Histogram-Aware Sorting for Enhanced Word-Aligned Compression in Bitmap Indexes

Owen Kaser[1], Daniel Lemire[2], Kamel Aouiche[2]

1- University of New Brunswick, Saint John
2- Université du Québec at Montréal (UQAM)

October 23, 2008

Bitmap indexes

```
SELECT * FROM
T WHERE x=a
AND y=b;
```

Above, compute

$$\boxed{\begin{array}{l}\{r \mid r \text{ is the row id of a row where } x = a\} \cap \\ \{r \mid r \text{ is the row id of a row where } y = b\}\end{array}}$$

Bitmap indexes

```
SELECT * FROM
T WHERE x=a
AND y=b;
```

- Bitmap indexes have a long history. (1972 at IBM.)

Above, compute

$$\boxed{\begin{array}{l}\{r \mid r \text{ is the row id of a row where } x = a\} \cap \\ \{r \mid r \text{ is the row id of a row where } y = b\}\end{array}}$$

Bitmap indexes

```
SELECT * FROM
T WHERE x=a
AND y=b;
```

- Bitmap indexes have a long history. (1972 at IBM.)
- Long history with DW & OLAP. (Sybase IQ since mid 1990s).

Above, compute

$$\boxed{\begin{array}{l}\{r \mid r \text{ is the row id of a row where } x = a\} \cap \\ \{r \mid r \text{ is the row id of a row where } y = b\}\end{array}}$$

Bitmaps and fast AND/OR operations

- Computing the union of two sets of integers between 1 and 64 (eg row ids, trivial table)...
- E.g., $\{1, 5, 8\} \cup \{1, 3, 5\}$?

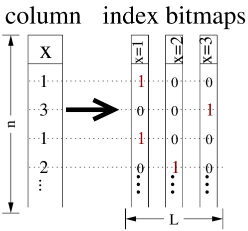
Bitmaps and fast AND/OR operations

- Computing the union of two sets of integers between 1 and 64 (eg row ids, trivial table)...
- E.g., $\{1, 5, 8\} \cup \{1, 3, 5\}$?
- Can be done in **one operation** by a CPU:
  `BitwiseOR( 10001001, 10101000)`

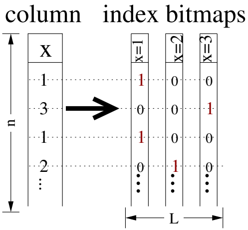
Bitmaps and fast AND/OR operations

- Computing the union of two sets of integers between 1 and 64 (eg row ids, trivial table)...
- E.g., $\{1, 5, 8\} \cup \{1, 3, 5\}$?
- Can be done in **one operation** by a CPU:
  `BitwiseOR( 10001001, 10101000)`
- Extend to sets from $1..N$ using $\lceil N/64 \rceil$ operations.

Bitmap compression

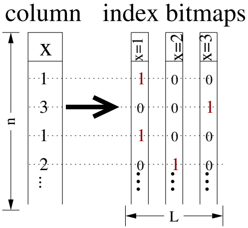


- A column with n rows and L distinct values $\Rightarrow$ nL bits



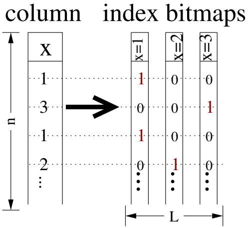
- A column with n rows and L distinct values $\Rightarrow nL$ bits
- E.g., $n = 10^6$, $L = 10^4 \rightarrow 10$ Gbits

Bitmap compression



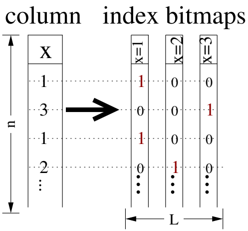
- A column with n rows and L distinct values $\Rightarrow nL$ bits
- E.g., $n = 10^6$, $L = 10^4 \rightarrow 10$ Gbits
- Uncompressed bitmaps are often impractical

Bitmap compression



- A column with n rows and L distinct values $\Rightarrow nL$ bits
- E.g., $n = 10^6$, $L = 10^4 \rightarrow 10$ Gbits
- Uncompressed bitmaps are often impractical
- Moreover, bitmaps often contain **long streams of zeroes**...

