Phase Wind-Up Analysis

Assessing Real-Time Kinematic Performance

Don Kim, Luis Serrano, and Richard Langley

MANY ADVANCES in GPS technology have occurred since the first test satellite was launched in February 1978. Perhaps the most significant for applications requiring very high accuracies in real time was the development of the technique known as RTK, or real-time kinematic.

In RTK positioning and navigation, a reference station transmits carrier-phase and pseudorange data over a radio link to one or more roving stations. At a rover, the reference station data is combined with the rover data, resolving carrier-phase ambiguities, and the rover's position is determined in real time. Either single- or dual-frequency GPS receivers can be used, with the dual-frequency systems typically affording faster ambiguity resolution and higher positioning accuracies over longer distances.

RTK systems, in common with other techniques, are susceptible to biases and errors such as ionospheric and tropospheric refraction along with line-of-sight-dependent phase-measurement effects including multipath, antenna phase-center variation, and carrier-phase phase wind-up. This latter phenomenon may not be familiar to all readers. It is a bias introduced into carrier-phase measurements by the rotation of a GPS receiver's antenna. There is also a contribution from the rotation of a GPS satellite's antenna as it orbits about the Earth.

In developing an RTK-based vehicle navigation system at the University of New Brunswick (UNB), we have observed a few instances where the phase wind-up due to rotation of the rover receiving antenna can significantly degrade system performance. In this month's column, we'll look at carrier-phase wind-up, introducing three wind-up observables that allowed us to perform qualitative assessments of its effects on the UNB RTK system. One motivation behind such an assessment is to determine whether or not we need to proceed to the next step of implementing algorithms to correct for the effects of phase wind-up. I am joined by Dr. Don Kim, the chief architect and developer of the UNB RTK system, and graduate student Luis Serrano.

INNOVATION INSIGHTS
with Richard Langley

Phase wind-up can significantly degrade system performace.

"Innovation" features discussions about recent advances in GPS technology and its applications as well as the fundamentals of GPS positioning. The column is coordinated by Richard Langley of the Department of Geodesy and Geomatics Engineering at the University of New Brunswick, who welcomes your comments and topic ideas. To contact him, see the "Columnist" section on page 10 of this issue.

A circularly polarized antenna's phase depends directly on the antenna's orientation with respect to the signal source. As a result, the observed carrier phase depends on the relative orientation of the transmitting and receiving antennas as well as the direction of the line of sight between them. Changing the receiver antenna orientation changes the reference direction and thus the measured phase. Similarly, changing the orientation of the transmitting antenna changes the direction of the electric field at the transmitting antenna and subsequently that at the receiving antenna. The result is also a change in the measured phase. As one or both of the antennas rotate, the phase change accumulates and is referred to as phase wind-up. It has also been called phase wrap-up and the (right-hand circular) polarization phase component.

In addition, the rotation of the receiving antenna causes an apparent change in the GPS carrier frequency. It is distinguished from the normal Doppler shift in that phase wind-up is carrier-frequency independent and does not affect ranging modulation group delay. Note that Doppler shift is the apparent change in carrier frequency caused by movement along a receiver's line-of-sight (LOS). A GPS receiver antenna can sense Doppler shift, which alters both the GPS carrier phase and ranging modulation group delay. The polarization-induced frequency shift is sometimes called rotational Doppler.

In this article, we will concentrate our discussion on the phase wind-up associated with rotation of receiving antennas.

Observable Effects

The different effects of phase wind-up can be observed depending on the relationship between the spin axis of the platform hosting the antenna and the antenna boresight.

A base-mounted antenna (the spin axis is aligned with the antenna boresight) has a phase that depends directly on the antenna's orientation. In this case, the phase shift is common to all signals, from different satellites, simultaneously received and therefore to all receiver channels. By double-differencing the measurements (between satellites and between receivers), therefore, this common phase shift can be removed.

For a rotating circumference-mounted antenna (the spin axis is not aligned with the antenna boresight), however, the phase term appears to be the sum of the common phase shift plus some small perturbation that increases as the depression angle deviates from the spin axis. All of the channels see the same steady phase wind-up term but each channel will have a different amount of spin modulation (phase and amplitude) determined by the elevation angle. This additional spin modulation cannot be cancelled by the double-differencing operation.

