ADM4690—Supply Chain Management
Risk Pooling Game—introduction

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1 The Risk Pooling Game

In the text-attached game, you are asked to simultaneously manage both a system with risk pooling (a centralized system) and a system without risk pooling (a decentralized system). The system records the profits of both systems, so you can compare the performance.

We explain the inventory models used in the game below:

- In the **Centralized Warehouse**, the supply chain consists of one supplier serving a warehouse, which in turn serves three retailers. The lead time between the supplier and the warehouse is 2. The lead time between the warehouse and the retailers is again 2. Inventory decisions at the warehouses and the three retailers are made by s single decision -maker whose sole objective is to minimize system-wide cost.
- In the **Decentralized Warehouse**, the supply chain consists of one supplier serving three three retailers directly. The lead time between the supplier and the retailers is 4. Inventory decisions at the three retailers are made by three independent decision-makers whose objective is to minimize his own cost.
- Each retailer is facing a demand randomly generalized by the computer
  1. The demand is normally distributed with changeable mean and standard deviation—the default value is $\mu = 25$ and $\sigma = 10$.
  2. You can also control the correlation of demands among different retailers.
- The initial inventories are also changeable subject to certain constraints—the default values can be seen directly from the scree.

  The constraints for both systems are
  1. Each retailer has the same initial inventory level
  2. Inventory-in-transit from the warehouse to the retailer must be the same for each of the retailers
  3. Inventory-in-transit from the warehouse to the retailer must be the same for both periods

In the centralized system

1. Warehouse inventory:
2. Store inventory:
3. Inventory to store:
4. Inventory to warehouse:

In the decentralized system

1. Store inventory
2. Inventory to store

- **Fill rate** is equal to the fraction of demand met over total demand—which is the **service level**.
- Cost structure
  1. There is a changeable holding cost per item per period—the default is 1.5
  2. There is a changeable purchase cost per item—the default is 1.5
  3. There is a changeable revenue per item—the default is 20
- Inventory Policy: There are policies available in the software, one is the (s, S) policy, and another is the periodic review policy.
  1. (s, S) policy:

$$s = \text{ROP} \quad = \quad \mu_D \mu_L + z_\alpha \sqrt{\mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2} \tag{1}$$

  2. periodic review policy:

$$I_{\max} \quad = \quad \mu_D (r + \mu_L) + z_\alpha \sqrt{(r + \mu_L)\sigma_D^2 + \mu_D^2 \sigma_L^2}$$

- Therefore in the centralized system, the single decision-maker needs to make two decisions:
  1.

$$\begin{aligned} &\underline{\text{Amount to order at warehouse}} \\ &= s - \underline{\text{warehouse echelon inventory}} \\ &= s - \underline{\text{inventory-in-transit to warehouse}} - \underline{\text{inventory at warehouse}} - \\ &\underline{\text{inventory-in-transit from warehouse to retailers}} - \underline{\text{inventory at retailers}} \end{aligned}$$

  In the default setting:

$$s = 12\mu_D + 3.46(1.33)\sigma_D$$

  where D is the demand at one retailer. This follows form the above formula (2), indeed,

$$\begin{aligned} s = \text{ROP} \quad &= \quad \mu_{D^e} \mu_{L^e} + z_\alpha \sqrt{\mu_{L^e} \sigma_{D^e}^2 + \mu_{D^e}^2 \sigma_{L^e}^2} \\ &= \quad (3\mu_D)(4) + z_\alpha \sqrt{(4)(3\sigma_D)} \\ &= \quad 12\mu_D + 2\sqrt{3} z_\alpha \sigma_D \\ &\approx \quad 12\mu_D + (3.46 \times 1.33)\sigma_D \end{aligned}$$

  where $z_\alpha = 1.33$ implies the service level is $1 - \alpha \approx 1 - 0.10 = 0.90$, i.e., 90%-service level.

2.

$$\underline{\text{Allocation at each retailer}}$$
$$= s - \underline{\text{inventory at retailer}} - \underline{\text{inventory-in-transit from warehouse to retailer}}$$

- Therefore in the decentralized system, there is one decision for each retailer needed to make:

$$\underline{\text{Amount to order at retailer}}$$
$$= s - \underline{\text{inventory at retailer}} - \underline{\text{inventory-in-transit from warehouse to retailer}}$$

In the default setting:

$$\begin{aligned} s = \text{ROP} &= \mu_D \mu_L + z_\alpha \sqrt{\mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2} && (2) \\ &= 4\mu_D + z_\alpha \sqrt{4}\sigma_D && (3) \\ &= 4\mu_D + 2(1.47)\sigma_D && (4) \end{aligned}$$

where $z_\alpha = 1.47$ implies the service level is $1 - \alpha \approx 1 - 0.07 = 0.93$, i.e., 93%-service level.

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$$\text{Profit} = \text{Revenue} - \text{COGS} - \text{Holding}$$

where COGS is the cost of goods sold = number of item sold $\times$ purchasing cost per item.