Supply Chain Management: Contract

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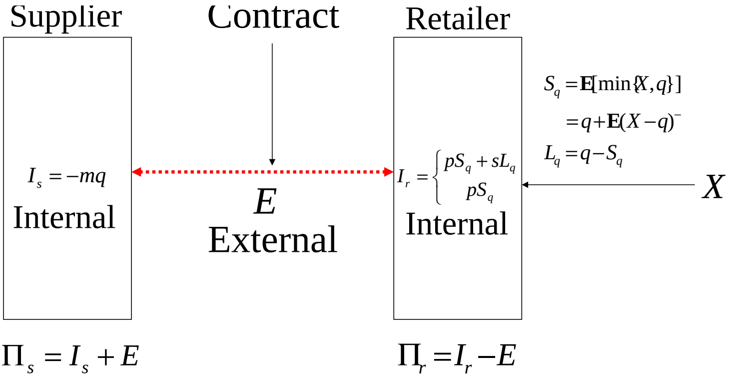
Section 1

Introduction

Supply Chain Contract

- Supply Chain Contracts are agreement between buyer and supplier on issues like
  - Pricing and volume discounts.
  - Minimum and maximum purchase quantities.
  - Delivery lead times.
  - Product or material quality.
  - Product return polices.
- We will use the Newsboy model to address the supply chain contract. Earlier we model the Newsboy problem as a cost-minimization problem. It can also be equivalently modeled as a profit-maximization problem, which is more convenient for the discussion of supply chain contract.

- The set up: To set up a uniform framework for discussion of supply chain contracts, we introduce the following notations.



$$\overbrace{\Pi = I_s + I_r}$$

An Illustrative Example of Supply Chain Contract II

- The profit of the retailer contains two parts: the internal part and the external part.

$$\max_q \; \Pi_r = I_r - E$$

- The profit of the supplier contains two parts: the internal part and the external part.

$$\max_q \; \Pi_r = I_s + E$$

- The profit of the system contains only internal parts:

$$\max_q \; \Pi_s + \Pi_r \;\; = \;\; I_s + I_r$$

Section 2

Price-only contract

The wholesale price contract I

- We first consider a *wholesale price contract*. There is one supplier and a retailer. The contract between the supplier and retailer can be modeled as a Stackelberg Game:
  1. The supplier offers the retailer an contract.
  2. If the retailer rejects, then the game ends.
  3. Otherwise, the retailer submits an order q to the supplier.
  4. The supplier produces the order at unit manufacturing price m and delivers to the retailer at unit whole-sale price w
  5. Season demands occurs. Unsold items are salvaged at unit price s.
  6. The payments are transferred between the two firms based on the agreed contract.

The wholesale price contract II

- The following assumption is reasonable:

Assumption

$$p > w > m > s$$

i.e., the retail price is greater than both the wholesale and salvage prices.

- Using the previous framework, the *wholesale* contract is specified by

$$E = wq$$

Three different points of view

- Retailer's View: The retailer wants to maximize its own profit.
- Supplier's View: The supplier wants to maximize its own profit.
- System's View: The system wants to maximize the overall profit.

Retailer's Problem I

- The retailer faces uncertain demand X with cumulative distribution function F_X and density function f_X.
- The retailer orders q (a decision variable) units from the supplier at wholesale price w.
- The retailer sells at retail price p per unit to customers.
- The salvage price is s.
- The realized expected sales be

$$S_q = \mathbb{E}_X[\min\{X, q\}] = q + \mathbb{E}_X[\min\{X - q, 0\}] = q + \mathbb{E}_X(X - q)^-$$

- The expected leftover inventory

$$L_q = \mathbb{E}_X[q - \min\{X, q\}] = q - S_q$$

Retailer's Problem II

- The retailer wants to choose an order quantity q to maximize her expected profit and in this case $I_r = pS_q + sL_q$:
- Therefore the retailer's problem is the following optimization problem

$$
\begin{aligned}
\max_q \; \Pi_r &= I_r - E \\
&= \left(\underbrace{pS_q}_{\text{expected revenue}} + \underbrace{sL_q}_{\text{expected salvage value}} \right) - wq \\
&= (p-s)S_q + (s-w)q \\
&= (p-s)\mathbb{E}_X(X-q)^- + (p-w)q
\end{aligned}
$$

Retailer's Problem III

- The optimal order quantity for the retailer is

$$q_r = F_X^{-1}\left(\frac{p-w}{p-s}\right)$$

Supplier's Problem

- The supplier produces q units to deliver to the retailer at wholesale price w.
- The retailer wants to choose q to maximize his expected profit:

$$\max_q \; \Pi_s = I_s + E = -mq + wq = (w-m)q$$

- The optimal order quantity q_s for the supplier is as much as possible under its capacity constraints.