Although the observed carrier phase from non-aligned antennas cannot be cancelled by the double-differencing operation, it has been generally ignored in GPS applications due to the relatively small effect. However, related to our recent work on an RTK-based vehicle navigation system, we have observed in a few cases that the phase wind-up can significantly degrade the performance of the system. In this article, we introduce three phase wind-up observables and use them to demonstrate qualitative assessment of the phase wind-up in several test scenarios.

Phase Wind-up Observables

To validate the effects of the phase wind-up in the carrier-phase measurements, we use the phase wind-up observables described in this section. For the present, these observables are not used for correcting the effects of the phase wind-up but for the qualitative assessment of its effects on the carrier-phase measurements. Potential use of these observables in correcting the effects of the phase wind-up in the carrier-phase measurements will be investigated in the near future.

Single-Difference Observable. The following equations are useful for qualitative assessment of the phase wind-up effects in the single- and double-differenced carrier-phase measurements.

The single-difference (between antennas) observation equation for carrier-phase measurements can be expressed as:

$$\begin{aligned}\Delta\Phi_i &= \Delta\rho + \lambda_i \Delta N_i + \Delta T - \Delta I_i + \Delta\tau + \Delta\varphi_{LB} \\ &\quad + \Delta\varphi_{RHCP_i} + \Delta\varphi_{PCV_i} + \Delta\varphi_{MP_i} + \Delta\varepsilon_i, \quad i = 1 \text{ or } 2,\end{aligned} \tag{1}$$

where Δ is the single-difference operator; ρ is the distance between the transmit and receive antenna phase centers; N is the integer ambiguity; T is the tropospheric delay; I is the ionospheric delay where $I_2 = (f_1^2 / f_2^2) I_1$; f and λ are the frequency and wavelength of the carrier-phase measurements, respectively; τ is the receiver clock bias; φ_{LB} is the antenna line bias; φ_{RHCP} is the phase wind-up; φ_{PCV} is the antenna phase center variation; φ_{MP} is the multipath contribution to the carrier-phase measurement; ε is receiver system noise; and *i* indicates the L1 or L2 signal.

The first step in obtaining the phase wind-up observables is highpass filtering the single-differenced carrier-phase measurements. In this case, we can remove constant components (such as the integer ambiguity) and reduce low-frequency components such as the tropospheric and ionospheric delay, the inter-frequency bias, the antenna line bias, the antenna phase-center variation, and multipath in **EQUATION (1)**. Differential (in the time domain) single-differenced carrier-phase measurements correspond to the output of the highpass filter as:

$$d\Delta\Phi_i = d\Delta\rho + d\Delta\varphi_{RHCP_i} + d\Delta\varepsilon_i', \quad i = 1 \text{ or } 2, \tag{2}$$

where $d\Delta\varepsilon_i'$ includes receiver noise and the residuals of low frequency components.

The second step in obtaining the phase wind-up observables is generating the geometry-free combination of L1 and L2 carrier-phase measurements as:

$$d\Delta\Phi_{GF} = d\Delta\Phi_1 - d\Delta\Phi_2 = d\Delta\varphi_{RHCP_{GF}} + d\Delta\varepsilon_{GF}' \tag{3}$$

As the geometry-free combination can remove common components in L1 and L2 carrier-phase measurements (namely, the tropospheric delay, the receiver clock bias, and the antenna line bias), the residuals of low-frequency components in $d\Delta\varepsilon_{GF}'$ will be further reduced.

The phase wind-up observable will be obtained by the integration of **EQUATION (3).** By using the same type of antennas (to reduce the antenna phase-center variation) and careful site selection (to reduce multipath), and for short baseline situations (to reduce the ionospheric delay), we can obtain:

$$\begin{aligned} &\int d\Delta\Phi_{GF}\,dt = \int d\Delta\varphi_{RHCP_{GF}}\,dt + \int d\Delta\varepsilon_{GF}'\,dt \\ &\therefore \quad \Delta\varphi_{RHCP_{GF}} \approx \int d\Delta\Phi_{GF}\,dt. \end{aligned} \tag{4}$$

As the phase wind-up observable in **EQUATION (4)** is obtained by the integration of **EQUATION (3)**, we cannot see the initial offset of the phase wind-up. Also, note that this observable shows the combined effects of the phase wind-up in the L1 and L2 carrier-phase measurements.

