Carrier-Phase Ambiguity Resolution

Handling the Biases for Improved Triple-Frequency PPP Convergence

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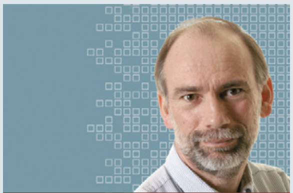


INNOVATION INSIGHTS
with Richard Langley

PPP? WHAT'S THAT? This acronym stands for precise point positioning and, although the technique is still in development, it has evolved to a stage where it can be considered another viable tool in the kitbag of GPS positioning techniques. It is now supported by a number of receiver manufacturers and several free online PPP processing services. You might think, looking at the name, that there's nothing particularly special about it. After all, doesn't any kind of positioning with GPS give you a precise point position including that from a handheld receiver or a satnav device? They key word here is *precise*.

The use of the word *precise*, in the context of GPS positioning, usually means getting positional information with precision and accuracy better than that afforded by the use of L1 C/A-code pseudorange measurements and the data provided in the broadcast navigation messages from the satellites. A typically small improvement in precision and accuracy can be had by using pseudoranges determined from the L2 frequency in addition to L1. This permits the real-time correction for the perturbing effect of the ionosphere. Such an improvement in positioning is embodied in the distinction between the two official GPS levels of service: the Standard Positioning Service provided through the L1 C/A-code and the Precise Positioning Service provided for "authorized" users, which requires the use of the encrypted P-code on both the L1 and L2 frequencies. Civil GPS users will have access to a similar level of service once a sufficient number of satellites transmitting the L2 Civil (L2C) code are in orbit. However, this capability will only provide meter-level accuracy. The PPP technique can do much better than this.

It can do so thanks to two additional precision aspects of the technique. The first is the use of more precise (and, again, accurate) descriptions of the orbits of the satellites and the behavior of their atomic clocks than those included in the navigation messages. Such data is provided, for example, by the International GNSS Service (IGS) through its global tracking network and analysis centers. These so-called precise products are typically used to process receiver data after collection in a post-processing mode, although real-time correction streams are now being provided by the IGS and some commercial entities.

Now, it's true that a user can get high precision and accuracy in GPS positioning using the differential technique where data from one or more base or reference stations is combined with data from the user receiver. However, by using precise products and a very thorough model of the GPS observables, the PPP technique does away with the requirement for a directly accessed base station.

The other precision aspect of PPP is its use of carrier-phase measurements rather than just pseudoranges. Carrier-phase measurements have a precision on the order of two magnitudes (a factor of 100) better than that of pseudoranges. But there is a catch to the use of carrier-phase measurements: they are ambiguous by an integer multiple of one cycle. Processing algorithms must resolve the value of this ambiguity and ideally fix it at its correct integer value. Unfortunately, it is difficult to do this instantaneously, and often many epochs of measurements are needed for a position solution to converge to a sufficiently high accuracy, say better than 10 centimeters. Researchers are actively working on reducing the convergence time, and in this month's column, we look at how using measurements from three satellite frequencies rather than just two can help.

"Innovation" is a regular feature that discusses advances in GPS technology and its applications as well as the fundamentals of GPS positioning. The column is coordinated by Richard Langley of the Department of Geodesy and Geomatics Engineering, University of New Brunswick. He welcomes comments and topic ideas. To contact him, see the "Contributing Editors" section on page 6.

While carrier-phase measurements typically have very low noise compared to pseudorange (code) measurements, they have an inherent integer cycle ambiguity: the carrier phase, interpreted as a range measurement, is ambiguous by any number of cycles. However, integer ambiguity fixing is now routinely applied to undifferenced GPS carrier-phase measurements to achieve precise positioning. Some implementations are even available in real time. This so-called precise point positioning (PPP) technique permits ambiguity resolution at the centimeter level.

With the new modernized satellites' capabilities, performing PPP with triple-frequency measurements will be possible and, therefore, the current dual-frequency formulation will not be applicable. There is also a need for a generalized formulation of phase biases for Radio Technical Commission for Maritime Services (RTCM) State Space Representation (SSR) needs. In this RTCM framework, the definition of a standard is important to allow interoperability between the two components of a positioning system: the network side and the user side.