Bitmap compression



- A column with n rows and L distinct values $\Rightarrow nL$ bits
- E.g., $n = 10^6$, $L = 10^4 \rightarrow 10$ Gbits
- Uncompressed bitmaps are often impractical
- Moreover, bitmaps often contain **long streams of zeroes**...
- Logical operations over these zeroes is a waste of CPU cycles.

How to compress bitmaps?

- Must handle **long streams of zeroes** efficiently $\Rightarrow$ Run-length encoding? (**RLE**)

How to compress bitmaps?

- Must handle **long streams of zeroes** efficiently ⇒ Run-length encoding? (**RLE**)
- RLE variants can focus on runs that align with machine-word boundaries.

How to compress bitmaps?

- Must handle **long streams of zeroes** efficiently $\Rightarrow$ Run-length encoding? (**RLE**)
- RLE variants can focus on runs that align with machine-word boundaries.
- Trade compression for speed.

How to compress bitmaps?

- Must handle **long streams of zeroes** efficiently ⇒ Run-length encoding? (**RLE**)
- RLE variants can focus on runs that align with machine-word boundaries.
- Trade compression for speed.
- Our **EWAH** extends Wu et al.'s word-aligned hybrid.

How to compress bitmaps?

- Must handle **long streams of zeroes** efficiently ⇒ Run-length encoding? (**RLE**)
- RLE variants can focus on runs that align with machine-word boundaries.
- Trade compression for speed.
- Our **EWAH** extends Wu et al.'s word-aligned hybrid.

0101000000000000	000. . . 000	000. . . 000	0011111111111100	. . .

⇒ dirty word, run of 2 "clean 0" words, dirty word. . .

Computational and storage bounds

- $n \longrightarrow$ number of rows, $c \longrightarrow$ number of 1s per row;

Computational and storage bounds

- $n \longrightarrow$ number of rows, $c \longrightarrow$ number of 1s per row;
- Construction in time $O(nc)$;

Computational and storage bounds

- $n \longrightarrow$ number of rows, $c \longrightarrow$ number of 1s per row;
- Construction in time $O(nc)$;
- Total size of bitmaps is also in $O(nc)$;

Computational and storage bounds

- $n \rightarrow$ number of rows, $c \rightarrow$ number of 1s per row;
- Construction in time $O(nc)$;
- Total size of bitmaps is also in $O(nc)$;
- Given two bitmaps B_1, B_2 of compressed size $|B_1|$ and $|B_2|\ldots$

Computational and storage bounds

- $n \rightarrow$ number of rows, $c \rightarrow$ number of 1s per row;
- Construction in time $O(nc)$;
- Total size of bitmaps is also in $O(nc)$;
- Given two bitmaps B_1, B_2 of compressed size $|B_1|$ and $|B_2|$...
- AND, OR, XOR in time $O(|B_1| + |B_2|)$.

Computational and storage bounds

- $n \to$ number of rows, $c \to$ number of 1s per row;
- Construction in time $O(nc)$;
- Total size of bitmaps is also in $O(nc)$;
- Given two bitmaps B_1, B_2 of compressed size $|B_1|$ and $|B_2|\ldots$
- AND, OR, XOR in time $O(|B_1| + |B_2|)$.
- Bounds do not depend on the number of bitmaps. Implementation scales to *millions* of bitmaps.

Improving compression by sorting the table

- **RLE**, **BBC**, **WAH**, **EWAH** are order-sensitive:
  they compress sorted data better;

Improving compression by sorting the table

- **RLE**, **BBC**, **WAH**, **EWAH** are order-sensitive: **they compress sorted data better**;
- But finding the *best* row ordering is NP-hard.

