It's Not All Bad

Understanding and Using GNSS Multipath

Andria Bilich and Kristine M. Larson

CAST YOUR MIND BACK 30 OR 40 YEARS. (Sorry, students, this exercise is for the older folks.) What was one of the most striking features of the suburban landscape? Virtually every house was topped with a Yagi TV antenna. The only way to receive TV signals before cable and satellite TV was directly from the transmitter tower. And, unless you had one of those fancy antenna rotors, reception wasn't always that great. Not only did we have to put up with weak signals, there was the problem of multipath. Besides a direct signal from the transmitter, the antenna could pick up a signal reflected off a nearby building, say, resulting in a delayed ghost image to the right of the main image on the TV screen. Even those out in the country weren't immune from multipath as a fluttery image might be seen caused by reflections from passing aircraft.

These days, with TV signals primarily delivered by cable and satellite, we don't *see* multipath much anymore. But we do hear it in our cars, from time to time, while listening to FM radio. (Students can tune back in now.) Although the FM "capture effect" provides some margin against multipath, it is not uncommon to lose stereo reception or to experience fading out of the signal while driving in built-up areas as a result of reflections.

INNOVATION INSIGHTS
with Richard Langley

SNR fluctuations are a telltale sign of multipath.

This same multipath phenomenon also affects GNSS signals. Unlike satellite TV antennas, the antennas feeding our GNSS receivers are omnidirectional. So we have the possibility of not only receiving a direct, line-of-sight signal from a GNSS satellite but also any indirect signal from the satellite that gets reflected off nearby buildings or other objects or even the ground. GNSS antenna and receiver manufacturers have developed techniques to minimize the impact of multipath on the GNSS observables. Nevertheless, there is typically some residual multipath afflicting the pseudorange and carrier-phase observables that limits the precision and accuracy of position determinations.

Telltale signs of multipath are the quasi-periodic fluctuations in the signal-to-noise ratios (SNRs) reported by some GNSS receivers, and in this month's column, we learn how an analysis of SNR values can be used to map and better understand the multipath environment surrounding an antenna.

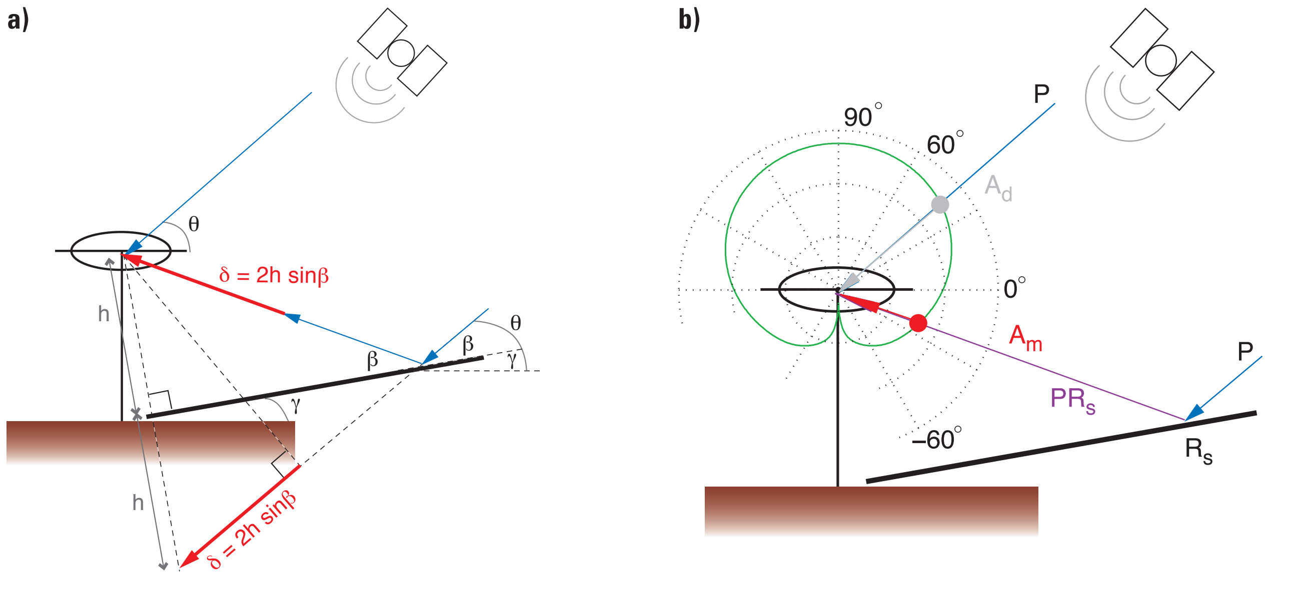
And, although an annoyance for most GNSS users, it turns out that multipath is not all bad. By analyzing the SNR fluctuations due to multipath, characteristics of the reflector can be deduced. If the reflector is the ground, then the amount of moisture in the soil can be measured. GNSS for measuring soil moisture? Who would have thought?

