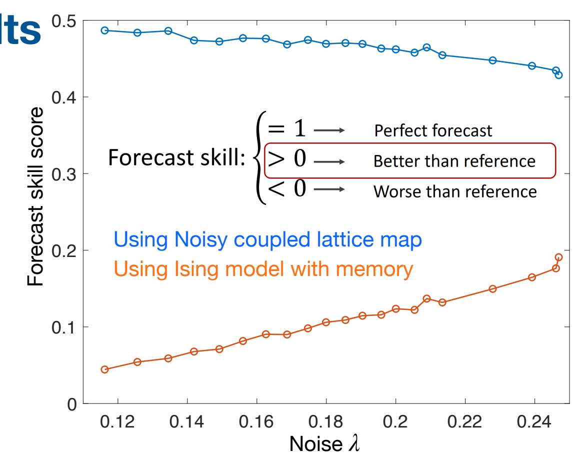
Assessing the accuracy of the dynamical Ising model



Results



- Inferred Ising parameters of coupled lattice maps at criticality fall very close to the critical line but lie in the disordered phase



- Forecast skill of the Ising model is better than the reference (average flip probability) and increases with noise

Discussions

- Equilibrium and dynamical properties of a wide range of coupled lattice maps are reproduced by the dynamical Ising model with memory
- The Ising representation results show that dynamical Ising model serves as a baseline model without requiring any details of the local dynamics
- Forecast skill shows information useful for prediction is obtained even with the simplest model

References

[1] A Noble, J Machta, A Hastings. Nat Comm, 6:6664, 2015

[2] V Nareddy, J Machta. PRE, 101:012122, 2020

More details are available in our reprint

https://arxiv.org/abs/2007.09195