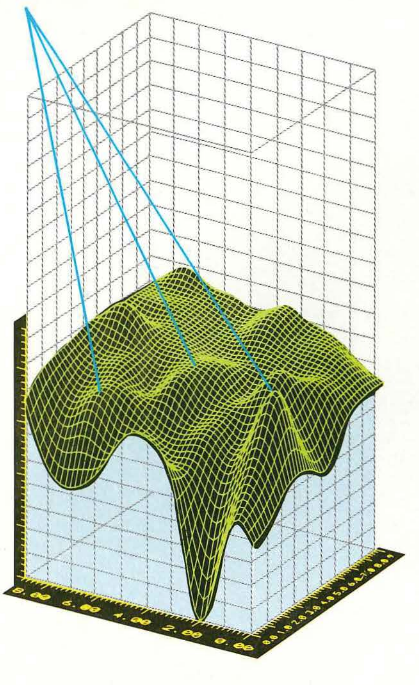

Relativity and GPS

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Relativistic effects in the Global Positioning System are surprisingly large, and users must carefully account for them, otherwise the system will not work properly. Important relativistic effects arise from relative motions of GPS satellites and users, and from the gravitational field of the earth. Even the earth's rotational motion requires significant relativistic corrections. This article describes these effects, quantifies them, and relates them to Einstein's fundamental principles: the constancy of the speed of light and the principle of equivalence. Our author is Neil Ashby, who is a professor of physics at the University of Colorado in Boulder.

"Innovation" is a regular column in GPS World *featuring discussions on recent advances in GPS technology and its applications, as well as on the fundamentals of GPS positioning. The column is coordinated by Richard Langley and Alfred Kleusberg of the Department of Surveying Engineering at the University of New Brunswick. We appreciate receiving your comments as well as suggestions of topics for future columns.*

Precise navigation and time transfer by means of the Global Positioning System relies on two things: the fundamental principles of special and general relativity, and the phenomenal accuracy and stability of modern atomic clocks. Clocks in Block I satellites have fractional stabilities of about one part in 10^{12} (one followed by 12 zeros or a million million) over a period of one day, while clock performance in Block II satellites can be 10 times better — the clocks are stable to about one part in 10^{13}. If a series of electromagnetic signals could be timed at this performance level while traveling exactly the same path from the earth to a mirror on Pluto and back, then the scatter in the resulting measurements of the size of the solar system would be no more than a meter or so!

To fully exploit such stability in the Global Positioning System, many relativistic effects must be taken into account. Otherwise, errors of hundreds of meters would quickly accumulate. Most users of GPS are unaware that both special and general relativity have such important consequences in the system (see the sidebar describing the distinction between the special and general theories). As the word *relativity* implies, these consequences depend on the coordinate system chosen to describe position and time measurements. It is remarkable that for users near the earth's surface, several large relativity effects cancel precisely, leaving a few small but significant effects for system designers to worry about in detail. These residual effects are sufficiently small that they can be considered separately; the total effect is then obtained by superimposing or adding up the individual effects.

CONSTANCY OF LIGHT SPEED

Einstein's revolutionary postulate, that in every inertial frame the speed of light has the same universally constant value, *c*, provides the physical basis for GPS operation. (For purposes of illustration in this article, an approximate value for the speed of light is used: $c \approx 30$ centimeters per nanosecond.) This can be seen from the following idealized experiment, which illustrates how relativity comes into GPS in a most fundamental way.

Suppose there exists a system of four or more synchronized clocks at known fixed positions in a vacuum, and suppose the clocks transmit timing pulses in synchrony at some known time. The pulses each spread out in spherical wave fronts from the transmitter positions, with the wave fronts expanding at speed *c*. A receiver measures arrival times of the pulses from the four transmitters and then, from the known positions and time of the transmissions, solves four propagation delay (pseudorange) equations to determine the receiver's position and clock time. Clearly, a 1-nanosecond timing error will result in a positional error of about 30 centimeters.

Navigation and timing using the real GPS are not this simple. The transmitters are in rapid motion, and receivers may move during signal propagation or between the reception of one timing pulse (actually a pseudorandom noise code transition) and the next. The earth's gravitational field causes clocks at different altitudes to beat at different rates, so atomic clocks at different altitudes would rapidly lose synchronism if there were no compensation.

Also, the diurnal rotation of the earth implies that users on the earth's surface cannot consider themselves to be in an inertial reference frame. In an inertial reference frame, a free particle will move with uniform velocity in a straight line; so loosely speaking, an inertial reference frame is one that is either at rest or moves with constant translational velocity but without rotational motion. Consider, for example, a light ray from a star passing over the zenith of an earth-fixed observer; let the local plumb line determine the z-axis of an earth-fixed reference frame. In this reference frame, the track of the ray is a spiral, not a straight line. This gives rise to the Sagnac effect (to be discussed later) and is sufficiently large that GPS must account for it to allow precise navigation and timing.