"Innovation" is a regular column that features discussions about recent advances in GPS technology and its applications as well as the fundamentals of GPS positioning. The column is coordinated by Richard Langley of the Department of Geodesy and Geomatics Engineering at the University of New Brunswick, who welcomes your comments and topic ideas. To contact him, see the "Contributing Editors" section on page 6.

We often hear "multipath" blamed as the last great source of unmodeled errors in GNSS observations, and therefore positions. But what is multipath? And what can we do about it? Can we remove multipath, or understand its temporal and spatial nature, or use it in new and novel ways? In this article, we address some of these outstanding multipath questions through the lens of the signal-to-noise ratio, or SNR. This article begins with background on the multipath phenomenon and discusses how carrier-phase multipath is related to SNR, an observable that is routinely collected by GNSS receivers but rarely used. The remainder of the article details a few new applications of SNR observations for multipath analysis. With this single observable type and a few assumptions about its relation to tracking loops and the environment surrounding the antenna, we can understand the multipath environment, remove multipath errors from carrier-phase measurements, and in some cases even transform this error into a new source of environmental information.

Multipath is exactly what it sounds like — a signal that travels along more than one path. When GNSS radio waves propagate from the GNSS satellite toward the receiving antenna, it is possible for the incoming signal to travel more than one path via reflection, diffraction, scattering, or a combination of these. Although all these phenomena contribute to multipath, in this article we limit multipath to reflections of a specular nature. Specular reflections occur when an electromagnetic wave hits an object (such as the surface of the Earth, a building, or a car) that is smooth relative to the signal wavelength. Upon reflection from the smooth surface, the outgoing energy is coherent, discrete, and sent in a single direction. From this point forward, multipath is taken to mean specular reflections from a large object.

When received by a GNSS antenna, this coherent reflected signal will disturb the tracking loops and distort the measured code and phase. The code and phase distortions occur because the



▲ **FIGURE 1 (a)** Forward-scatter multipath geometry, where the red arrows indicate the longer path traveled by the multipath signal relative to the direct signal. See Table 1 for definition of terms. **(b)** Signal amplitudes after including antenna gain pattern (green line) effects and attenuation upon reflection at a surface; see Table 2 for definition of terms.

GNSS receiver tracks a composite signal, which is the sum of the direct or line-of-sight signal and one or more multipath signals. The composite signal is biased from the direct signal simply because the multipath signal travels a longer path length than the desired direct signal. GNSS tracking and positioning rely upon the assumption of direct line-of-sight between satellite and receiver, thus tracking a composite signal will result in mismeasurement of the carrier and code ranges.

Why is multipath still an unsolved problem with GNSS positioning? As discussed below, multipath is a site-specific phenomenon — each GNSS site or satellite or vehicle will have a unique multipath-generating environment. Multipath is also dynamic — errors evolve with motion of the GNSS satellites and change as the reflecting surfaces (such as growing vegetation, moving cars, dry or damp ground) around the receiving antenna also change. Multipath errors cannot be simply differenced away — multipath at one station will not cancel out upon differencing with observables from another station. Nor can multipath always be "averaged out" — with real-time or rapid static GNSS positioning, the spatial and temporal complexity of site-specific multipath environments can adversely affect position determination.

Symbol	Term	Units
δ	Path delay	meters
h	Antenna-reflector distance	meters
ψ	Multipath relative phase	radians
θ	Satellite elevation angle	radians
β	Angle of reflection at surface	radians
γ	Tilt angle of reflecting surface	radians
λ	Signal carrier wavelength	meters