Improving compression by sorting the table

- **RLE**, **BBC**, **WAH**, **EWAH** are order-sensitive: **they compress sorted data better**;
- But finding the *best* row ordering is NP-hard.
- Lexicographic sorting is
  - **fast**, even for very large tables.

Improving compression by sorting the table

- **RLE**, **BBC**, **WAH**, **EWAH** are order-sensitive: **they compress sorted data better**;
- But finding the *best* row ordering is NP-hard.
- Lexicographic sorting is
  - **fast**, even for very large tables.
  - **easy**: `sort` is a Unix staple.

Improving compression by sorting the table

- **RLE**, **BBC**, **WAH**, **EWAH** are order-sensitive: **they compress sorted data better**;
- But finding the *best* row ordering is NP-hard.
- Lexicographic sorting is
  - **fast**, even for very large tables.
  - **easy**: `sort` is a Unix staple.
- Substantial index-size reductions (often 2.5 times)

k-of-N encoding

- With L bitmaps, you can represent L values by mapping each value to **one bitmap**;

1-of-N	value
100000	cat
010000	dog
001000	dish
000100	fish
000010	cow
100000	cat
000001	pony

k-of-N encoding

1-of-N	**2-of-N**
100000	1100
010000	1010
001000	1001
000100	0110
000010	0101
100000	1100
000001	0011

- With L bitmaps, you can represent L values by mapping each value to **one bitmap**;
- Alternatively, you can represent $\binom{L}{2} = L(L-1)/2$ values by mapping each value to a **pair of bitmaps**;

k-of-N encoding

1-of-N	**2-of-N**
100000	1100
010000	1010
001000	1001
000100	0110
000010	0101
100000	1100
000001	0011

- With L bitmaps, you can represent L values by mapping each value to **one bitmap**;
- Alternatively, you can represent $\binom{L}{2} = L(L-1)/2$ values by mapping each value to a **pair of bitmaps**;
- More generally, you can represent $\binom{L}{k}$ values by mapping each value to a ***k*-tuple of bitmaps**;

k-of-N encoding

1-of-N	**2-of-N**
100000	1100
010000	1010
001000	1001
000100	0110
000010	0101
100000	1100
000001	0011

- With L bitmaps, you can represent L values by mapping each value to **one bitmap**;
- Alternatively, you can represent $\binom{L}{2} = L(L-1)/2$ values by mapping each value to a **pair of bitmaps**;
- More generally, you can represent $\binom{L}{k}$ values by mapping each value to a ***k*-tuple of bitmaps**;
- At query time, you need to load k bitmaps in a look-up for one value;

k-of-N encoding

1-of-N	**2-of-N**
100000	1100
010000	1010
001000	1001
000100	0110
000010	0101
100000	1100
000001	0011

- With L bitmaps, you can represent L values by mapping each value to **one bitmap**;
- Alternatively, you can represent $\binom{L}{2} = L(L-1)/2$ values by mapping each value to a **pair of bitmaps**;
- More generally, you can represent $\binom{L}{k}$ values by mapping each value to a ***k*-tuple of bitmaps**;
- At query time, you need to load k bitmaps in a look-up for one value;
- You trade **query-time performance** for **fewer bitmaps**;

k-of-N encoding

1-of-N	**2-of-N**
100000	1100
010000	1010
001000	1001
000100	0110
000010	0101
100000	1100
000001	0011

- With L bitmaps, you can represent L values by mapping each value to **one bitmap**;
- Alternatively, you can represent $\binom{L}{2} = L(L-1)/2$ values by mapping each value to a **pair of bitmaps**;
- More generally, you can represent $\binom{L}{k}$ values by mapping each value to a ***k*-tuple of bitmaps**;
- At query time, you need to load k bitmaps in a look-up for one value;
- You trade **query-time performance** for **fewer bitmaps**;
- Often, **fewer bitmaps translates into a smaller index, created faster**.