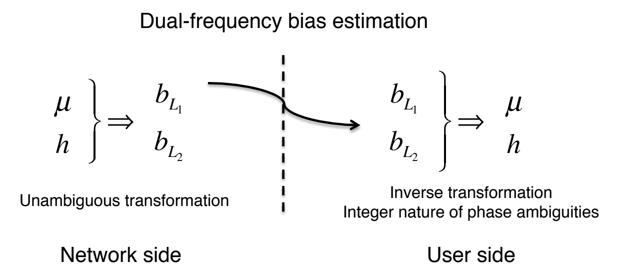
Classical Formulation

In this section, we review the formulation of the observation equations. We will use the following constants in the equations:

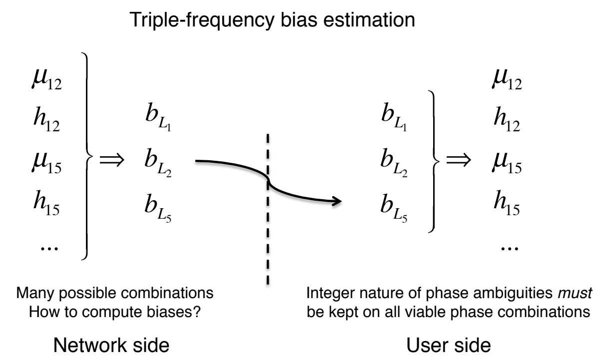
$$\gamma = \frac{f_1^2}{f_2^2}, \quad \lambda_1 = \frac{c}{f_1}, \quad \lambda_2 = \frac{c}{f_2}$$

where f_1 and f_2 are the two primary frequencies transmitted by all GPS satellites and c is the vacuum speed of light. For the GPS L1 and L2 bands, $f_1 = 154f_0$ and $f_2 = 120f_0$, where $f_0 = 10.23$ MHz.

The pseudorange (or code)



▲ **FIGURE 1** Phase biases estimation in the dual-frequency case.



▲ **FIGURE 2** Phase biases estimation in the triple-frequency case.

measurements, P_1 and P_2, are expressed in meters, while phase measurements, L_1 and L_2, are expressed in cycles. In the following, we use the word "clock" to mean a time offset between a receiver or satellite clock and GPS System Time as determined from either code or phase measurements on different frequencies or some combination of them.

The code and phase measurements are modeled as:

$$\begin{aligned}
P_1 &= D_1 && +\Delta h_p && +\left(e+\Delta\tau_p\right) && \\
P_2 &= D_2 && +\Delta h_p && +\gamma\left(e+\Delta\tau_p\right) && \\
\lambda_1 L_1 &= D_1+\lambda_1 W && +\Delta h && -\left(e+\Delta\tau\right) && -\lambda_1 N_1 \\
\lambda_2 L_2 &= D_2+\lambda_2 W && +\Delta h && -\gamma\left(e+\Delta\tau\right) && -\lambda_2 N_2
\end{aligned} \qquad (1)$$

where:

- D_1 and D_2 are the geometrical propagation distances between the emitter and receiver antenna phase centers at f_1 and f_2 including troposphere elongation, relativistic effects and so on.
- W is the contribution of the wind-up effect (in cycles).
- e is the code ionosphere elongation in meters at f_1. This elongation varies with the inverse of the square of the carrier frequency and is applied with the opposite sign for phase.
- $\Delta h = h_i - h^j$ is the difference between receiver i and emitter j ionosphere-free phase clocks. Δh_p is the corresponding term for code clocks.
- $\Delta\tau = \tau_i - \tau^j$ is the difference between receiver i and emitter j offsets between the phase clocks at f_1 and the ionosphere-free phase clocks. By construction, the corresponding quantity at f_2 is $\gamma\Delta\tau$. Similarly, the corresponding quantity for the code is $\Delta\tau_p$ (time group delay).
- N_1 and N_2 are the two carrier-phase ambiguities. By definition, these ambiguities are integers. Unambiguous phase measurements are therefore $L_1 + N_1$ and $L_2 + N_2$.

Equations (1) take into account all the biases related to delays and clock offsets. The four independent parameters, Δh, $\Delta\tau$, Δh_p, and $\Delta\tau_p$, are equivalent to the definition of one clock per observable. However, our choice of parameters emphasizes the specific nature of the problem by identifying reference clocks for code and phase (Δh_p and Δh) and the corresponding hardware offsets ($\Delta\tau_p$ and $\Delta\tau$). These offsets are assumed to vary slowly with time, with limited amplitudes.

The measured widelane ambiguity, $\tilde{N}_w$, (also called the Melbourne-Wübbena widelane) can be written as:

$$\left\langle \tilde{N}_w \right\rangle = N_w + \mu_i - \mu^j \qquad (2)$$

where N_w is the integer widelane ambiguity, μ^j is the constant widelane delay for satellite j and μ_i is the widelane delay for receiver i (which is fairly stable for good quality geodetic receivers). The symbol $\langle\ \rangle$ means that all quantities have been averaged over a satellite pass.