▲ **TABLE 1** Multipath Geometry Terms

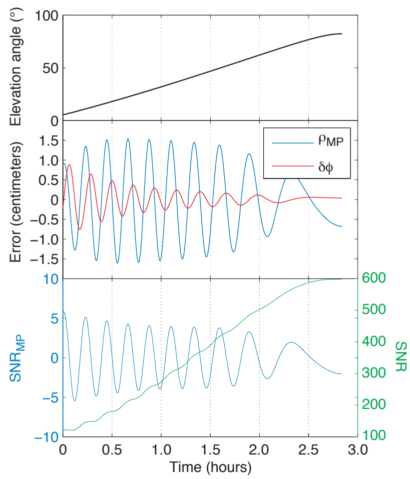
Simplified Multipath Model

On the most basic level, multipath errors are driven by the geometric relationships between the receiving point (the GNSS receiver antenna), the sending point (the GNSS satellite antenna), and the reflecting object. We illustrate these geometric relationships using simple ray tracing; for a more involved ray-tracing technique, see the paper "Development and Testing of a New Ray-Tracing Approach to GNSS Carrier-Phase Multipath Modelling" listed in Further Reading.

The geometric relationships between the satellite, receiving antenna, and reflecting objects dictate the additional path length traveled by the multipath signal, and how this path length changes as the satellite moves.

In an ideal, multipath-free world, this geometry is described only by the line-of-sight betwxeen satellite and receiver, which we describe via the azimuth and elevation angle of the satellite relative to the receiver. The geometry becomes more complicated when a reflecting/multipath object is introduced. **TABLE 1** introduces some multipath terms and **FIGURE 1** shows how these factors combine to create a forward-scatter multipath environment where a single reflected signal is received by the GNSS antenna. This illustration shows an antenna receiving two signals from one GNSS satellite, the desired direct ray and a second ray that reflects off a tilted, planar object before reception. For this example, we assume all angles are coplanar and disregard the third dimension.

Using the multipath terms listed in Table 1 and the geometric relationships depicted in Figure 1a, the additional distance traveled by the reflected/multipath signal relative to the direct one is the



▲ **FIGURE 2** Simulated carrier-phase error, code error, and SNR (recorded direct-plus-multipath SNR in green; SNR due to multipath alone in blue in linear amplitude units for a horizontal surface 1.0 meters below the antenna, assuming $R_s = 0.2$ reflection coefficient and a choke ring antenna gain pattern.

path delay. The phase of the multipath signal (again, relative to the direct signal) is the angular equivalent of path delay:

$$\psi = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda}2h\sin\beta \qquad [1]$$

Already, we see that the path delay and multipath relative phase are a function of the antenna-reflector distance (*h*) and the angle of reflection from the surface (*β*), and that the same multipath object will result in different multipath phases for different GNSS signals due to the dependence on *λ*.

As discussed below, the time-varying nature of multipath is key to understanding and mitigating its effects. Thus we examine the multipath frequency, that is, the rate of change of the multipath phase:

Symbol	Term	Units
A_m	Multipath signal amplitude	volts
A_d	Direct signal amplitude	volts
R_s	Attenuation coefficient of reflecting surface	unitless
P	Incoming signal power	volts

▲ **TABLE 2** Multipath Signal Terms

$$\frac{d\psi}{dt} = \frac{2\pi}{\lambda}2h\cos\beta\frac{d\beta}{dt}. \qquad [2]$$

If we assume a single stationary reflecting object, the only time-varying factor in Figure 1 is the satellite — as the satellite moves relative to the receiving antenna, the reflection point also moves, changing the path delay and multipath relative phase. Substituting the angular relationships (see Figure 1a) between the satellite, receiver, and reflecting object into the previous equation makes this more obvious:

$$\omega = \frac{d\psi}{dt} = \frac{2\pi}{\lambda} 2h\cos(\theta - \gamma)\frac{d\theta}{dt} \quad [3]$$

But how is "multipath frequency" related to quantities measured by our GNSS receivers: the code range, carrier phase, and signal-to-noise ratio (SNR)? To answer that question, we must introduce another set of multipath quantities, which describe the dominant signal strength factors (**TABLE 2**) for the direct and multipath signals; we ignore thermal noise, cable losses, etc. The amplitude of the direct signal (A_d) is equivalent to the GNSS signal strength as it is received and is affected by the antenna gain pattern (Figure 1b). The multipath signal comes through the antenna gain pattern at a different angle; by design, most GNSS antennas will apply less gain at angles consistent with common multipath geometries, such as below the antenna horizon. The multipath signal will also experience some amount of attenuation upon reflection; the combination of attenuation and antenna gain yields the amplitude of the multipath signal (A_m). Note that the broadcast GNSS signals are right-hand circularly polarized (RHCP), which are largely converted to left-hand polarization upon reflection. Thus the simplified "gain pattern" introduced here must incorporate both RHCP and LHCP patterns.