Double-Difference Observable. The double difference (between antennas and satellites) observation equation for carrier-phase measurements can be expressed as:

$$\begin{aligned} \nabla\Delta\Phi_i &= \nabla\Delta\rho + \lambda_i\nabla\Delta N_i + \nabla\Delta T - \nabla\Delta I_i \\ &\quad + \nabla\Delta\varphi_{RHCP_i} + \nabla\Delta\varphi_{PCV_i} + \nabla\Delta\varphi_{MP_i} + \nabla\Delta\varepsilon_i, \quad i = 1 \text{ or } 2, \end{aligned} \tag{5}$$

where $\nabla\Delta$ is the double difference operator. The receiver clock bias and the antenna line bias are removed in **EQUATION (5)**. With the same considerations for the antenna type, site selection, and baseline length as for the single-difference approach, we can obtain a simpler equation as:

$$\nabla\Delta\Phi_i = \nabla\Delta\rho + \lambda_i\nabla\Delta N_i + \nabla\Delta\varphi_{RHCP_i} + \nabla\Delta\varepsilon_i', \quad i = 1 \text{ or } 2. \tag{6}$$

As $\nabla\Delta T$ is dependent on the height difference of the receiver antennas as well as baseline length, we have to assume the height difference to be small to reduce the tropospheric delay to a negligible size.

Let's assume that ambiguity parameters are resolved in the double-differenced carrier-phase measurements. In fact, this assumption is normally acceptable under the situations considered above (with insignificant residuals of low-frequency components). Then, by generating the geometry-free combination of L1 and L2 ambiguity-fixed carrier-phase measurements, the phase wind-up observable will become:

$$\begin{aligned} \nabla\Delta\Phi_{GF} &= \nabla\Delta\Phi_{L1} - \nabla\Delta\Phi_{L2} - \lambda_1\nabla\Delta N_1 + \lambda_2\nabla\Delta N_2 \\ &= \nabla\Delta\varphi_{RHCP_{GF}} + \nabla\Delta\varepsilon_{GF}' \\ \therefore \quad &\nabla\Delta\varphi_{RHCP_{GF}} \approx \nabla\Delta\Phi_{GF}. \end{aligned} \tag{7}$$

By estimating $\nabla\Delta\rho$ in **EQUATION (6)**, we may separate the phase wind-up on each separate L1 and L2 carrier-phase measurement as:

$$\nabla\Delta\varphi_{RHCP_i} = \nabla\Delta\Phi_i - \nabla\Delta\hat{\rho} - \lambda_i\nabla\Delta N_i, \quad i = 1 \text{ or } 2. \tag{8}$$

Test Scenarios

To assess the effects of phase wind-up, we performed a rooftop experiment at Head Hall on the University of New Brunswick Fredericton campus using a 3-axis (3D) motion table that uses computer-controlled stepper motors and controllers (see photo). The results support the validation of the phase wind-up observables. We also investigated the significance of the phase wind-up effects on an RTK-based vehicle navigation system using double-differenced carrier-phase measurements.

3D Motion Table Test. As illustrated in the photo, we set up a base station antenna in static mode and mounted two antennas on the 3D motion table capable of rotating 360 degrees around the yaw axis and 40 degrees around the pitch and roll axes.

With this system set-up, we could analyze two different antenna combinations — one static and one rotating antenna and two rotating antennas. The first combination of a static and a rotating antenna is a typical test set-up for assessing phase wind-up. The second combination of a pair of rotating antennas was used to demonstrate that the effects of the phase wind-up are cancelled in the single- and double-differenced carrier-phase measurements for co-rotating aligned antennas.

Static-Rotating Antenna Pair. We started the phase wind-up simulation with a non-aligned case. For the simulation, we selected a 40-degree pitch angle and rotated the antennas clockwise and counter clockwise around the yaw axis. Then we reset the pitch angle to align the antennas' spin axes and boresights, and rotated again clockwise and counter clockwise around the yaw axis.

