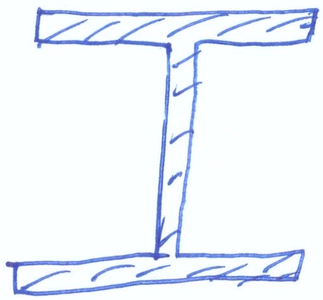
Elastic and Plastic Section Properties of



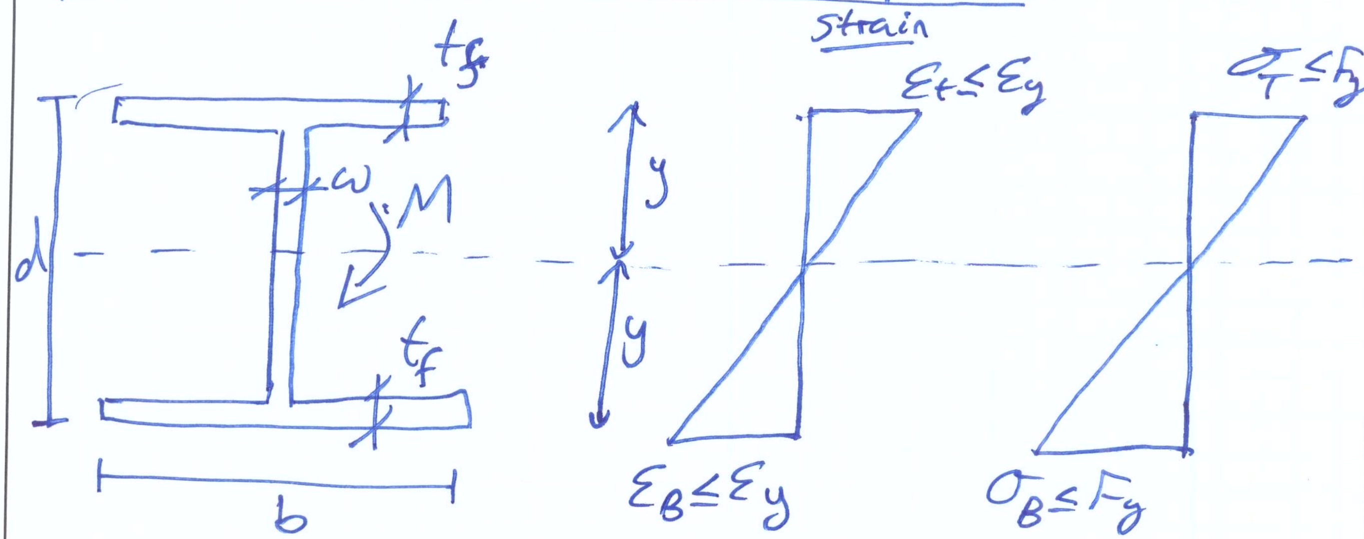
"I" shape
(W section)