Under the simplified model of GNSS receiver response to tracking direct plus short-delay (smaller than 1.5 code chips) reflected signals, the multipath relative phase and signal amplitudes describe both the code and carrier-phase multipath errors, respectively denoted ρ_{MP} and $\delta\phi$:

$$\rho_{MP} = \frac{\delta A_m \cos\psi}{A_d + A_m \cos\psi} \quad [4]$$

$$\tan(\delta\phi) = \frac{A_m \sin\psi}{A_d + A_m \cos\psi} . \quad [5]$$

These equations are derived from code and carrier tracking behavior in the presence of multipath. Look in Further Reading for precise derivations and additional background material.

In addition to carrier phase and code observables, GNSS receivers routinely record SNR (or the related carrier-to-noise-density ratio — C/N_0) for each satellite. As the term indicates, SNR is a ratio of signal power to the noise floor of the GNSS observation, and has conventionally been used only for comparison of signal strengths between channels and satellites or to assess interference. Like code and carrier-phase multipath errors, SNR is a function of multipath phase and signal strengths:

$$SNR^2 = A_d^2 + A_m^2 + 2A_d A_m \cos\psi . \quad [6]$$

If we remove the effects of the direct signal, the remaining SNR is due only to multipath and is reduced to a simple function of multipath signal amplitude, relative phase, and a time-invariant phase offset:

$$SNR_{MP} = A_m \cos(\psi + \phi_0) . \quad [7]$$

Note that the equations for code multipath, carrier-phase multipath, and SNR contain the cosine or sine of the multipath relative phase, ψ. Therefore all three GNSS observables will have quasi-sinusoidal behavior driven by ω. To illustrate this, **FIGURE 2** gives an example for a rising satellite reflecting off horizontal ground 1.0 meters below the antenna. All three GNSS observables oscillate at the same frequency; however, pseudorange error and SNR are in phase while carrier-phase error is 90 degrees out of phase.

In this article, we use SNR observations to understand and quantify multipath effects. We choose SNR over the other observable types because multipath effects on SNR have the most unambiguous relationship to multipath. Typical levels of pseudorange noise will swamp all but the most extreme of multipath errors; carrier-phase data are more precise, but extracting multipath from these data requires first modeling clocks, orbits, and atmospheric delays. SNR data are directly related to carrier-phase multipath, are largely independent of the above effects, and are determined independently for individual satellites. Unfortunately, not all GNSS receivers provide SNR data with the requisite precision and accuracy to clearly observe the multipath relationships; see "Scientific Utility of the Signal-to-Noise Ratio (SNR) Reported by Geodetic GPS Receivers" in Further Reading for information on high-utility SNR. When SNR data are of sufficient quality, they can provide a unique and direct window on the multipath errors affecting the code and carrier observations.

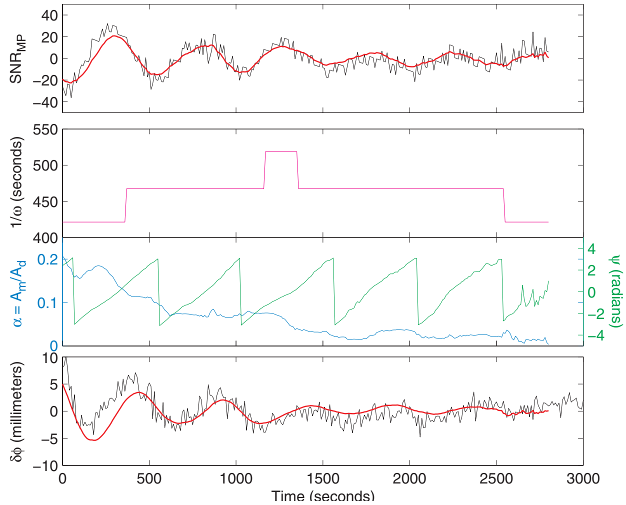
SNR Multipath Applications

A number of new scientific applications of SNR data are evolving to exploit the above multipath relationships. In the following sections, we describe three different SNR-multipath applications and provide relevant (although not exhaustive) references. All of these applications draw upon the above relationships and require precise and accurate SNR data that conform to the simplified multipath model described above.