The top panel in **FIGURE 1** shows the changes of azimuth due to the rotation of the antennas. Height changes at around 400 seconds of elapsed time in the middle panel were caused by resetting the pitch angel. The azimuth and height were determined by processing conventional double-differenced carrier-phase measurements. The bottom panel shows the phase wind-up observations in **EQUATION (4)** derived using the geometry-free combination of the differential single-differenced carrier-phase measurements. Although the rotating antennas will experience the same phase changes (in cycles) at both L1 and L2 frequencies, the phase wind-up observables in distance units must be consistent with the wavelength difference between the two frequencies. This means that there should be a peak-to-peak effect of approximately 5 centimeters in the observations. The bottom panel in Figure 1 confirms the predicted effect.

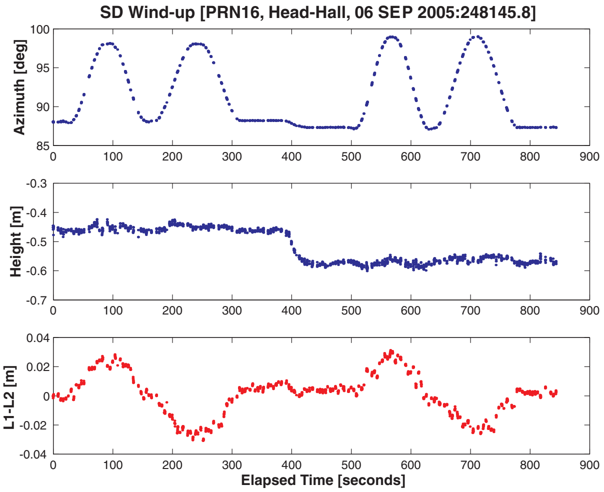
Two Co-Rotating Antennas. The effects of phase wind-up on the carrier-phase measurements obtained from two rotating antennas are illustrated in **FIGURE 2**. This set-up represents the case where the boresights of two antennas are aligned with each other although not necessarily with the spin axis. As expected, the bottom panel shows that the effects of phase wind-up are cancelled by single differencing between antennas. The reasoning for this cancellation is that two co-rotating antennas experience exactly the same rotational effects regardless of the alignment of the spin axis and the antennas' common boresights.

We also analyzed the effects of phase wind-up in the double-differenced carrier-phase measurements using **EQUATION (7)**. As illustrated in **FIGURE 3**, the effects of phase wind-up are significantly reduced in the double-differenced carrier-phase measurements compared with the single-differenced ones in Figure 1. Also, we can notice a slight difference between the non-aligned (first two rotations) and aligned (last two rotations) results in the bottom panel. Although not easily discerned because of the scale of the y-axis, the phase wind-up observations of the non-aligned double-differenced carrier-phase measurements are noisier than those of the aligned ones.

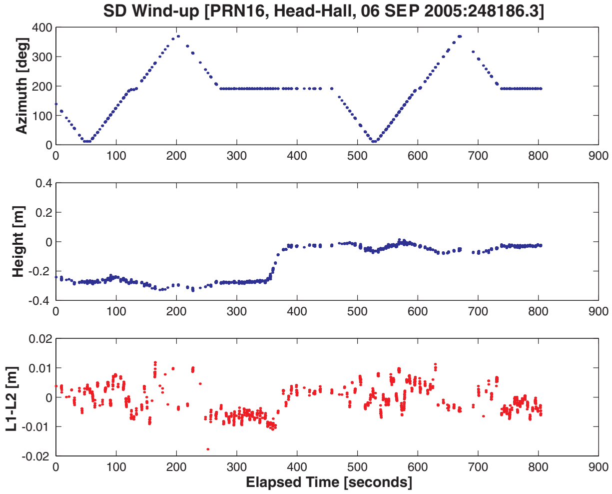
The reasoning for the difference is that the non-aligned carrier-phase measurements will pick up additional residual effects of phase wind-up. Also, antenna phase-center variation, if any, can be picked up by the rotation of non-aligned antennas. Multipath may contaminate the non-aligned carrier-phase measurements as well. Therefore, in the first two rotations, we may see the combined effects of three different error sources including the residual spin modulation of the phase wind-up, antenna phase-center variation, and multipath. In fact, it is the combined effects of several error sources rather than the isolated effects of the phase wind-up that can compromise an RTK system in reality. We discuss this again in the next section.