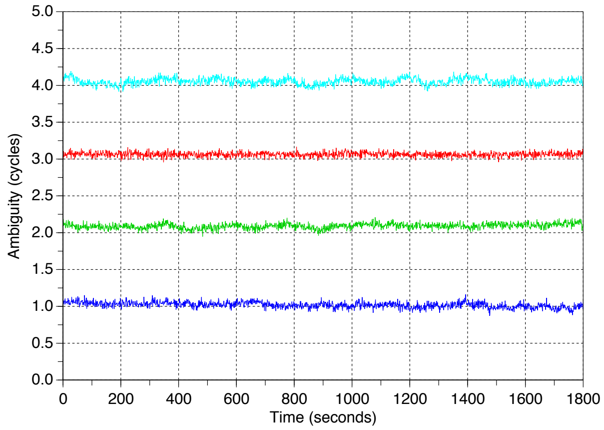
Integer widelane ambiguities are then easily identified from averaged measured widelanes corrected for satellite widelane delays. Once integer widelane ambiguities are known, the ionosphere-free phase combination can be expressed as

$$Q_c = D_c + \lambda_c W + h_i - h^j - \lambda_c N_1 \qquad (3)$$

where $Q_c = \left(\gamma\lambda_1 L_1 - \lambda_2\left(L_2 + N_w\right)\right)/\left(\gamma - 1\right)$ is the ionosphere-free phase combination computed using the known N_w ambiguity, D_c is the propagation distance, h_i is the receiver clock and h^j is the satellite clock. N_1 is the remaining ambiguity associated to the ionosphere-free wavelength λ_c (10.7 centimeters).

The complete problem is thus transformed into a single-frequency problem with wavelength λ_c and without any ionosphere contribution. Many algorithms can be used to solve Equation (3) using data from a network of stations. If D_c is known with sufficient accuracy (typically a few centimeters, which can be achieved using a good floating-point or real-valued ambiguity solution), it is possible to simultaneously solve for N_1, h_i and h^j. The properties of such a solution have been studied in detail. A very interesting property of the h^j satellite clocks is, in particular, the capability to directly fix (to the correct integer value) the N_1 values of a receiver that was not part of the initial network.

The majority of the precise-point-positioning ambiguity-resolution (PPP-AR) implementations are based on the identification and use of the two quantities μ^j and h^j. These quantities may be called widelane biases and integer phase clocks, a decoupled clock model or uncalibrated phase delays, but they are all of the same nature.



▲ **FIGURE 3** Ambiguity residuals for the extra-widelane 5-2 combination.

A Real-Time PPP-AR Implementation

A PPP-AR technique was successfully implemented by the Centre National d'Etudes Spatiales (CNES) in real time in the so-called PPP-Wizard demonstrator in 2010 and has been subsequently improved. In this demonstrator and in the framework of the International GNSS Service (IGS) Real-Time Service (RTS) and the RTCM, the GPS and GLONASS constellation orbits and clocks are computed. Additional biases for GPS ambiguity resolution are computed and broadcast to the user. The demonstrator also provides an open-source implementation of the method on the user side, for test purposes. Centimeter-level positioning accuracy in real time is obtained on a routine basis.

Limitations of the Bias Formulations. The current formulation works but it has several drawbacks:

- The chosen representation is dependent on the implemented method. Even if the nature of the biases is the same, their representation may be different according to the underlying methods, and this makes it difficult for a standardization of the bias messages.
- The user side must implement the same method as the one used on the network side. Otherwise, the user side would have to convert the quantities from one method to another, leading to potential bugs or misinterpretations.
- It is limited to the dual-frequency case. There are only two quantities to be computed in the dual-frequency case (μ^j_{12} and h^j_{12}), but in the triple-frequency case, there are many more possible combinations. For example, one can have (this is a non-exhaustive list) μ^j_{12}, μ^j_{15}, μ^j_{25}, h^j_{12}, h^j_{15}, h^j_{25}, where the indices refer to different pairs of frequencies, and other ionosphere-free combinations such as phase widelane-only or even phase ionosphere-free and geometry-free combinations are possible.

New RTCM SSR Model

The new model, as proposed by the RTCM Special Committee 104 SSR working group for phase bias messages is based on the idea that the phase bias is inherent to each frequency. Thus, instead of making specific combinations, one phase bias per phase observable is identified and broadcast.