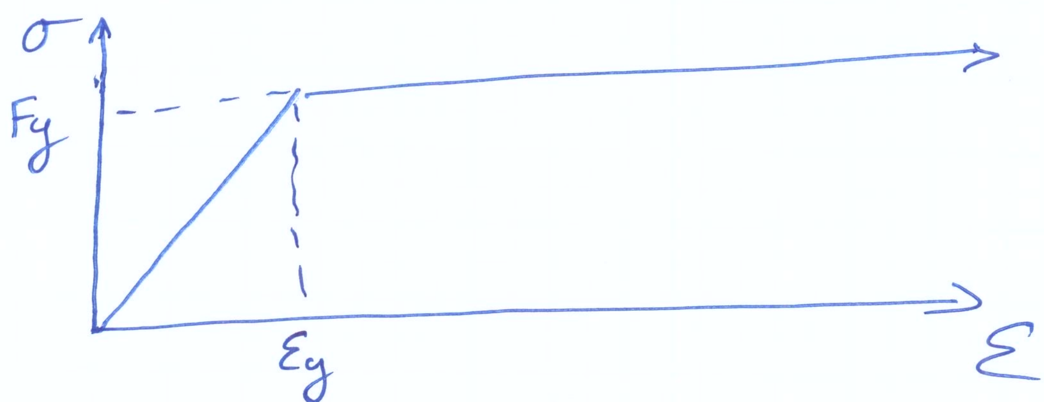
S_x, M_y

Z_x, M_p

k

Elastic + Plastic Section Properties





- Linear strain
- Linear Stress

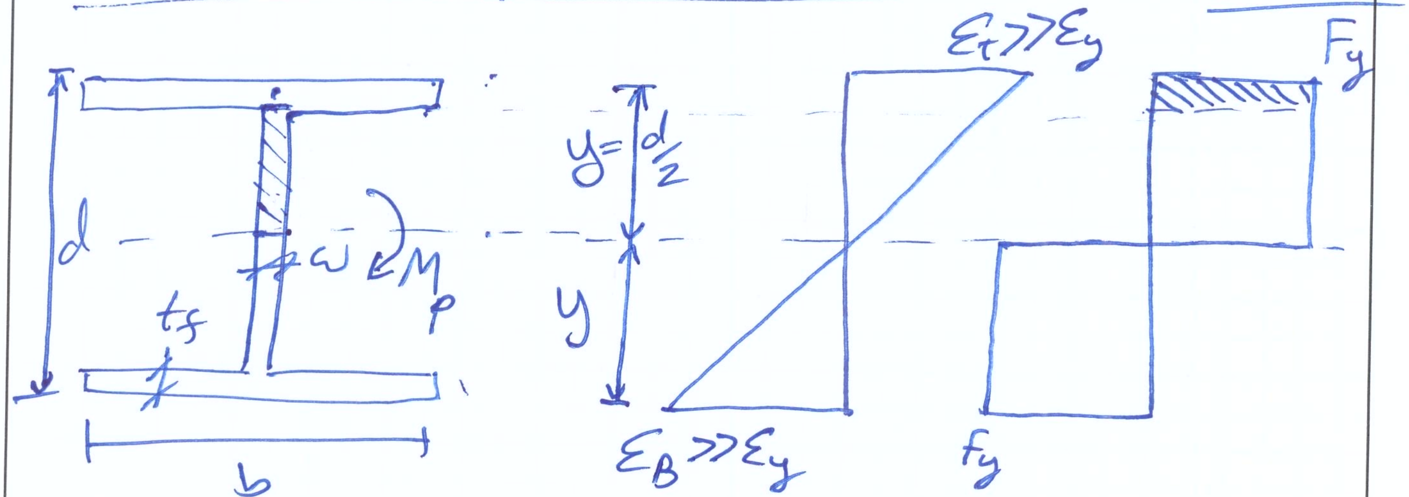
} Elastic

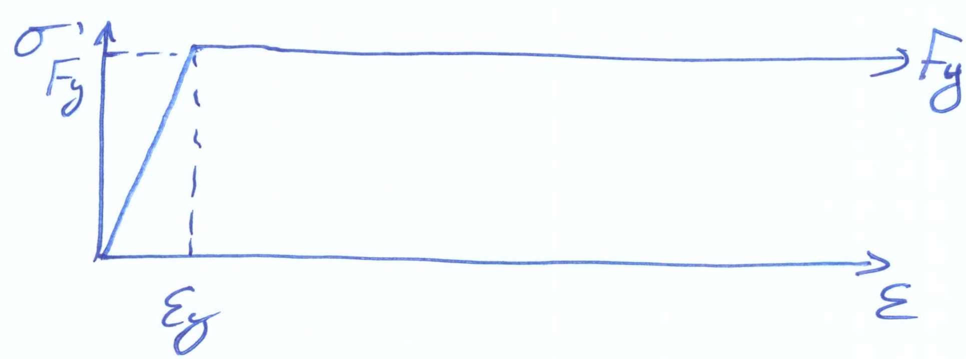
$$\sigma = \frac{M\,y}{I} \rightarrow M = \frac{\sigma\,I}{y} = \sigma S$$

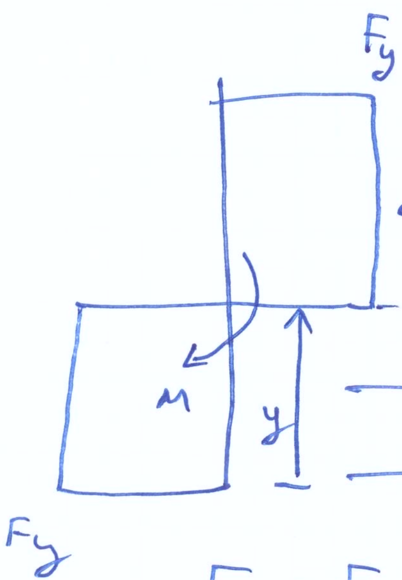
$$I_x = \frac{1}{12}\left[b d^3 - (b-w)(d-2f)^3\right]$$

- S_x ✓
- $M_y = F_y S_x$ ✓

Plastic Shape Properties







$F_{FT} = (b\,t_f)(F_y)$

$F_{wT} = \left[\frac{(d-2t_f)}{2}\,w\right] F_y$

$F_{wB} = \left[\frac{(d-2t_f)}{2}\,w\right] F_y$

$F_{FB} = (b\,t_f)(F_y)$

$F_{FT} = F_{FB}$, $F_{wT} = F_{wB}$

$\Sigma M_{PNA} = 0$

$$M_p = F_{FT}\, y_{FT} + F_{WT}\, y_{WT} + F_{FB}\, y_{FB} + F_{WB}\, y_{WB}$$

$$M_p = 2 F_{FT}\, y_{FT} + 2 F_{WB}\, y_{WB}$$

$$y_{FT} = \frac{d}{2} - \frac{t_f}{2}$$

$$y_{WT} = \left(\frac{d}{2} - t_f\right)/2$$

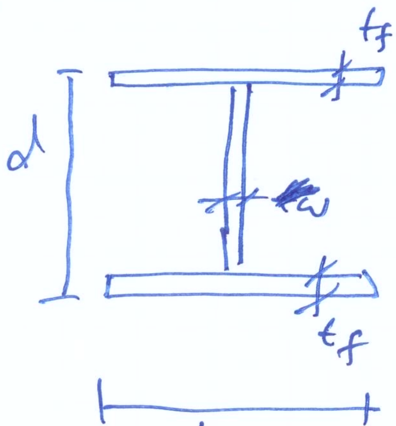
Let's define: $M_p = F_y Z_x$

Plastic section modulus.