Vehicle Test. We had carried out a vehicle test to examine the overall performance of the UNB RTK system. The reference station was set up at UNB and (for this test) the reference station and mobile station data were post-processed. During the more than one-hour test, we drove the vehicle on highways and streets in and around Fredericton, New Brunswick. **FIGURE 4** shows the trajectory of the vehicle.

The performance of the UNB RTK system was very good most of the time during the test. However, we found several places where the system had problems. In some cases, this took place when the vehicle changed its direction **(FIGURE 5)** — note the gaps in the solutions. So we began to investigate what caused the system problems. We selected an area where the vehicle was being



▲ **FIGURE 1** Single-differenced phase wind-up observations using one static and one rotating antenna



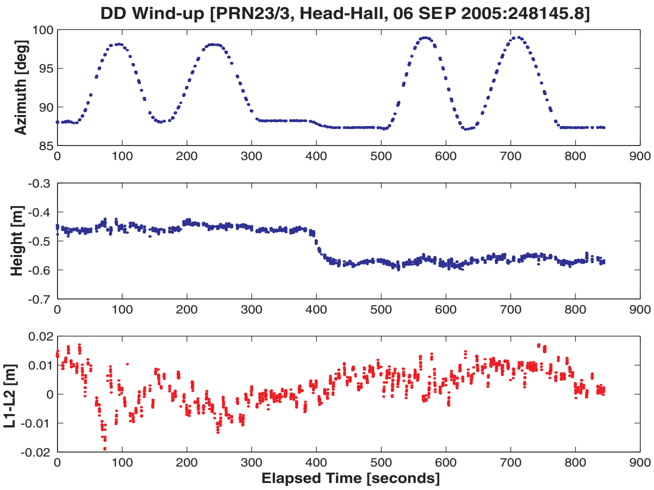
▲ **FIGURE 2** Single-differenced phase wind-up observations using a pair of rotating antennas

driven downhill and changing its direction **(FIGURE 6)**. In this case, we can expect that the vehicle experiences phase wind-up due to non-aligned boresights. Note that during the first 8 seconds the vehicle stopped and waited for a green light to make a left turn.

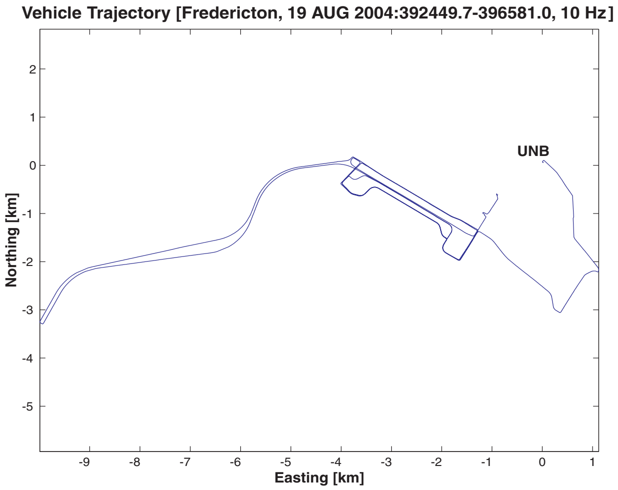
The phase wind-up observations obtained from the single-differenced geometry-free combination in **EQUATION (4)** are illustrated in **FIGURE 7** . The bottom panel clearly shows the effects of phase wind-up. As the vehicle changed its direction about 180 degrees (see the top panel and Figure 5), the phase wind-up observations should pick up a half of the wavelength difference between L1 and L2 carrier phases. For PRN 18 in Figure 7, we see this change of 2.5 centimeters. On the other hand, we saw an additional change of around 1–2 centimeters for PRN 22. This additional change comes from the combined effects of the three different error sources: the residual spin modulation of the phase wind-up, antenna phase-center variation, and multipath. As we were not aware of this happening when we performed the vehicle test, we did not prepare any external aid to isolate individual error sources. Thus it is not clear which error source is dominant in this case.