It is noted that this convention was adopted a long time ago for code biases. Indeed, in the RTCM framework, and unlike the standard differential code bias (DCB) convention where code biases are undifferenced but combined, the RTCM SSR code biases are defined as undifferenced and uncombined. The general model for uncombined code and phase biases is therefore:

$$\begin{aligned}
P_1' &= P_1 + \Delta b_{P_1} = D + e + \Delta h_p \\
P_2' &= P_2 + \Delta b_{P_2} = D + \gamma e + \Delta h_p \\
\lambda_1 L_1' &= \lambda_1 \left(L_1 + \Delta b_{L_1} \right) = D - e + \Delta h_p - \lambda_1 N_1 \\
\lambda_2 L_2' &= \lambda_2 \left(L_2 + \Delta b_{L_2} \right) = D - \gamma e + \Delta h_p - \lambda_2 N_2
\end{aligned} \tag{4}$$

Time group delays, τ, and phase clocks, h, in Equation (1) are replaced by code and phase biases (Δb_P and Δb_L respectively). RTCM SSR code and phase biases correspond to the satellite part of these biases. The prime notation denotes the "unbiasing" process of the measurements. Here, the clock definition is crucial. As the biases are uncombined, they are referenced to the clocks. The convention chosen for the standard is natural: it is the same as the one used by IGS, that is, Δh_p in our notation.

This new model can be extended to the triple-frequency case very easily, as it does not involve explicit dual-frequency combinations:

$$
\begin{aligned}
P_1' &= P_1 + \Delta b_{P_1} = D + e + \Delta h_p \\
P_2' &= P_2 + \Delta b_{P_2} = D + \gamma_2 e + \Delta h_p \\
C_5' &= C_5 + \Delta b_{C_5} = D + \gamma_5 e + \Delta h_p \\
\lambda_1 L_1' &= \lambda_1 \left(L_1 + \Delta b_{L_1} \right) = D - e + \Delta h_p - \lambda_1 N_1 \\
\lambda_2 L_2' &= \lambda_2 \left(L_2 + \Delta b_{L_2} \right) = D - \gamma_2 e + \Delta h_p - \lambda_2 N_2 \\
\lambda_5 L_5' &= \lambda_5 \left(L_5 + \Delta b_{L_5} \right) = D - \gamma_5 e + \Delta h_p - \lambda_5 N_5
\end{aligned} \tag{5}
$$

This new model simplifies the concept of phase biases for ambiguity resolution. This representation is very attractive because no assumption is made on the method used to identify phase biases on the network side. All the implementations are valid if they respect this proposed model. It also allows convenient interoperability if the network and user sides implement different ambiguity resolution methods.

TABLE 1 summarizes the different messages used for PPP-AR in the context of RTCM SSR:

Parameter Nature	RTCM SSR message	Quantity
GPS/GLONASS orbits/ clocks	1060/1066	D, h_p
GPS code biases	1059/1065	b_P
GPS phase biases	1265	b_L

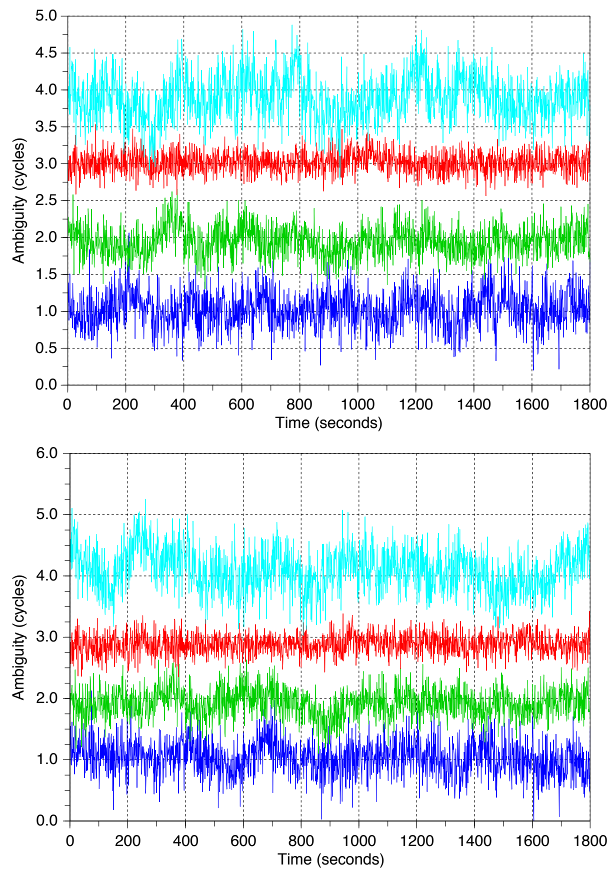
▲ **TABLE 1** RTCM SSR messages for PPP-AR.