System's Problem

- The system wants to choose q to maximize its expected overall profit:

$$\begin{aligned}\max_q \,[\Pi_s + \Pi_r] &= (p-s)S_q + (s-w)q + wq \\ &= (p-s)S_q + sq \\ &= (p-s)\mathbb{E}_X(X-q)^- + (p-m)q\end{aligned}$$

- The optimal order quantity for the system is

$$q = F_X^{-1}\left(\frac{p-m}{p-s}\right)$$

Compare the three views

- Let us compare the optimal order quantities between the retailer and the system, assuming the supplier has unlimited supplying capacity.
- It can shown that the system optimal order quantity is always greater than that of the retailer optimal:

$$q = F_X^{-1}\left(\frac{p-m}{p-s}\right) > q_r = F_X^{-1}\left(\frac{p-w}{p-s}\right)$$

- The claim above follows by noting that $P(X \leq q_r) = \frac{p-w}{p-s} \underbrace{<}_{\text{Assumption } 1} \frac{p-m}{p-s} = P(X \leq q)$.

Double marginalization I

- Note that $q = q_r$ if and only if $w = m$. This fact illustrates the so-called *double marginalization* phenomenon, first identified by Spengler (1950), of the whole-sale price contract: *system optimal can be achieved only if the supplier earns no profit—this is impossible in reality!*.
- To summarize, the wholesale price contract has the following properties:
  - Simple
  - Double marginalization
- The reason for double marginalization is: the two agents, the supplier and the retailer are both *selfish* (a rational behavior, what do you think!) players in this game. They seek to maximize their own profits without caring for the system performance.

Double marginalization II

- Now the question arises on whether there exists supply chain contract which achieves system optimal under the assumption that both supplier and retailer are selfish. A supply chain is called *coordinated* if there exists a supply chain contract that is system optimal. We will discuss some coordinated supply chain contracts below.

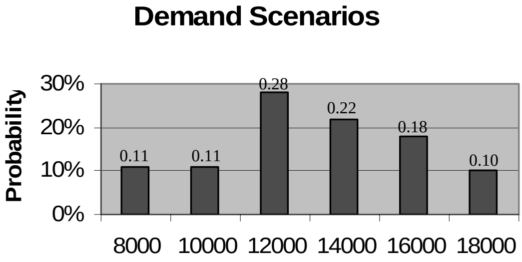
Subsection 1

A Case—Swimsuit Production

- Given a simple one-supplier-one-retailer supply chain model with the cost structures:

$$
\begin{aligned}
m &= \$35: \text{ production unit cost} \\
w &= \$80: \text{ supplier wholesale price} \\
p &= \$125: \text{ retailer selling price} \\
s &= \$20: \text{ retailer salvage price}
\end{aligned}
$$

- The demand facing the supply chain is given below:



A Case—Swimsuit Production II

- Demand in table

X	P(X)
8,000	0.11
10,000	0.11
12000	0.28
14,000	0.22
16,000	0.18
18,000	0.10

- What are the expected optimal profits and order quantities for the supplier, the retailer and the system?

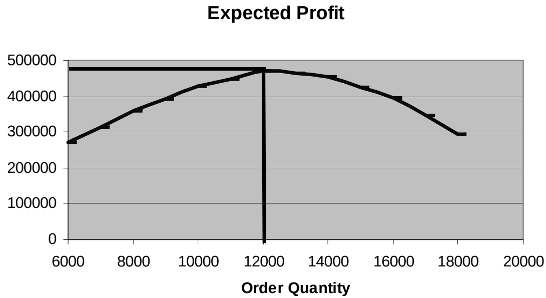
Analysis of the Case I

- Retailer's optimal order quantity is 12,000 units (also showed in the graph below), obtained as

$$\Pr(X \le q_r) \ge \frac{p-w}{p-s} = \frac{125-80}{125-20} = \frac{45}{105} \approx 0.4286$$