The idealized navigation experiment discussed earlier avoids the problem of synchronizing the transmitting clocks in GPS satellites. The constancy of the speed of light provides an experimental procedure for synchronizing clocks placed at rest at different points in a given inertial frame. Having chosen some clock as a reference, and knowing the distance *L* of any other clock from the reference, we can synchronize this clock by accounting for the time, *L/c*, needed for a signal to propagate from the reference, starting at a known time. This is called *Einstein synchronization* and, again, is based on fundamental relativity principles.

In the following sections, the main relativistic effects will be discussed in turn and related to fundamental relativity principles.

TIME DILATION

First, clocks in relative motion suffer (relativistic) *time dilation,* sometimes also called the *second-order* or *transverse Doppler effect.* Given two identical clocks, if one is moved to some distant point and then brought back and compared with the clock that remained at rest, it will be found that less time has elapsed on the traveling clock. This

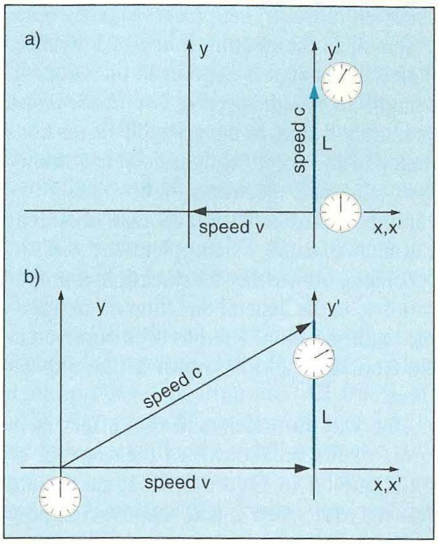

Figure 1. Thought experiment showing that moving clocks beat more slowly than clocks that remain at rest. **a.** The top part of the figure shows the thought experiment from the viewpoint of the moving frame. **b.** The bottom part of the figure shows the thought experiment viewed from the laboratory or rest frame.

is also a straightforward consequence of the principle of the constancy of the speed of light.

Figure 1 is a sketch of a thought experiment (sometimes called by the German word Einstein used, *Gedankenexperiment*) involving the application of the barest minimum of relativity principles, which shows how time dilation comes about. The figure shows two inertial frames in relative motion with their x,x′ axes parallel and having their positive senses in the direction in which the moving frame (the x′,y′ frame) travels relative to the laboratory frame (the x,y frame). Suppose an observer in the moving frame has a rigid rod of length L that is oriented perpendicular to the direction of relative motion. For lengths with such orientation, there is no reason to think that there is any dependence of the length L on the history of accelerations by which the relative motion was produced. Therefore, by symmetry, observers in the two different reference frames will agree on the value of this length. To perform the experiment, let the moving observer send a light signal from the bottom end of the rod to the top (see Figure 1a). A time L/c is measured in the moving frame of reference.

However, viewing the process from the laboratory or "rest" frame requires a different picture: Figure 1b. Here we see that the length of the path traveled by the light is numerically longer than the path length L, because the path is slantwise. But the speed of light is the same in the two frames, so the light appears to take a longer time in the rest frame than in the moving frame, and so more time must elapse on the clock at the rod's endpoint in the rest frame in order to come up with the correct length of the rod. The moving clock must therefore beat more slowly; its frequency is redshifted (see the sidebar describing the Doppler effect). This is time dilation.

The phenomenon of time dilation is the basis for the famous twin paradox: Given twins, let one move at high speed out to a distant star and back. A comparison of the twins would show the traveling twin to be younger. But to the traveling twin, why should the twin remaining behind not be younger, because he or she will appear to have moved away and back with high speed? The resolution of this paradox requires a somewhat complicated discussion. Here we shall only point out that the traveling twin has to be decelerated and then accelerated in the other direction in order to return to the nontraveling twin. This requires a change of reference frame and a resynchronization of the traveler's clock, which is discussed in more detail further on.

Although GPS satellite velocities are small compared with c, time dilation is a huge effect in the system. Analysis of the right-angled triangle in Figure 1b shows that for two otherwise identical clocks, the fractional frequency shift of the moving clock is $-v^2/2c^2$. Atomic clocks in GPS satellites have speeds of the order of 3.9 kilometers per second. The fractional slowing due to time dilation of such clocks is then about -8.3 parts in 10^{11}. After 12 hours, such a clock would be slow by about 3,600 nanoseconds, leading to an unacceptable positional error of about one kilometer if no relativistic corrections are made.