- A **Gray Code** (GC) minimizes bit transitions: $00, 01, \mathbf{11}, 10$

Gray-code order, when $k > 1$

- A **Gray Code** (GC) minimizes bit transitions: $00, 01, \mathbf{11}, 10$
- Pinar et al. propose to sort whole index by $<_{\mathrm{GC}}$, Gray-code ordering. Practical?

Gray-code order, when $k > 1$

- A **Gray Code** (GC) minimizes bit transitions: $00, 01, \mathbf{11}, 10$
- Pinar et al. propose to sort whole index by $<_{\mathrm{GC}}$, Gray-code ordering. Practical?
- We contribute an easy/fast way to achieve GC-like results using lexicographic sort.

- A **Gray Code** (GC) minimizes bit transitions: $00, 01, \mathbf{11}, 10$
- Pinar et al. propose to sort whole index by $<_{\mathrm{GC}}$, Gray-code ordering. Practical?
- We contribute an easy/fast way to achieve GC-like results using lexicographic sort.
- Empirical improvement in index size: typically 0–4%.

Gray-code order, when $k > 1$

- A **Gray Code** (GC) minimizes bit transitions: $00, 01, \mathbf{11}, 10$
- Pinar et al. propose to sort whole index by $<_{\mathrm{GC}}$, Gray-code ordering. Practical?
- We contribute an easy/fast way to achieve GC-like results using lexicographic sort.
- Empirical improvement in index size: typically 0–4%.
- Paper has details.

Experimental environment

- Mac Pro with 2 dual-core CPUs 2 GiB RAM (no thrashing)
- GNU GCC 4.0.2 (C++)—32-bit binaries

Experimental environment

- Mac Pro with 2 dual-core CPUs 2 GiB RAM (no thrashing)
- GNU GCC 4.0.2 (C++)—32-bit binaries
- **Source code under GPL**: http://code.google.com/p/lemurbitmapindex/ (Linux and MacOS)

- Mac Pro with 2 dual-core CPUs 2 GiB RAM (no thrashing)
- GNU GCC 4.0.2 (C++)—32-bit binaries
- **Source code under GPL**: http://code.google.com/p/lemurbitmapindex/ (Linux and MacOS)
- Mix of real and synthetic data:
  1. up to 877 M rows, 22 GB, 4 M attributes.
  2. experiments using 4–10 columns

Coding suggestions: choosing k

1. If $k > 1$, bitmaps are denser and a query processes k of them;

Coding suggestions: choosing k

1. If $k > 1$, bitmaps are denser and a query processes k of them; cost can grow with $n_i^{(k-1)/k}$ (**big jump from 1 to 2**).

Coding suggestions: choosing k

1. If $k > 1$, bitmaps are denser and a query processes k of them; cost can grow with $n_i^{(k-1)/k}$ (**big jump from 1 to 2**).
2. If $k > 1$, we (usually) get smaller indexes.

Coding suggestions: choosing k

1. If $k > 1$, bitmaps are denser and a query processes k of them; cost can grow with $n_i^{(k-1)/k}$ (**big jump from 1 to 2**).
2. If $k > 1$, we (usually) get smaller indexes.

Potentially high query-time penalty for $k > 1$, at best modest space gains.

Coding suggestions: choosing k

1. If $k > 1$, bitmaps are denser and a query processes k of them; cost can grow with $n_i^{(k-1)/k}$ (**big jump from 1 to 2**).
2. If $k > 1$, we (usually) get smaller indexes.

Potentially high query-time penalty for $k > 1$, at best modest space gains.

Default choice of k

Our index-construction algorithm handles the extremely sparse ($k = 1$) indexes nicely. $k = 1$ **looks like a good choice.**

When sorting, column order matters

- The first column(s) gain more from the sort (column 1 is primary sort key);

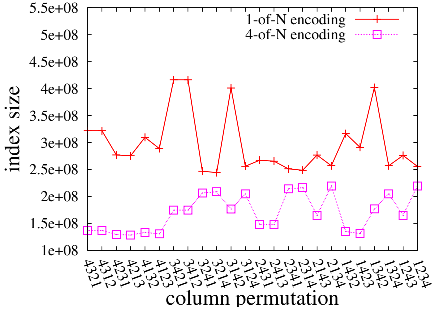
When sorting, column order matters

- The first column(s) gain more from the sort (column 1 is primary sort key);
- Conceptually, we may wish to reorder columns, eg swap columns 1 & 3.