So

$$q_r = 12,000$$



Analysis of the Case II

- Retailer's expected profit is \$470,700, obtained as

$$\begin{aligned}\Pi_r &= (p-s)\mathbb{E}_X(X-q)^- + (p-w)q \\ &= 105\mathbb{E}_X(X-12000)^- + 45(12000) \\ &= 105[0.11 \times (8000-12000) + 0.11 \times (10000-12000)] \\ &\quad +45(12000) = 470,700\end{aligned}$$

- Supplier profit is \$540,000, obtained as

$$\Pi_s = (w-m)q_r = (80-35)12000 = 540,000$$

- Supply Chain Profit is \$1010,700, obtained as

$$\Pi = \Pi_r + \Pi_s = 470,700 + 540,000 = 1010,700$$

Analysis of the Case III

- However, the optimal system order quantity is

$$\mathtt{Pr}(X \leq q) \geq \frac{p-m}{p-s} = \frac{125-35}{125-20} = \frac{90}{105} \approx 0.86$$

So

$$q = 16,000$$

with a total profit of 1014,500, obtained as

$$\begin{aligned}
& (p-s)\mathbb{E}_X(X-q)^- + (p-m)q \\
= \; & 105[0.11 \times (8000 - 16000) + 0.11 \times (10000 - 16000) \\
& +0.28 \times (12000 - 16000) + 0.22 \times (14000 - 16000)] \\
& +90(16,000) \\
= \; & 105(-3100) + 90(16000) = 1114,500
\end{aligned}$$

Analysis of the Case IV

- This means the wholesale price contract did not achieve the system optimal. Now the question is: *Is there anything that the retailer and supplier can do to increase the profit of both?*

Section 3

The Buy-back Contract

The Buy-back Contract I

- The Buy-back Contract: The contract is specified by three parameters (q, w, b), where $b > s$. The supplier charges the retailer w per unit purchased, but pays the retailer b per unit for any unsold items. Therefore
$$E = wq - bL_q$$
- Now the new profit distribution picture is given by (and in this case $I_r = pS_q$: no need to salvage the leftover at the retailer, the supplier will buy back them, and $I_s = -mq + sq$: since the supplier will salvage the unsold at price s

The Buy-back Contract II

-

$$
\begin{array}{|lcl|}
\hline
\Pi_r & = & I_r - E = I_r - (wq - bL_q) \\
 & = & (p-b)\mathbb{E}_X(X-q)^- + (p-w)q \\
 & & \\
\hline
 & & \\
\Pi_s & = & (s-m)q + wq - bL_q \\
 & = & (b-s)\mathbb{E}_X(X-q)^- + (w-m)q \\
 & & \\
\hline
 & & \\
\Pi_s + \Pi_r & = & (p-s)\mathbb{E}_X(X-q)^- + (p-m)q \\
 & & \\
\hline
\end{array}
$$

The Buy-back Contract III

- Under this contract, the system order quantity is

$$q = F_X^{-1}\left(\frac{p-m}{p}\right).$$

- Now the retailer has the incentive to order more, his optimal order quantity becomes

$$q_r = \underbrace{F_X^{-1}\left(\frac{p-w}{p-b}\right)}_{\text{buy-back order}} > \underbrace{F_X^{-1}\left(\frac{p-w}{p-s}\right)}_{\text{wholesale order}}$$

The Buy-back Contract IV

- Given all the parameters $p > w > m > s$, we can choose b such that the retailer order up to the optimal system quantity q:

$$
\begin{aligned}
\frac{p-w}{p-b} &= \frac{p-m}{p} \\
&\Downarrow \\
b &= \frac{p(w-m)}{p-m}
\end{aligned}
$$

The Swimsuit Production Case—continued I

- Suppose the supplier offers to buy unsold swimsuits from the retailer for $b = \$55$. Under this buy-back contract, we want to know what the expected optimal profits and order quantities for the supplier, the retailer and the system are?
- We can apply the formulas from the the previous discussion answer the questions above.
  - Retailer's optimal order quantity is 14,000 units (also showed in the graph below), obtained as

$$
\begin{aligned}
\texttt{Pr}(X \leq q_r) &= \frac{p-w}{p-b} = \frac{125-80}{125-55} \\
&= \frac{45}{70} \approx 0.643
\end{aligned}
$$