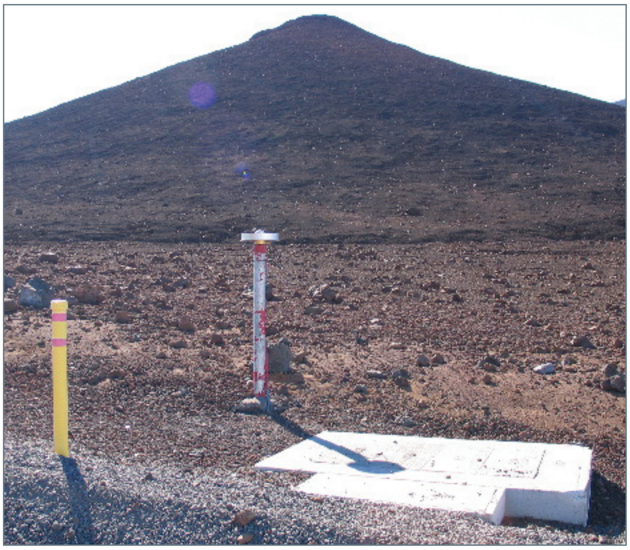
Multipath Corrections. Recall that the multipath errors in GNSS observables are simply a function of signal amplitudes and the relative phase between direct and multipath signals. It stands to reason that if these amplitudes and phases can be estimated, we can model and remove multipath errors from our code and carrier observations. SNR data allow us to do just that. After extracting the direct signal (A_d) to reveal the SNR due only to multipath (SNR_{MP}), this remaining time series depends only on A_m and ψ. As shown in Figure 2 and Equation 7, SNR due to multipath oscillates with a constituent frequency ω, which is the time derivative of ψ, and has an amplitude envelope equivalent to A_m. Therefore, from SNR due to multipath we can estimate multipath relative phase and multipath amplitude as a function of time.

This idea of modeling SNR data to estimate multipath parameters as time-varying quantities was first explored in a multi-antenna differential environment. This concept was extended to undifferenced SNR data so that carrier-phase errors at single-antenna GPS stations could be modeled and removed. In our implementation, we used wavelet analysis to first separate the direct amplitude from the multipath signal, then estimated the frequency content $\omega(t)$ of SNR_{MP} as a function of time. Using ω as the primary input to an adaptive least-squares algorithm, we then estimated multipath amplitude and relative phase as a function of time. Substituting these A_d, A_m, and ψ estimates into Equation 5 for carrier-phase multipath yielded a multipath-error correction profile.

A simple example from the Salar de Uyuni, a large salt flat in Bolivia, illustrates the process. For PRN8 observed during September 2002 with an antenna about 1.4 meters above the salt surface, the SNR due to multipath has very clear oscillations with a constituent frequency of approximately 0.0021 Hz (470 second period) (see **FIGURE 3**). Using frequency estimates as an input, the adaptive estimation algorithm estimates direct and multipath signal amplitudes as well as the multipath relative phase, which is approximately linear with time due to the relatively constant frequency estimate. Figure 3 shows that the modeled SNR_{MP} closely matches the SNR data, and the carrier phase correction profile closely matches the phase errors.



▲ **FIGURE 3** SNR modeling example from the Salar de Uyuni data set, PRN8, ascending arc, in seconds since the beginning of the satellite pass. Real data are given in black, while estimated quantities are colored lines; estimation uses SNR due only to multipath, i.e., after the direct signal has been removed, in linear amplitude units. The goal of SNR modeling is to generate a phase-multipath correction profile, shown in the bottom panel as a red line overlaying phase residuals.



▲ **FIGURE 4** Mauna Kea GPS station MKEA, facing northwest

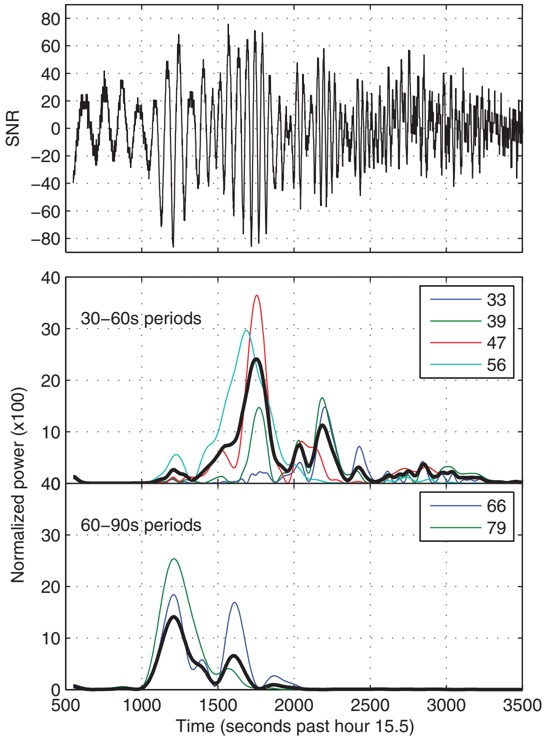
SNR-based phase-error estimation techniques show great promise for removing multipath errors from phase data. For the Salar de Uyuni test session, we derived SNR-based carrier-phase corrections for all satellites in view. By applying these corrections, we achieved a reduction in carrier-phase postfit residual root-mean-square error of up to 20 percent for static positioning, and 1–7 dB reduction in spectral power at multipath periods for kinematic positions.