When sorting, column order matters

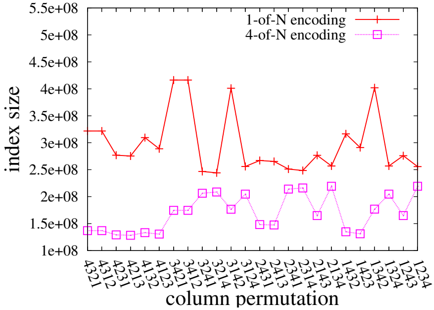
- The first column(s) gain more from the sort (column 1 is primary sort key);
- Conceptually, we may wish to reorder columns, eg swap columns 1 & 3.
- Column order is crucial!

Netflix: 24 column orderings



- The first column(s) gain more from the sort (column 1 is primary sort key);
- Conceptually, we may wish to reorder columns, eg swap columns 1 & 3.
- Column order is crucial!
- Finding the best ordering quickly remains open.

Netflix: 24 column orderings



Progress toward choosing column order

- Paper models "gain" of putting a given column first.
- Idea: order columns greedily (by max gain).

Progress toward choosing column order

- Paper models “gain” of putting a given column first.
- Idea: order columns greedily (by max gain).
- Experimentally, this approach is not promising: **the best orderings don’t seem to depend on gain.**

Progress toward choosing column order

- Paper models “gain” of putting a given column first.
- Idea: order columns greedily (by max gain).
- Experimentally, this approach is not promising: **the best orderings don’t seem to depend on gain.**
- Factors:
  - skews of columns
  - number of distinct values
  - k
  - density of column’s bitmaps

What usually works for dimension ordering?: $k=1$

For 1-of-N bitmaps, a density-based approach was okay:

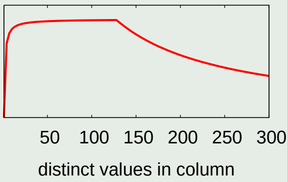
What usually works for dimension ordering?: $k=1$

For 1-of-N bitmaps, a density-based approach was okay:

Ordering rule, $k = 1$: "sparse but not too sparse"

Order columns by decreasing

$$\min\left(\frac{1}{n_i}, \frac{1 - 1/n_i}{4w - 1}\right), \text{ where}$$



- $n_i \rightarrow$ the number of distinct values in column i,
- $w \rightarrow$ the word size.

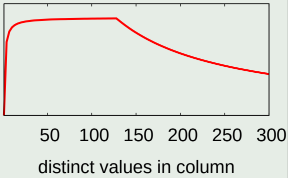
What usually works for dimension ordering?: $k=1$

For 1-of-N bitmaps, a density-based approach was okay:

Ordering rule, $k = 1$: "sparse but not too sparse"

Order columns by decreasing

$$\min\left(\frac{1}{n_i}, \frac{1 - 1/n_i}{4w - 1}\right), \text{ where}$$



- $n_i \rightarrow$ the number of distinct values in column i,
- $w \rightarrow$ the word size.

See 30–40% size reduction, merely knowing dimension sizes (n_i).

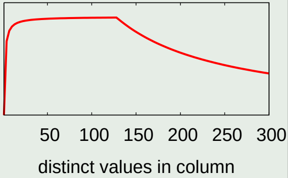
What usually works for dimension ordering?: k=1

For 1-of-N bitmaps, a density-based approach was okay:

Ordering rule, $k = 1$: "sparse but not too sparse"

Order columns by decreasing

$$\min\left(\frac{1}{n_i}, \frac{1 - 1/n_i}{4w - 1}\right), \text{ where}$$



- $n_i \rightarrow$ the number of distinct values in column i,
- $w \rightarrow$ the word size.