The Swimsuit Production Case—continued II

- So

$$q_r = 14,000,$$

- and

$$\begin{aligned}
\mathbb{E}_X(X-q)^- &= \mathbb{E}_X(X-14000)^- \\
&= 0.11 \times (8000-14000) \\
&\quad +0.11 \times (10000-14000) \\
&\quad +0.28 \times (12000-14000) \\
&= -1660
\end{aligned}$$

- Retailer's expected profit is \$513,800, obtained as

$$\begin{aligned}
\Pi_r &= (p-b)\mathbb{E}_X(X-q)^- + (p-w)q \\
&= 70(-1660) + 45(14000) \\
&= 513,800
\end{aligned}$$

The Swimsuit Production Case—continued III

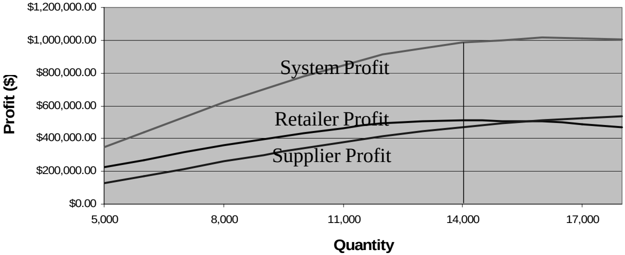
- Supplier profit is $538,700, obtained as

$$\begin{aligned}\Pi_s &= (b-s)\mathbb{E}_X(X-q_r)^- + (w-m)q_r \\ &= (55-20)(-1660) + (45)14000 = 571,900\end{aligned}$$

- Supply Chain Profit is $1085,700, obtained as

$$\Pi = \Pi_r + \Pi_s = 513,800 + 571,900 = 1085,700$$

The Swimsuit Production Case—continued IV



Section 4

The Revenue-sharing Contract

The Revenue-sharing Contract I

- The Revenue-sharing Contract: The contract is specified by three parameters (q, w, φ), where $\varphi > \frac{w}{p}$. The supplier charges the retailer at a lower wholesale price w per unit purchased, and the retailer gives $1 - \varphi$ percent of his revenue to the supplier. Therefore assume the retailer does not share the salvage revenue with the retailer

$$E = wq + (1 - \varphi)pS_q$$

- Now the new profit distribution picture is given by (and in this case $I_r = pS_q + sL_q$ and $I_s = -mq$)

The Revenue-sharing Contract II

-

$$
\begin{array}{|lcl|}
\hline
\Pi_r & = & I_r - E = I_r - wq - (1-\varphi)pS_q \\
 & = & (\varphi p - s)\mathbb{E}_X(X-q)^- + (\varphi p - w)q \\
\hline
\Pi_s & = & I_s + E = -mq + wq + (1-\varphi)pS_q \\
 & = & (1-\varphi)p\mathbb{E}_X(X-q)^- + (w - m + (1-\varphi)p)q \\
\hline
\Pi_s + \Pi_r & = & (p-s)\mathbb{E}_X(X-q)^- + (p-m)q \\
\hline
\end{array}
$$

The Revenue-sharing Contract III

- Under this contract, the system order quantity is

$$q = F_X^{-1}\left(\frac{p-m}{p-s}\right).$$

- Now the retailer has the incentive to order more, his optimal order quantity becomes

$$q_r = \underbrace{F_X^{-1}\left(\frac{\varphi p - w}{\varphi p - s}\right)}_{\text{revenue-sharing order}} > \underbrace{F_X^{-1}\left(\frac{p-w}{p-s}\right)}_{\text{wholesale order}}$$

The Revenue-sharing Contract IV

- Given all the parameters $p > w > m > s$, we can choose φ such that the retailer order up to the optimal system quantity q:

$$
\begin{aligned}
\frac{\varphi p - w}{\varphi p - s} &= \frac{p - m}{p - s} \\
&\Downarrow \\
\varphi &= \frac{w}{m}\left(s + \frac{(p-s)(w-s)}{m-s}\right)
\end{aligned}
$$