Power Spectral Maps. Sadly, the complex and time-varying nature of multipath error cannot always be removed. In those cases, a better understanding of the multipath environment (the direction of and distance to reflecting objects) may aid the GNSS analyst. With this information, an analyst could discern the effect of multipath on position solutions, or de-weight multipath-corrupted observations, or simply choose one solution strategy (static, real-time kinematic or RTK, long vs. short occupation, etc.) over another to minimize or avoid multipath effects. For example, short duration but high-frequency multipath errors would be unimportant to someone solving for a single position using 24 hours of data, but that same multipath source could wreak havoc in an RTK survey. A method to evaluate the multipath environment at different frequencies and with a sense or orientation is therefore of great value.

As with the phase-error modeling example above, we accomplish multipath characterization via the frequency content of SNR oscillations, but this time backing out the distance, h (see Equation 3). This distance is directly related to the multipath frequency — nearby objects yield low-frequency errors, distant objects lead to high-frequency errors. By relating the distance, h, to angles (θ,γ) describing the direction and orientation of reflecting objects (Figure 1a), we can fully describe the multipath environment.

In this application, dubbed power spectral mapping, a wavelet transform is applied to each satellite's SNR time series to extract multipath power estimates over a range of frequencies or height values. The 3-D power vs. frequency vs. spatial coordinate data cube is then sliced into frequency bands of interest (i.e., height ranges), and all data contributing to a frequency band are combined. The signal power is assigned to the satellite's location and projected onto a "sky plot." This type of plot has four quadrants for north, south, east and west; concentric rings indicate satellite elevation angle; the center of the plot is the zenith while the outer ring is the horizon. This combination and projection process forms a map depicting the multipath characteristics of a GPS site.

These maps can help the analyst determine the source of multipath errors. For example, at first glance the permanent International GNSS Service (IGS) GPS station MKEA (see **FIGURE 4**) on Mauna Kea volcano in Hawaii seems to be multipath-free as it is surrounded by nothing but jagged rocky ground — uneven ground (relative to the GNSS wavelength) should create a diffuse multipath signature. The SNR data tell a different story, with strong coherent oscillations (see **FIGURE 5**) over a range of frequencies. By conducting wavelet analysis for all satellites in view, the combined power spectral maps (see **FIGURE 6**) show very strong reflections coming from the south-southeast and northwest, the location of volcanic cinder cones. Although



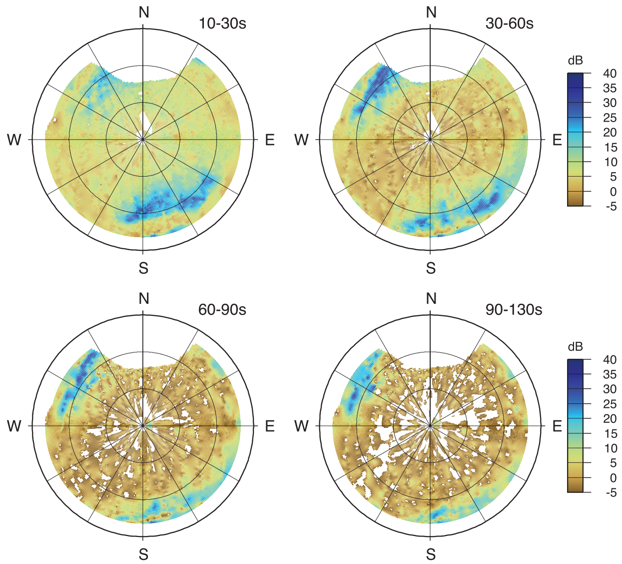
▲ **FIGURE 5** Example SNR profile from MKEA (top panel) as a function of time, in linear amplitude units after direct signal contributions have been removed. The bottom panels show wavelet power at different periods (colored lines), which are averaged together to form the wavelet power over 30–60 and 60–90 seconds-period bands of interest (heavy black lines).

rocky, these cinder cones generate strong multipath reflections. The sloped hillsides can be broken into a set of discrete reflectors at different distances, creating multipath oscillations at different frequencies over each satellite pass. For a more in-depth discussion of MKEA multipath and other power spectral map examples, see "Mapping the GPS Multipath Environment Using the Signal-to-Noise Ratio (SNR)," listed in Further Reading.