Bias Estimation in the Dual-Frequency Case. The new phase biases identification in the dual-frequency case is straightforward. There are two biases (b_{L_1}, b_{L_2}) to be estimated using two combinations (μ and h). The problem to be solved is described in **FIGURE 1**.

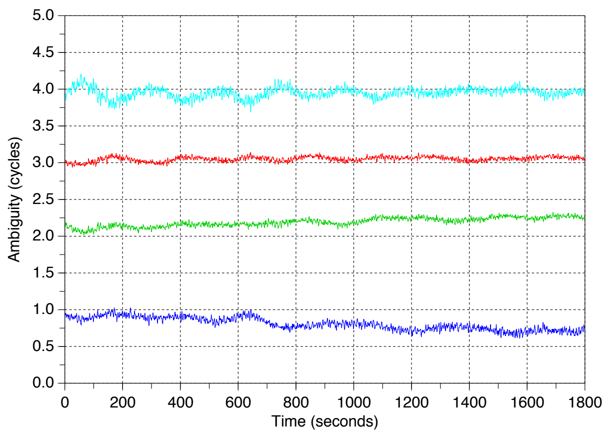
It can be solved very easily on the network side by means of a 2 × 2 matrix inversion:

$$
\begin{pmatrix} b_{L_1} \\ b_{L_2} \end{pmatrix} = \frac{1}{\gamma_2 \lambda_1 - \lambda_2} \begin{pmatrix} -\lambda_2 & 1 \\ -\gamma_2 \lambda_1 & 1 \end{pmatrix} \begin{pmatrix} \mu - \alpha_{21} b_{P_1} - \alpha_{22} b_{P_2} \\ (\gamma_2 - 1)(h - h_P) \end{pmatrix} \tag{6}
$$

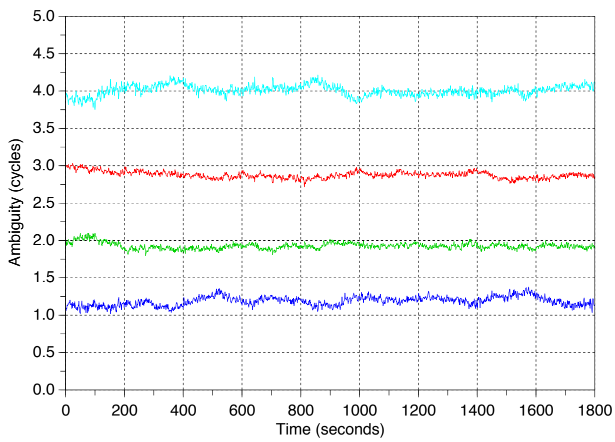
with



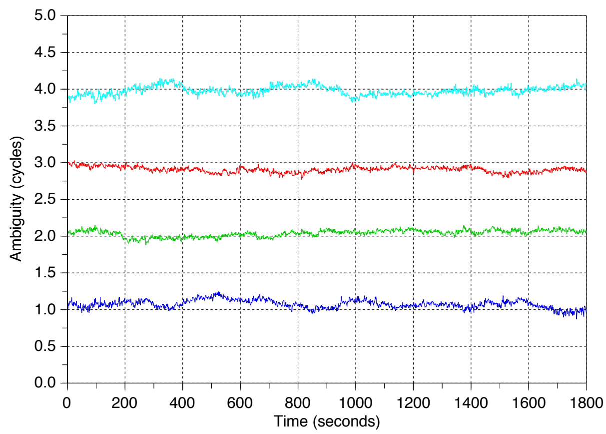
▲ **FIGURE 4** Ambiguity residuals for widelane combinations; top: 1-2 widelane, bottom: 1-5 widelane.



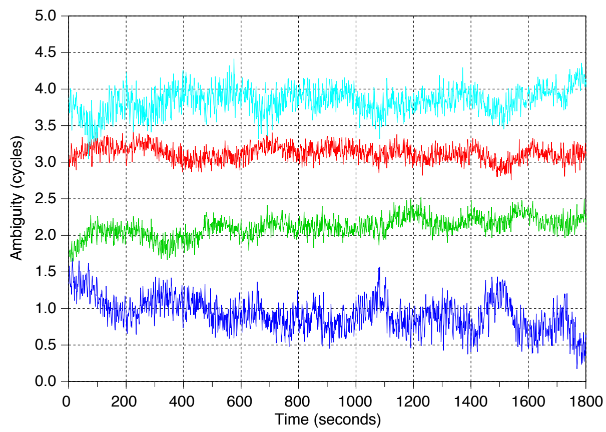
▲ **FIGURE 5** Ambiguity residuals for widelane-only 1-2-5 ionosphere free combinations.



