1. Every month, the amount of money invested in the stock XCO can double with probability $\frac{1}{4}$, stay the same with probability $\frac{1}{4}$, or halve with probability $\frac{1}{2}$. If you invest $100 today on XCO, after two months, and assuming you cannot sell your stock investment during this period:

(a) What is the probability distribution of the net money value of your investment?
(b) What is the average or expected net money value of your investment?
(c) What is the standard deviation of the net money value of your investment?

Answer:

(a) To derive the probability distribution of the net money value of the investment after 2 months, it is useful to construct a tree diagram.

With this tree we can now state the probability distribution of the money value of the investment after 2 months.
<table>
<thead>
<tr>
<th>Outcome (x)</th>
<th>Probability (P(x))</th>
<th>xP(x)</th>
<th>x² P(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$400</td>
<td>1/16</td>
<td>$25</td>
<td>10000</td>
</tr>
<tr>
<td>$200</td>
<td>2/16</td>
<td>$25</td>
<td>5000</td>
</tr>
<tr>
<td>$100</td>
<td>5/16</td>
<td>$31.25</td>
<td>3125</td>
</tr>
<tr>
<td>$50</td>
<td>2/8</td>
<td>$12.5</td>
<td>625</td>
</tr>
<tr>
<td>$25</td>
<td>1/4</td>
<td>$6.25</td>
<td>156.25</td>
</tr>
<tr>
<td>Total</td>
<td>1</td>
<td>$100</td>
<td>18906.25</td>
</tr>
</tbody>
</table>

(b) The average or expected net money value of your investment after 2 months is $100, so on average you neither win or lose with this investment.

(c) The standard deviation of the net money value of your investment after 2 months is

\[
\sqrt{18906.25 - (100)^2} = 94.37
\]

2. In a large metropolitan area, past records revealed that 30 percent of all the high school graduates go to college. From 15 graduates selected at random, what is the probability that:

(a) exactly 8 will go to college?
(b) exactly 1 will go to college?

**Answer:** The number of graduates that go to college out of the 15 randomly selected is a Binomial random variable with parameters \( n = 15 \) and \( p = 0.30 \). Using the \( n = 15 \) Table on page 834 of the Textbook we get

(a) \( P(8) = 0.035 \)
(b) \( P(1) = 0.031 \)

3. David’s gasoline station offers 4 cents off per gallon if the customer does not use a credit card and pays in cash. Past evidence indicates that 40% of all customers pay in cash. During a one-hour period twenty customers buy gasoline at this station.

(a) What is the probability that no more than seven customers pay in cash?
(b) What is the probability that more than ten customers do not pay in cash?

**Answer:** The number of customers that pay gasoline with cash out of the 20 randomly buying gas in a one-hour period is a Binomial random variable with parameters \( n = 20 \) and \( p = 0.40 \). Using the \( n = 15 \) Table on page 834 of the Textbook we get

(a)

\[
P(X \leq 7) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7)
= 0.000 + 0.000 + 0.0003 + 0.012 + 0.035 + 0.075 + 0.124 + 0.166
= 0.415
\]

(b) \( P(\text{more than 10 customers do not pay in cash}) = P(9 \ or \ less \ customers \ pay \ in \ cash) \)

\[
= 0.000 + 0.000 + 0.0003 + 0.012 + 0.035 + 0.075 + 0.124 + 0.166 + 0.180 + 0.160
= 0.755
\]