Soil Moisture. Manuel Martin-Neira is credited with introducing the idea, in 1993, that reflected GPS signals could be used for scientific studies. Since then, GPS reflection studies for ocean altimetry and winds, soil moisture, and snow sensing have all been discussed in the literature. These studies typically use an antenna pointed to optimize Earth reflections and specifically designed to track reflected (LHCP) signals. This means that antennas designed to suppress ground reflections, such as those used by the geophysical, geodetic, and surveying communities, are not used.

Motivated by our studies showing that multipath effects could clearly be seen in geodetic-quality data collected with multipath-suppressing antennas, we proposed that these same GPS data could be used to extract a multipath parameter that would correlate with changes in the reflectance of the ground surface. In our initial study, we used data from an existing IGS GPS site in Tashkent, Uzbekistan, and concentrated on SNR reflectance changes caused by rain and subsequent drying of the soil. While the correlation between the SNR data and precipitation models was strong, we lacked proper ground instrumentation to demonstrate that we were measuring true soil moisture changes.

Subsequently, together with other colleagues, we carried out an experiment designed to more rigorously demonstrate the link between GPS SNR and soil moisture. Specifically, we were interested in using GPS reflection parameters to determine the soil's volumetric water content — the fraction of the total volume of soil that is occupied by water, an important input to climate and meteorological models. Traditional soil moisture sensors (water content reflectometers) were buried in the ground at multiple depths (2.5 and 7.5 centimeters) at a site just south of the University of Colorado in Boulder. Precipitation data were also collected. Using a fixed frequency, Equation 7 was used to model the SNR data and estimate an amplitude and phase



▲ **FIGURE 6** GPS L1 power spectral maps for MKEA SNR data for four different frequency bands (given as periods in upper right-hand corner of each plot). Figure is reproduced from "Mapping the GPS Multipath Environment Using the Signal-to-Noise Ratio (SNR)."

offset on each day. **FIGURE 7** shows phase estimates converted to water content for six satellites that pass over the same ground south of the GPS antenna. We specifically concentrated on these six satellites because they transmit the new L2C signal, which yields superior SNR data compared to the L1 C/A-code signal.

Figure 7 shows excellent agreement between in situ sensors and the GPS multipath parameters. Soil moisture values rise within hours of a precipitation event, and then drop over approximately one week as the soil dries. It is important to note that the GPS SNR data are sensing much larger spatial regions (hundreds of square meters) whereas the soil probes measure values over a very small soil region (100 centimeters square). Climate scientists desire soil moisture measurements that have large footprints, and SNR data from some existing GPS stations are uniquely poised to provide this scale of soil moisture measurements.

Conclusions

Under the simplified multipath model discussed here, SNR data have a defined relationship to both carrier-phase and pseudorange multipath errors. Although SNR is traditionally used only as a measure of signal tracking, we have demonstrated some applications that use this common but underutilized observable to identify potential multipath sources, model and remove phase multipath errors, or retrieve soil moisture content from ground reflections. All of these applications are predicated upon accurate and precise SNR measurements, which conform to the simplified multipath model. Not all receivers are created equal in this respect, thus care must be taken in selecting reliable SNR data for analysis.

Acknowledgments

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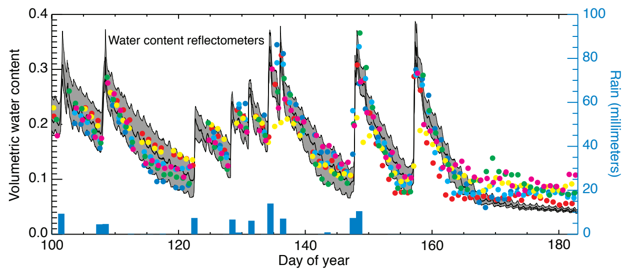

▲ **FIGURE 7** Variation in volumetric water content (VWC) from multiple GPS satellites (colored dots) and water content reflectometers buried at 2.5-centimeter depth (data range given by grey shaded region). Daily precipitation totals in blue. Figure is reproduced from "Use of GPS Receivers as a Soil Moisture Network for Water Cycle Studies."