FIGURE 8 shows the phase wind-up observations obtained from the double-differenced geometry-free combination in **EQUATION (7)**. The bottom panel clearly shows a significant effect of phase wind-up. Assuming that there are no significant contributions from antenna phase-center variation and multipath, the double-differenced phase wind-up observations in the bottom panel should provide evidence of the residual spin modulation of the phase wind-up. However, it is not clear which error source is most likely in this case. Therefore, we could not assess how significant the residual effect of the phase wind-up was. Instead, as illustrated in the bottom panel, we could confirm the significance of the combined effects of the three different error sources.

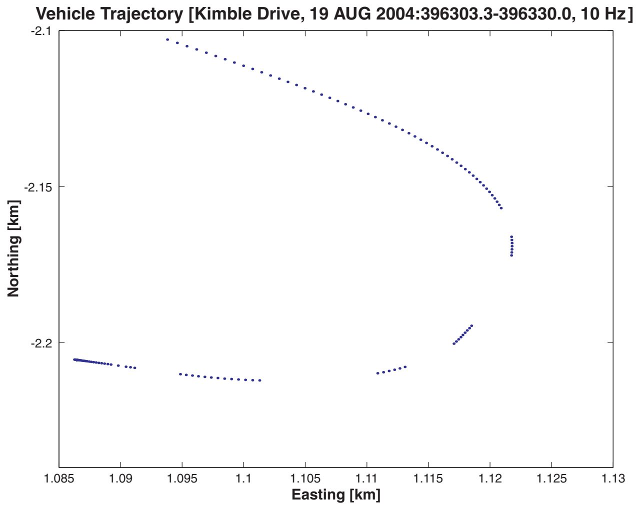
We further tried to assess the residual effects of the phase wind-up on L1 and L2 double-differenced carrier-phase measurements using **EQUATION (8)**. The residuals of the L1 double-differenced carrier-phase measurements during the first 8 seconds (when the vehicle was waiting for the green light) show no bias. On the other hand, there was a bias of 2 centimeters in the



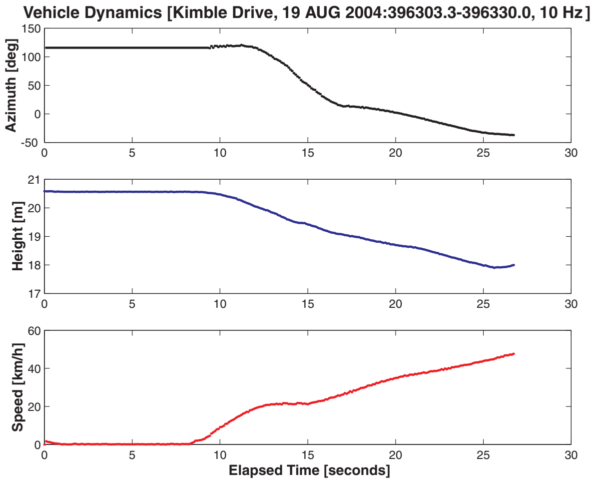
▲ **FIGURE 3** Double-differenced phase wind-up observations using one static and one rotating antenna



▲ **FIGURE 4** Vehicle trajectory determined by post-processing the test data



▲ **FIGURE 5** An example showing bad performance of the RTK system (solution gaps)



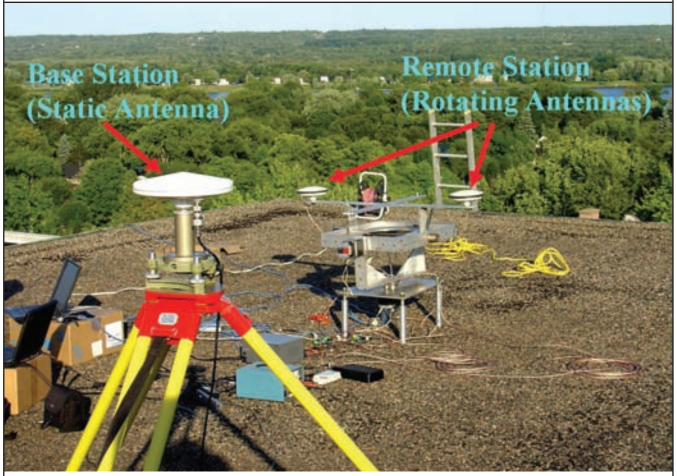
▲ **FIGURE 6** Vehicle dynamic information illustrating direction change (top), downhill travel (middle), and speed (bottom)