The Swimsuit Production Case—continued I

- Suppose the supplier offers to decrease the wholesale price to $2 = \$60$, and in return, the retailer provides $1 - \varphi = 15\%$ of the revenue to the supplier. Under this revenue-sharing contract, we want to know what the expected optimal profits and order quantities for the supplier, the retailer and the system are?
- We can apply the previous formulas to answer the questions above.
    - Retailer's optimal order quantity is 14,000 units (also showed in the graph below), obtained as

$$
\begin{aligned}
\texttt{Pr}(X \leq q_r) &\geq \frac{\varphi p - w}{\varphi p - s} \\
&= \frac{0.85(125) - 60}{0.85(125) - 20)} \\
&= \frac{46.25}{86.25} \approx 0.536
\end{aligned}
$$

The Swimsuit Production Case—continued II

- So

$$q_r = 14,000,$$

- and

$$\begin{aligned}
\mathbb{E}_X(X-q)^- &= \mathbb{E}_X(X-14000)^- \\
&= 0.11 \times (8000-14000) \\
&\quad +0.11 \times (10000-14000) \\
&\quad +0.28 \times (12000-14000) \\
&= -1660
\end{aligned}$$

- Retailer's expected profit is \$504,325, obtained as

$$\begin{aligned}
\Pi_r &= (\varphi p - s)\mathbb{E}_X(X-q)^- + (\varphi p - w)q \\
&= 86.25(-1660) + 46.25(14000) \\
&= 504,325
\end{aligned}$$

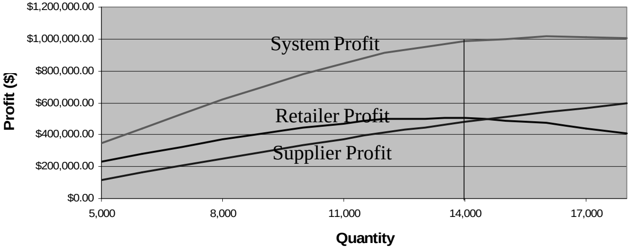
The Swimsuit Production Case—continued III

- Supplier profit is $581,375, obtained as

$$\begin{aligned}\Pi_s &= (1-\varphi)p\mathbb{E}_X(X-q)^- + (w-m+(1-\varphi)p)q \\ &= 18.75(-1660) + (43.75)14000 = 581,375\end{aligned}$$

- Supply Chain Profit is $985,700, obtained as

$$\Pi = \Pi_r + \Pi_s = 504,325 + 581,375 = 1085,700$$



Section 5

Some other contracts

Some other contracts I

- There are some other contracts widely used in practice, we briefly talk about some of them without going to details.
  - The Quantity-flexibility Contract: The contract is specified by three parameters (q, w, δ). The supplier charges the retailer wholesale price w per unit purchased, and the retailer is compensated by the supplier a full refund of unsold items $(w-s)\mathbb{E}\left[\min\{(q-X)^+, \delta q\}\right]$ as long as the number of leftovers is no more than a certain quantity δq. Therefore

$$E = wq - (w-s)\mathbb{E}\left[\min\{(q-X)^+, \delta q\}\right]$$

Some other contracts II

- The Sales-rebate Contract: The contract is specified by four parameters (q, w, r, t). The supplier charges the retailer wholesale price w per unit purchased, but then gives the retailer an r rebate per unit sold above a threshold t. Therefore

$$E = \begin{cases} wq & \text{if } q \geq t \\ (w-r)q + r\left(t + \int_t^q F(y)dy\right) & \text{if } q > t \end{cases}$$

- The Quantity-discount Contract: The contract is specified by parameters $(q, w(q))$. The supplier charges the retailer $w(q)$ per unit purchased depending on how much is ordered. Therfore

$$E = w(q)q$$

Section 6

Discusion and comparison

Discussions I

- All the contracts try to coordinate the newsboy by dividing the supply chain's profits based on different criteria as so to share risk.

Contracts	order quantity	retailer profit	supplier profit	System profit
Wholesale	12,000	470,700	540,00	1010,700
Buy-Back	14,000	513,800	571,900	1085,700
Revenue-Sharing	14,000	504,325	581,375	1085,700
Global	16,000			1114,500

Discussions II

- Revenue-sharing/buy-back and quantity-flexibility gives the retailer some downside protection: if the demand is lower than q, the retailer gets some refund.
- The sales-rebate gives the retailer some upside incentive: if the demand is greater than q, the retailer effectively purchases the units sold above t for less than their cost of production.
- The quantity-discount adjusts the retailer's marginal cost curve so that the supplier earns progressively less on each unit.

- The cost to administrator is different.
- The risk associated with each contract is different.