▲ **FIGURE 6** Ambiguity residuals for the N_1 combination using a fixed 1-2 widelane.



▲ **FIGURE 7** Ambiguity residuals for the N_1 combination using a fixed 1-5 widelane.



▲ **FIGURE 8** Ambiguity residuals for the N_1 combination using a fixed 2-5 widelane.

$$\alpha_{21} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} / \lambda_1$$

$$\alpha_{22} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} / \lambda_2$$

Note: All the quantities denote the satellite part of the Δ operator defined above.

Bias Estimation in the Triple-Frequency Case. The triple-frequency bias identification is tricky due to the need, using only three biases, to keep the integer nature of phase ambiguities on all viable ionosphere-free combinations, and in particular combinations that were not used in the identification process. At this level, one cannot make assumptions on what kind of combinations will be employed by a user. The problem to be solved is described in **FIGURE 2**.

As an example, a naïve solution would be to identify the extra-widelane phase biases, μ_{25}^j, using the dual-frequency widelane approach, and then identify the b_{L_5} bias. Given the large wavelength of the extra-widelane combination, such identification would be very easy. However, the corresponding bias would be only helpful for extra-widelane ambiguity identification, and its noise would prevent its use for widelane 15 (L1/L5) ambiguity resolution or other useful combinations available in the triple-frequency context.

Each independent phase bias can be directly estimated in a filter; however, in order to keep ascending compatibility with the dual-frequency case during the deployment phase of the new modernized satellites, we have chosen to stay in the old framework, that is, to work with combinations of biases. The resolution method is the following:

- The widelane biases, that is, the identification of all the $b_{Li} - b_{Lj}$ quantities, are solved. For this computation and in order to have an accurate estimate of these biases, the two MW-widelane biases μ_{12} and μ_{15} are used coupled to an additional phase bias, which is given by the triple-frequency ionosphere-free phase combination with the integer widelane ambiguities already fixed. This last combination using only phase measurements is much more accurate than MW-widelanes. The system to be solved is redundant and the noise of the different equations has to be chosen carefully.
- The remaining bias (b_{L1}) is estimated using the traditional ionosphere-free phase combination of L1 and L2.

This computation has been implemented in the CNES real-time analysis center software, and since September 15, 2014, CNES broadcasts phase biases compatible with this triple-frequency concept on the IGS CLK93 real-time data stream.

▲ **FIGURE 9** Network used for the triple-frequency PPP study.

Real Data Analysis

To prove the validity of the concept, at CNES, we compute several ambiguity combinations using real data. The process is the following:

- Look for good receiver locations having a large number of GPS Block IIF satellites (transmitting the L5 signal) in view for a period of time exceeding 30 minutes, and choose among them, one participating in the IGS Multi-GNSS (MGEX) experiment. The station CPVG (Cape Verde) in the Reseau GNSS pour l'IGS et la Navigation (REGINA) network was chosen for the time span on September 28, 2014, between 19 and 20 hours UTC. During this period, four Block IIF satellites were visible simultaneously (PRNs 1, 6, 9, 30) for a total of 14 GPS satellites in view.
- Record a compatible phase-bias stream. The CLK93 stream is recorded during the time span of the experiment.
- Perform a PPP solution using the measurements, CLK93 corrections and biases to estimate the propagation distance, the troposphere delay and the receiver clock and phase ambiguity estimates according to Equation (5).
- For different ambiguity estimates, compute and plot the obtained residuals.

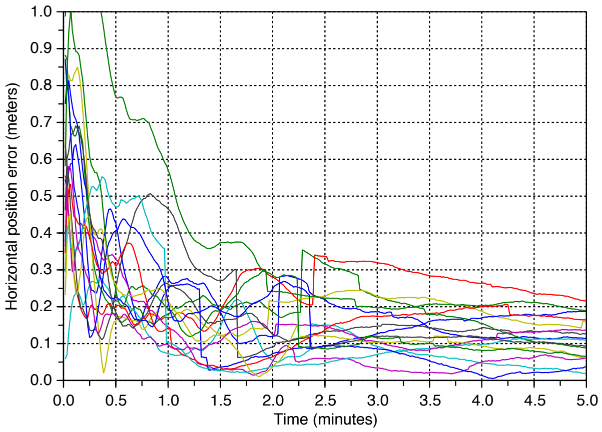
We present in the following graphs various ambiguity residuals for the four Block IIF satellites in view. The values of each ambiguity are offset by an integer value for clarity purposes.