See 30–40% size reduction, merely knowing dimension sizes (n_i).
See also [Canahuate et al., 2006] for related work.

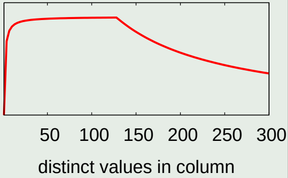
What usually works for dimension ordering?: k=1

For 1-of-N bitmaps, a density-based approach was okay:

Ordering rule, $k = 1$: "sparse but not too sparse"

Order columns by decreasing

$$\min\left(\frac{1}{n_i}, \frac{1 - 1/n_i}{4w - 1}\right), \text{ where}$$



- $n_i \rightarrow$ the number of distinct values in column i,
- $w \rightarrow$ the word size.

See 30–40% size reduction, merely knowing dimension sizes (n_i).
See also [Canahuate et al., 2006] for related work.
Situation worse for $k > 1$. Details

Future directions

- Need better **mathematical modelling** of bitmap compressed size in sorted tables;

Future directions

- Need better **mathematical modelling** of bitmap compressed size in sorted tables;
- Study the effect of **word length** (16, 32, 64, 128 bits);

Future directions

- Need better **mathematical modelling** of bitmap compressed size in sorted tables;
- Study the effect of **word length** (16, 32, 64, 128 bits);
- Apply to **Column**-oriented DBMS [Stonebraker et al., 2005];

Future directions

- Need better **mathematical modelling** of bitmap compressed size in sorted tables;
- Study the effect of **word length** (16, 32, 64, 128 bits);
- Apply to **Column**-oriented DBMS [Stonebraker et al., 2005];
- Consider encodings that can efficiently support **range** queries [Chan and Ioannidis, 1999].

Questions?

?

Canahuate, G., Ferhatosmanoglu, H., and Pinar, A. (2006).
Improving bitmap index compression by data reorganization.
http://hpcrd.lbl.gov/~apinar/papers/TKDE06.pdf (checked 2008-05-30).

Chan, C. Y. and Ioannidis, Y. E. (1999).
An efficient bitmap encoding scheme for selection queries.
In *SIGMOD'99*, pages 215–226.

Pinar, A., Tao, T., and Ferhatosmanoglu, H. (2005).
Compressing bitmap indices by data reorganization.
In *ICDE'05*, pages 310–321.

Sharma, V. (2005).
Bitmap index vs. b-tree index: Which and when?
online: http://www.oracle.com/technology/pub/articles/sharma_indexes.html.

Stonebraker, M., Abadi, D. J., Batkin, A., Chen, X., Cherniack, M., Ferreira, M., Lau, E., Lin, A., Madden, S.,

O'Neil, E., O'Neil, P., Rasin, A., Tran, N., and Zdonik, S. (2005).
C-store: a column-oriented dbms.
In *VLDB '05*, pages 553–564.

Wu, K., Otoo, E. J., and Shoshani, A. (2006).
Optimizing bitmap indices with efficient compression.
ACM Transactions on Database Systems, 31(1):1–38.

What usually works for dimension ordering?: $k > 1$

Density formula ($n_i \rightarrow \sqrt[k]{n_i}$) recommends poorly when $k > 1$. Our experiments on synthetic data give some guidance:

What usually works for dimension ordering?: $k > 1$

Density formula ($n_i \to \sqrt[k]{n_i}$) recommends poorly when $k > 1$. Our experiments on synthetic data give some guidance:

When $k > 1$, order columns by

1. descending skew
2. descending size

(And do the reverse when $k = 1$.)

What usually works for dimension ordering?: $k > 1$

Density formula ($n_i \rightarrow \sqrt[k]{n_i}$) recommends poorly when $k > 1$. Our experiments on synthetic data give some guidance:

When $k > 1$, order columns by

1. descending skew
2. descending size

(And do the reverse when $k = 1$.)

