Minding Your *I*s and *Q*s

A radio signal may be represented, in general, as

$$x(t) = A\cos(\omega t + \varphi)$$

where A is the instantaneous amplitude, ω is the instantaneous carrier frequency in radians per second, and φ is the instantaneous phase offset. Depending on the modulation type, the amplitude, frequency, and phase can be functions of time.

How can we determine the amplitude and phase of an unknown signal if we only know its frequency? A common approach starts by multiplying the signal by a pair of reference signals

$$\cos(\omega t) \text{ and } \cos(\omega t + 90°) = -\sin(\omega t)$$

These reference signals are 90° out of phase with each other. In other words, they are orthogonal or in phase quadrature. The resulting signals are

$$I = A\cos(\omega t + \varphi)\cos(\omega t)$$
$$Q = -A\cos(\omega t + \varphi)\cos(\omega t)$$

Using a bit of trigonometry, we find that

$$I = \frac{1}{2}A\left[\cos(\omega t + \varphi - \omega t) + \cos(\omega t + \varphi + \omega t)\right]$$
$$= \frac{1}{2}A\left[\cos(\varphi) + \cos(2\omega t + \varphi)\right]$$
$$Q = \frac{1}{2}A\left[\sin(\omega t + \varphi - \omega t) - \sin(\omega t + \varphi + \omega t)\right]$$
$$= \frac{1}{2}A\left[\sin(\varphi) - \sin(2\omega t + \varphi)\right]$$

with the signal products consisting of a zero frequency (direct current) component and one with a frequency equal to twice the signal frequency. If these signals are low-pass filtered, we obtain

$$I = \frac{1}{2}A\cos\varphi$$
$$Q = \frac{1}{2}A\sin\varphi$$

I and *Q* are known as the in-phase and quadrature-phase (or quadra-phase) signal components, respectively because *I* is maximized when the unknown signal is phase aligned with $\cos(\omega t)$ and *Q* is maximized when the unknown signal is phase aligned with $-\sin(\omega t)$.

The amplitude of the unknown signal can then be determined using the vector magnitude of the *I* and *Q* components:

$$A = 2\sqrt{I^2 + Q^2}$$

and the phase can be determined using the four-quadrant arctangent of the ratio of the *Q* and *I* components:

$$\varphi = \tan^{-1}\left(\frac{Q}{I}\right)$$

If a radio receiver provides both in-phase and quadrature-phase local oscillator signals, then an amplitude modulated (AM) radio signal can be demodulated using the vector magnitude of the filtered *I* and *Q* intermediate-frequency products. In fact, signals using any of the common modulation techniques can be demodulated by manipulating the *I* and *Q* products. The approach works whether using analog or digitized signals.

In a digital GPS receiver, after suitable down conversion of the received signal and digitization, in-phase and quadrature-phase sampled data are produced using *I* and *Q* replica carriers (including carrier Doppler shift) from a numerical (digitally) controlled oscillator. The *I* and *Q* signals are correlated separately with early, prompt, and late replica pseudo-random noise codes (plus code Doppler) synthesized by the code generator. Just two (prompt and dithered) replicas can be used if the dithered replica can be selected to be early or late. After integration of the resulting signals, baseband processing occurs using code and carrier tracking loop discriminators and loop filters to provide correction signals for the code and phase tracking loops and to provide measurements of the replica code phase and replica carrier Doppler phase, which can be converted into pseudorange and carrier-phase measurements for positioning, navigation, and timing solutions.

A transmitted radio signal itself may include separate *I* and *Q* components. For example, the GPS L1 signal is modulated with the C/A-code and the navigation message on the in-phase channel and the P(Y)-code and the navigation message on the quadrature channel. — R.B.L.

■ The acquisition/pull-in steps integrate during 1 millisecond. A longer integration time or variable integration time might be desirable.

■ The tracking loops currently integrate during 20 milliseconds for code and 1 millisecond for carrier phase. Again, a longer integration time or variable integration time could be desirable.

■ Lastly, the tracking loops have not been optimized and could be greatly improved.

Since the presentation of the paper on which this article is based at ION GNSS 2005, work has already begun on version 2.00 of the software in which the code has been restructured to merge the DOS version with Linux. Although it is not yet ready for release as a package, it can be found at SourceForge in the CVS section of project OSGPS. See the OpenSource GPS project Web site, *www.home. earthlink.net/~cwkelley/* for access information. As of early January 2006, version 2.00 appears to be working well with minor debugging needed.

Comments and suggestions are most