$$\boxed{Z_x = M_p / F_y}$$

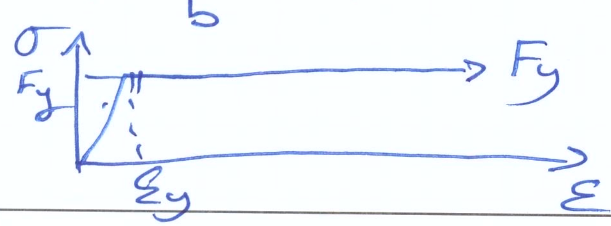
k = shape factor = Z/S

Example



$t_f = 20\,mm$
$w = 10\,mm$
$b = 200\,mm$
$d = 400\,mm$
$F_y = 350\,MPa$

Find:
- M_y, S_x
- M_p, Z_x
- Shape factor (k)



By Alan Lloyd

$$I_x = \frac{1}{12}\left[bd^3 - (b-w)(d-2t_f)^3\right] = \frac{1}{12}\left[(200)(400)^3 - (200-10)(400-40)^3\right]$$

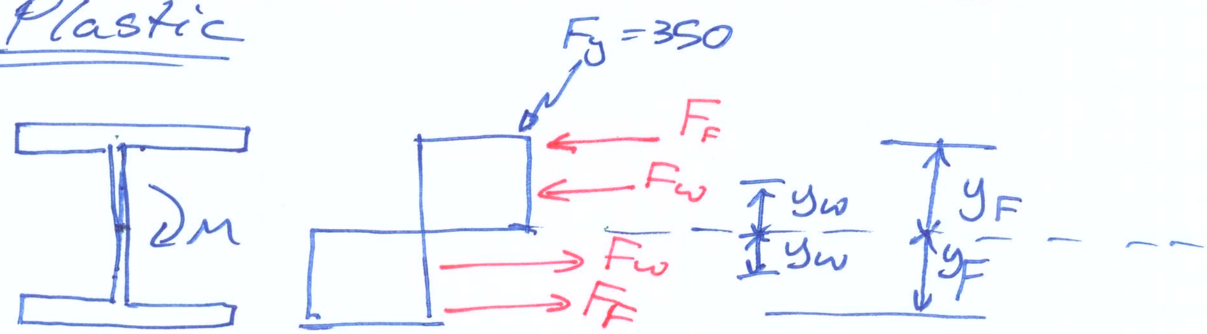
$$\underline{I_x = 327.95 \times 10^6 \text{ mm}^4}$$

$$S_x = \frac{I_x}{y} = \frac{327.95 \times 10^6 \text{ mm}^4}{400/2} = \boxed{S_x = 1639.7 \times 10^3 \text{ mm}^3}$$

$$M_y = F_y S_x = (350 \text{ MPa})(1639.7 \times 10^3)$$

$$\boxed{\begin{aligned} M_y &= 573.9 \times 10^6 \text{ N.mm} \\ M_y &= 573.9 \text{ kN.m} \end{aligned}}$$

Plastic



$$F_F = (A_F)F_y = F_y(t_F\, b) = (350 \text{ MPa})(20\text{mm} \times 200\text{mm})$$

$$F_F = ~~1260 \times 10^3 \text{ N}~~ \quad \underline{1400 \times 10^3 \text{ N} = F_F}$$

$$F_w = \frac{(A_w)}{2}F_y = F_y(w)(d-2t_f)/2$$

$$F_w = (350)(10)(400 - 2\times 20)/2$$

$$\underline{F_w = 1260 \times 10^3 \text{ N}}$$

$$y_F = \frac{d}{2} - \frac{t_f}{2} = \frac{400}{2} - \frac{20}{2} = \underline{190 \text{ mm} = y_F}$$

$$y_w = \left[\frac{(d-2t_f)}{2}\right] \times \frac{1}{2} = \left(\frac{400 - 20\times 2}{2}\right) \times \frac{1}{2} = \underline{90 \text{ mm} = y_w}$$

$$M_p = 2F_F\, y_F + 2F_W \times y_W$$

$$M_p = 2\left[(1400\times10^3 N)(190mm) + (1260\times10^3 N)(90mm)\right]$$

$$M_p = 758.8\times10^6 \ N\cdot mm$$

$$M_p = 758.8 \ kN\cdot m$$

$$Z_x = M_p/F_y = \frac{758.8\times10^6 \ N\cdot mm}{350 \ MPa} = Z_x =$$

$$Z_x = 2168\times10^3 mm^3$$

Shape factor

$$k = Z/S = \frac{2168\times10^3}{1639.7\times10^3} = k = 1.32$$