Open issues, $k > 1$

1. How do we balance skew & size factors?

What usually works for dimension ordering?: $k > 1$

Density formula ($n_i \rightarrow \sqrt[k]{n_i}$) recommends poorly when $k > 1$. Our experiments on synthetic data give some guidance:

When $k > 1$, order columns by

1. descending skew
2. descending size

(And do the reverse when $k = 1$.)

Open issues, $k > 1$

1. How do we balance skew & size factors?
2. What **other properties** of the histograms are needed?

Gray-code order

Lex.	order	
0	1	1
0	1	1
1	**0**	1
1	0	1
1	**1**	**0**
1	1	0
1	1	**1**
1	1	1
1	1	1

Gray-code		
0	1	1
0	1	1
1	1	**0**
1	1	0
1	1	**1**
1	1	1
1	1	1
1	**0**	1
1	0	1

- **Gray-code** (GC) order is an alternative to lexicographical order (defined only for bit arrays);

Gray-code order

Lex.	order	
0	1	1
0	1	1
1	**0**	1
1	0	1
1	**1**	**0**
1	1	0
1	1	**1**
1	1	1
1	1	1

Gray-code		
0	1	1
0	1	1
1	1	**0**
1	1	0
1	1	**1**
1	1	1
1	1	1
1	**0**	1
1	0	1

- **Gray-code** (GC) order is an alternative to lexicographical order (defined only for bit arrays);
- May improve compression more than lex. sort ($k > 1$);

Lex. order		
0	1	1
0	1	1
1	**0**	1
1	0	1
1	**1**	**0**
1	1	0
1	1	**1**
1	1	1
1	1	1

Gray-code		
0	1	1
0	1	1
1	1	**0**
1	1	0
1	1	**1**
1	1	1
1	1	1
1	**0**	1
1	0	1

- **Gray-code** (GC) order is an alternative to lexicographical order (defined only for bit arrays);
- May improve compression more than lex. sort ($k > 1$);
- [Pinar et al., 2005] process a materialized bitmap index.

Gray-code order

Lex. order		
0	1	1
0	1	1
1	**0**	1
1	0	1
1	**1**	**0**
1	1	0
1	1	**1**
1	1	1
1	1	1

Gray-code		
0	1	1
0	1	1
1	1	**0**
1	1	0
1	1	**1**
1	1	1
1	1	1
1	**0**	1
1	0	1

- **Gray-code** (GC) order is an alternative to lexicographical order (defined only for bit arrays);
- May improve compression more than lex. sort ($k > 1$);
- [Pinar et al., 2005] process a materialized bitmap index.
- Slow, if uncompressed index does not fit in RAM.

Gray-code order

Lex. order		
0	1	1
0	1	1
1	**0**	1
1	0	1
1	**1**	**0**
1	1	0
1	1	**1**
1	1	1
1	1	1

Gray-code		
0	1	1
0	1	1
1	1	**0**
1	1	0
1	1	**1**
1	1	1
1	1	1
1	**0**	1
1	0	1

- **Gray-code** (GC) order is an alternative to lexicographical order (defined only for bit arrays);
- May improve compression more than lex. sort ($k > 1$);
- [Pinar et al., 2005] process a materialized bitmap index.
- Slow, if uncompressed index does not fit in RAM.
- GC order is not supported by DBMSes or Unix utilities.