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KRISTINE M. LARSON received a B.A. in engineering sciences from Harvard University in 1985 and a Ph.D. in geophysics from the Scripps Institution of Oceanography, University of California at San Diego, in 1990. Since 1990, she has been a faculty member in the Department of Aerospace Engineering Sciences at the University of Colorado at Boulder. The primary focus of her work is developing and improving GPS applications for measuring plate tectonics, episodic slip, volcanic deformation, ice-sheet motion, timing, seismic waves, soil moisture, and snow depth.

Manufacturers

The Salar de Uyuni and Mauna Kea data sets were obtained from **Ashtech** (now **Magellan Professional**, *www.promagellangps.com*) Z-12 receivers using **Allen Osborne Associates** (acquired by **ITT Communications Systems**, *www.cs.itt.com*) AOAD/M_T element antennas while the soil moisture experiment data set was from a **Trimble** *(www.trimble.com)* NetRS receiver fed by a model TRM29659.00 choke ring antenna with SCIT radome. ⊕

FURTHER READING

■ Multipath Basics and Mitigation Techniques

"Introduction to Multipath: Why is Multipath Such a Problem for GNSS? by A. Bilich in *GPS World's* online Tech Talk <http://techtalk.sidt.gpsworld.com/?p=14>, posted January 19, 2008.

"GPS Receiver Architectures and Measurements" by M.S. Braasch and A.J. van Dierendonck in *Proceedings of the IEEE*, Vol. 87, No. 1, January 1999, pp. 48–64.

"Conquering Multipath: The GPS Accuracy Battle" by L.R. Weill in *GPS World*, Vol. 8, No. 4, April 1997, pp. 59–66.

"Multipath Effects" by M.S. Braasch in *Global Positioning System: Theory and Applications*, edited by B.W. Parkinson, J.J. Spilker Jr., P. Axelrad, and P. Enge, Vol. 1, Chp. 14, American Institute of Aeronautics and Astronautics, Washington, D.C., 1996.

■ Multipath Ray Tracing

"Development and Testing of a New Ray-Tracing Approach to GNSS Carrier-Phase Multipath Modelling" by L. Lau and P.A. Cross in *Journal of Geodesy*, Vol. 81, No. 11, pp. 713–732, 2007 (doi: 10.1007/s00190-007-0139-z).

■ Assessing and Modeling Multipath Using Signal-to-Noise Ratios

"Mapping the GPS Multipath Environment Using the Signal-to-Noise Ratio (SNR)" by A. Bilich and K. M. Larson in *Radio Science*, Vol. 42, RS6003, 2007 (doi:10.1029/2007RS003652).

"Scientific Utility of the Signal-to-Noise Ratio (SNR) Reported by Geodetic GPS Receivers" by A. Bilich, P. Axelrad, and K. M. Larson in *Proceedings of ION GNSS 2007*, the 20th International Technical Meeting of the Satellite Division of The Institute of Navigation, Fort Worth, Texas, September 26–28, 2007, pp 1999-2010.

"Modeling GPS Phase Multipath with SNR: Case Study from Salar de Uyuni, Bolivia" by A. Bilich, K. M. Larson, and P. Axelrad in *Journal of Geophysical Research*, Vol. 113, B04401, 2008 (doi:10.1029/2007JB005194).

■ Using GPS to Estimate Soil Moisture

"Using GPS Receivers to Measure Soil Moisture Fluctuations: Initial Results" by K.M. Larson, E. E. Small, E. Gutmann, A. Bilich, P. Axelrad, and J. Braun in *GPS Solutions*, Vol. 12, No. 3, pp. 173–177, 2008 (doi: 10.1007/s10291-007-0076-6).

"Use of GPS Receivers as a Soil Moisture Network for Water Cycle Studies by K.M. Larson, E. E. Small, E. D. Gutmann, A. L. Bilich, J. J. Braun, and V. U. Zavorotny in *Geophysical Research Letters*, Vol. 35, L24405, 2008 (doi:10.1029/2008GL036013).

■ Measuring Reflected GPS Signals from Space

"Reflecting on GPS: Sensing Land and Ice from Low Earth Orbit" by S.T. Gleason in *GPS World*, Vol. 18, No. 10, October 2007, pp. 44–49.

"A Passive Reflectometry and Interferometry System (PARIS): Application to Ocean Altimetry" by M. Martin-Neira in *ESA Journal*, Vol. 17, No. 4, 1993, pp. 331–355.