Melbourne-Wübbena Extra-Widelane. FIGURE 3 represents the MW extra-widelane (between frequencies L2 and L5) ambiguity estimation using our process. The MW extra-widelane ambiguity has a wavelength of 5.86 meters. The noise of the combination expressed in cycles is very low, and the integer nature of ambiguities in this combination is clearly visible.

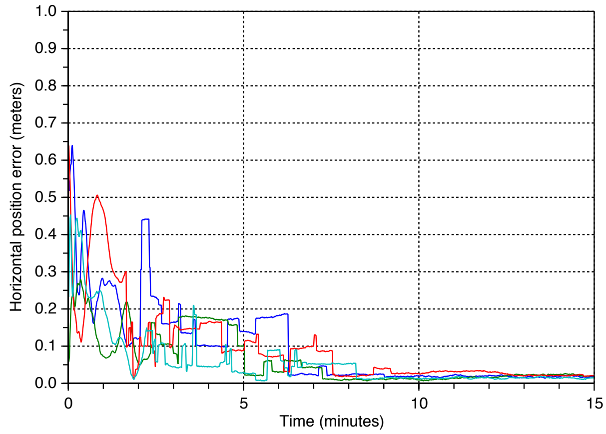
Melbourne-Wübbena Widelanes. FIGURE 4 represents the MW-widelanes (the regular 1-2 and 1-5 combinations). Here again, the integer nature of the four ambiguities is clearly visible.

Widelane-Only Ionosphere-Free Phase. In the triple-frequency context, there is a possibility of forming an ionosphere-free combination of the three phase observables. This combination has an important noise amplification factor (>20), but would allow us to perform decimeter-accuracy PPP using only the solved widelane integer ambiguities and if the corresponding phase biases are accurate. In addition, it can be shown that the wavelength of the widelane ambiguity when the extra-widelane ambiguity is solved is about 3.4 meters. It means that the remaining widelane using this combination can be solved if the position is accurate enough (a few tens of centimeters) and the extra-widelane is known. FIGURE 5 shows such a case, that is, the residuals of the widelane ambiguity using this combination and assuming that the extra-widelane is already solved for.

Such a case where the solution is the most biased is shown (the dark blue curve). This behavior is mainly due to the



▲ **FIGURE 10** Widelane-only triple-frequency PPP convergence (horizontal position error).



▲ **FIGURE 11** All ambiguities triple-frequency PPP convergence (horizontal position error).

difficulty in estimating the phase biases on this combination accurately using only a few Block IIF satellites. We hope that in the future the increasing number of modernized satellites will help such bias estimation.

N_1 Ionosphere-Free Phase. FIGURES 6 to 8 show the three possible ambiguity estimates using the ionosphere-free phase combination with two measurements (we assume that the corresponding widelane has already been solved). In each case, the computed biases allow us to easily retrieve the integer nature of the N_1 ambiguity.

Application to Triple-Frequency PPP

The results presented above show that the integer ambiguity nature of phase measurements is conserved for various useful observable combinations and prove the validity of the model. Another experiment has been carried out to estimate the impact of ambiguity convergence in the triple-frequency context. For that, in order to maximize the observability of the GPS Block IIF constellation and thus the accuracy of the biases, a network of ten stations across Europe has been chosen for the phase biases computation (see FIGURE 9). The station REDU (in green) was the test station to be positioned. The test occurred on January 10, 2015, around 11:00 UTC. At that time, four Block IIF satellites were visible simultaneously (PRNs 1, 3, 6, 9) for a total of 10 satellites in view.

The PPP-Wizard open source client was used to perform PPP in real time. The advantage of this implementation is that it directly follows the uncombined observable formulation described in Equations (5). The strategy for ambiguity resolution is a simple bootstrap approach.