Size improvement is small (usually $< 4\%$), but it's essentially free:

1. What Pinar et al. do: expensive GC sort after encoding
   eg: [Tax, Cat, Girl, Cat] $\rightarrow$ **sort([1100, 0110, 1001, 0110])**;

Gray-code sorting, cheaply

Size improvement is small (usually $< 4\%$), but it's essentially free:

1. What Pinar et al. do: expensive GC sort after encoding
   eg: [Tax, Cat, Girl, Cat] $\rightarrow$ **sort([1100, 0110, 1001, 0110])**;
2. Instead, sort the table lexicographically;
   eg: [Tax, Cat, Girl, Cat] $\rightarrow$ [Cat, Cat, Girl, Tax]

Gray-code sorting, cheaply

Size improvement is small (usually $< 4\%$), but it's essentially free:

1. What Pinar et al. do: expensive GC sort after encoding
   eg: [Tax, Cat, Girl, Cat] $\rightarrow$ **sort([1100, 0110, 1001, 0110])**;
2. Instead, sort the table lexicographically;
   eg: [Tax, Cat, Girl, Cat] $\rightarrow$ [Cat, Cat, Girl, Tax]
3. Map ordered values to k-tuples of bitmaps ordered as Gray codes: Cat: **0011**, Dog: **0110**, Girl: **0101**, Tax: **1100**;

   Lex ascending sequence: Cat, Dog, Girl, Tax.
   GC ascending sequence: 0011, 0110, 0101, 1100 for codes

Gray-code sorting, cheaply

Size improvement is small (usually $< 4\%$), but it's essentially free:

1. What Pinar et al. do: expensive GC sort after encoding
   eg: [Tax, Cat, Girl, Cat] $\rightarrow$ **sort([1100, 0110, 1001, 0110])**;
2. Instead, sort the table lexicographically;
   eg: [Tax, Cat, Girl, Cat] $\rightarrow$ [Cat, Cat, Girl, Tax]
3. Map ordered values to k-tuples of bitmaps ordered as Gray codes: Cat: **0011**, Dog: **0110**, Girl: **0101**, Tax: **1100**;

   Lex ascending sequence: Cat, Dog, Girl, Tax.
   GC ascending sequence: 0011, 0110, 0101, 1100 for codes
   eg: [Cat, Cat, Girl, Tax] $\rightarrow$ **[0011, 0011, 0101, 1100]**
   (generates a GC-sorted result without expensive GC sorting).

Gray-code sorting, cheaply

Size improvement is small (usually $< 4\%$), but it's essentially free:

1. What Pinar et al. do: expensive GC sort after encoding
   eg: [Tax, Cat, Girl, Cat] $\rightarrow$ **sort([1100, 0110, 1001, 0110])**;
2. Instead, sort the table lexicographically;
   eg: [Tax, Cat, Girl, Cat] $\rightarrow$ [Cat, Cat, Girl, Tax]
3. Map ordered values to k-tuples of bitmaps ordered as Gray codes: Cat: **0011**, Dog: **0110**, Girl: **0101**, Tax: **1100**;

   Lex ascending sequence: Cat, Dog, Girl, Tax.
   GC ascending sequence: 0011, 0110, 0101, 1100 for codes
   eg: [Cat, Cat, Girl, Tax] $\rightarrow$ **[0011, 0011, 0101, 1100]**
   (generates a GC-sorted result without expensive GC sorting).
4. Easily extended for > 1 columns.

Gray-code sorting, cheaply

Size improvement is small (usually $< 4\%$), but it's essentially free:

1. What Pinar et al. do: expensive GC sort after encoding
   eg: [Tax, Cat, Girl, Cat] $\rightarrow$ **sort([1100, 0110, 1001, 0110])**;
2. Instead, sort the table lexicographically;
   eg: [Tax, Cat, Girl, Cat] $\rightarrow$ [Cat, Cat, Girl, Tax]
3. Map ordered values to k-tuples of bitmaps ordered as Gray codes: Cat: **0011**, Dog: **0110**, Girl: **0101**, Tax: **1100**;

   Lex ascending sequence: Cat, Dog, Girl, Tax.
   GC ascending sequence: 0011, 0110, 0101, 1100 for codes
   eg: [Cat, Cat, Girl, Tax] $\rightarrow$ **[0011, 0011, 0101, 1100]**
   (generates a GC-sorted result without expensive GC sorting).
4. Easily extended for > 1 columns.

In our tests, this is as good as a Gray-code bitmap index sort [Pinar et al., 2005], but technically much easier.