L2 double-differenced carrier-phase measurements. Considering the vehicle's dynamics at this time (static mode and fixed orientation), the most likely error source for this bias is multipath. As the vehicle turns to the left (from the 9-second epoch of elapsed time), the geometry between a reflector and a satellite changes and hence the effect of multipath is changed. Also, due to the antenna rotation, the carrier-phase measurements pick up the residual spin modulation of the phase wind-up and, if any, the antenna phase-center variation as well.

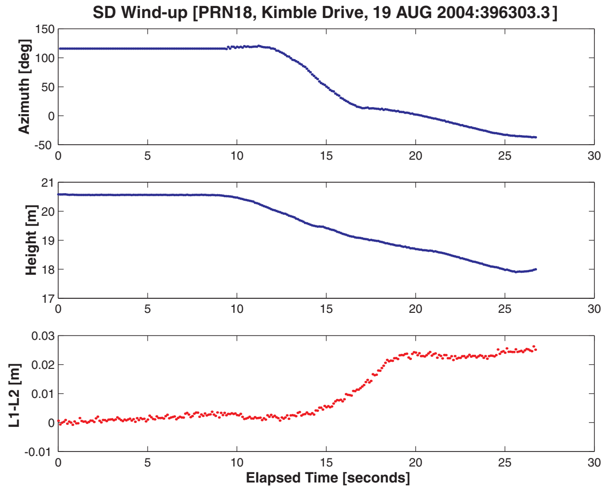
Concluding Remarks

RTK technology is able to provide positioning and navigation solutions in real time with high accuracy and precision — normally better than a few centimeters at a 95 percent confidence level. However, an RTK system can suffer from reliability problems when operating conditions are sub-optimal or if certain error sources are not properly compensated. In our recent work on an RTK-based vehicle navigation system, we have often observed that phase wind-up can degrade the performance of our system. In this article, we introduced three phase wind-up observables and demonstrated qualitative assessment of the phase wind-up using these observables.

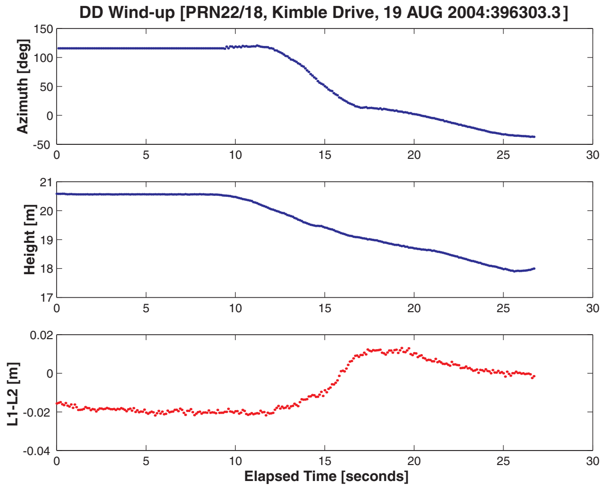
In situations where there are no significant contributions from antenna phase-center variation and multipath, the double-difference



▲ **ROOF-TOP TEST** set-up for the validation of phase wind-up effects



▲ **FIGURE 7** Single-differenced phase wind-up observations for PRN 18



▲ **FIGURE 8** Double-differenced phase wind-up observations for PRNs 18 and 22

phase wind-up observable can provide evidence of any residual spin modulation of the phase wind-up. Although we could not assess directly the effects of the residual spin modulation in our tests, we confirmed that these effects could be significant in the double-differenced carrier-phase measurements.

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Manufacturer

The phase wind-up tests discussed in this article used *OEM4* GPS receivers produced by **NovAtel Inc.** *(www.novatel.com)*.

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