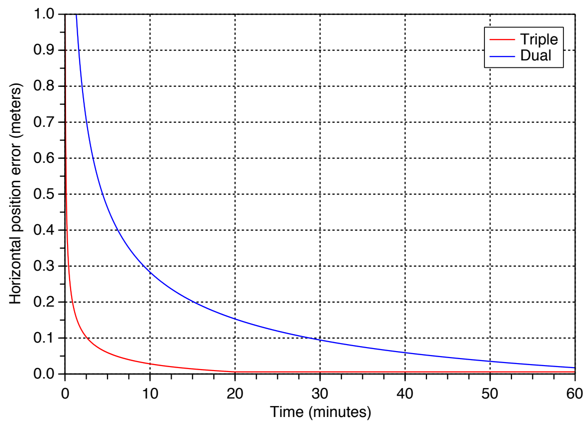
Convergence of the Widelane-Only Solution. In this test, a PPP solution was performed, but only the fixing of the widelane ambiguities was implemented. As noted in the previous section, the wavelength of the widelane ambiguity when the extra-widelane ambiguity is solved is about 3.4 meters, so it is expected that all the widelanes can be fixed in a very short time. Despite the amplification factor of about 20 of the equivalent unambiguous phase combination, we expect to obtain an accuracy of about 10 centimeters with such a solution.

FIGURE 10 shows the convergence time of several PPP runs in this context (16 different runs of five minutes are superimposed), in terms of horizontal position error.

The extra-widelanes are fixed instantaneously; the remaining widelanes are fixed in about two minutes on average to be below 30 centimeters (this is represented by the different sharp reductions of the errors). This new configuration, available in the triple-frequency context, is very interesting as it provides an intermediate class of accuracy, which converges very quickly and which is suitable for applications that do not demand centimeter accuracy. Another interesting aspect of this combination is the gap-bridging feature. In PPP, gap-bridging is the functionality that allows us to recover the integer nature of the ambiguities after a loss of the receiver measurements over a short period of time (typically a pass through a tunnel or under a bridge). This is done usually by means of the estimation of a geometry-free combination (ionosphere delay estimation) during the gap. Realistic maximum gap duration in the dual-frequency case is about one minute. In the triple-frequency case, the wavelength of the geometry-free combination involving the widelane (if the extra-widelane is fixed) is 1.98 meters. With such a large wavelength, the gaps are much easier to fill, and we can safely extend the gap duration to several minutes. In addition, the widelane combinations are wind-up independent, so there is no need to monitor a possible rotation of the antenna during the gap, as in the dual-frequency case.

Overall Convergence (All Ambiguities). Another PPP convergence test has been carried out with all ambiguities fixing activated (four different runs of 15 minutes are superimposed). Results are shown in FIGURE 11.

The centimeter accuracy is obtained in this configuration within eight minutes, which is a significant improvement in comparison to the dual-frequency case. Further improvement of this convergence time is expected with an increase in the number of Block IIF satellites and, subsequently, GPS IIIA satellites.



▲ **FIGURE 12** Realistic PPP convergence comparison between dual- and triple-frequency contexts (horizontal position error).

Convergence Time Comparison Between the Dual- and Triple-Frequency Contexts. Thanks to these new results, a realistic picture for PPP convergence in the dual- and triple-frequency contexts can be drawn. To do so, polynomial functions have been fitted over the data points obtained in the previous studies. Two data sets were used:

- Standard dual-frequency convergence (GPS only, 10 satellites in view).
- Triple-frequency convergence (GPS only, 10 satellites in view, four Block IIF satellites).

FIGURE 12 represents the comparison between the two polynomials (horizontal error).

Conclusion

The new phase-bias concept proposed for RTCM SSR has been successfully implemented in the CNES IGS real-time analysis center. This new concept represents the phase biases in an uncombined form, unlike the previous formulations. It has the advantage of the unification of the different proposed methods for ambiguity resolution, and it prepares us for the future; for example, for a widely available triple-frequency scenario. The validity of this concept has been shown; that is, the integer ambiguity nature of phase measurements is conserved for various useful observable combinations.

In addition, we have also shown that the triple-frequency context has a significant impact on ambiguity convergence time. The overall convergence time is drastically reduced (to some minutes instead of some tens of minutes) and there is an intermediate combination (widelane-only) that has some interesting properties in terms of convergence time, accuracy and gap-bridging for non-demanding centimeter-level applications.

Acknowledgments

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CORRECTION In the Innovation Insights introduction to last month's column, I said that "Interestingly, Maxwell used 20 equations to describe his theory but Oliver Lodge managed to boil them down to the four we are familiar with today." It was not Oliver Lodge but rather Oliver Heaviside who gave us the equations we now use. Oliver Lodge, on the other hand, carried out experiments in the generation and detection of radio waves (like Heinrich Hertz) and was involved in the development of key patents in wireless telegraphy. — *RBL*