THE PRINCIPLE OF EQUIVALENCE

A second important relativistic effect on satellite clocks arises from the gravitational field of the earth. This can be understood in simple terms directly from Einstein's principle of equivalence. For our purposes, the equivalence principle means that fictitious gravitational fields induced by acceleration cannot be distinguished over small regions from real gravitational fields. For example, the outward centripetal force felt by a small bug riding on a carousel is no different in principle from a gravitational force. To an observer in a rocket ship with acceleration $g = 9.8$ meters per second squared straight upward, there is an apparent gravitational field of strength g directed downward. Similarly, people in an elevator in "free fall" (accelerating downward at 9.8 meters per second squared) do not feel their weight, because in their reference frame there is a new gravitational field that cancels that of the earth.

Figure 2 depicts a thought experiment performed in a rocket ship with acceleration g. Suppose that an observer in the rocket sends photons of definite frequency from a transmitter down low to a receiver up high. The process is easy to describe when viewed from a nearby, nonaccelerated reference frame. By the time the photons arrive at the receiver, the receiver has picked up speed. The received photons are then effectively from a receding source and must be Doppler-shifted in frequency toward the red end of the spectrum. The nonaccelerated observer thus attributes the red shift to a longitudinal Doppler shift from a receding source. If the height that the photons rise is L, it takes a time L/c for them to travel upward, so the speed picked up is $v = gL/c$ and the fractional longitudinal Doppler shift is $-v/c$ or $-gL/c^2$. The observer in the rocket is unaware of the acceleration and attributes the observed red shift to the change of gravitational potential, gL, experienced by the photons (see "GPS and the Measurement of Gravity" in the October 1993 issue of *GPS World* for an introduction to the concept of the gravitational potential). This argument can be generalized to show that photons experiencing a change of gravitational potential $\Delta\Phi$ suffer a fractional frequency shift $-\Delta\Phi/c^2$. This also means that clocks at different potentials in a gravitational field must beat at different rates.

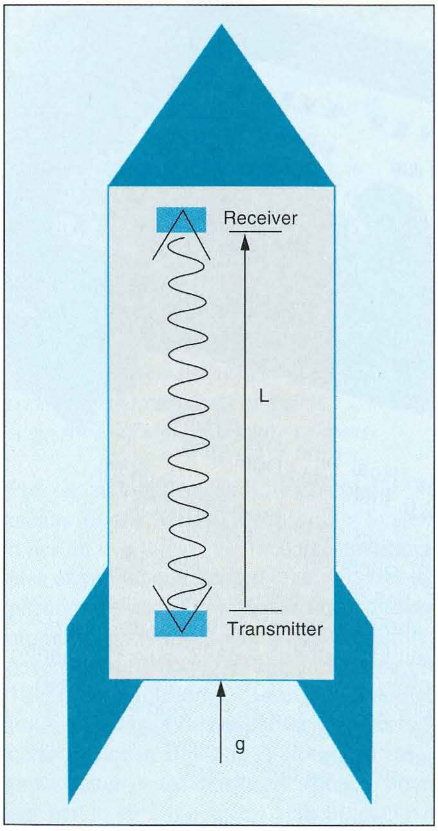

Figure 2. A signal traveling upward in a gravitational field is shifted lower in frequency.

For signals transmitted using atomic clocks in GPS satellites, the gravitational potential decreases as the signals fall to earth, so the frequency of the signals appears blueshifted. The effect is approximately twice as large as that due to time dilation arising from orbital motion and is of the opposite sign. For GPS, the effect amounts to a few parts in 10^{10} — another huge effect! The combined frequency shift at receivers on earth, of the second-order Doppler and gravitational frequency shifts, is toward higher frequencies.

SAGNAC EFFECT

A third relativistic effect arises from the motion of clocks at the earth's surface, due to the earth's spin. During the propagation time of a transmitted signal, observers fixed on the earth's surface move as the earth rotates. This is best viewed from a geocentric frame of reference whose axes are not rotating, but which point toward the fixed stars. Taking the point of view of observers in this frame (sometimes called the *Earth-Centered Inertial frame,* or ECI frame), it is easy to visualize the motion of an earth-fixed observer and to imagine taking into account this motion during the propagation time of the signal. To the earth-fixed observer, however, the process appears quite different. In the rotating frame of reference, light will not appear to go in all directions in straight lines with speed c. The frame is not an inertial frame, so the principle of the constancy of the speed of light does not strictly apply. Instead, electromagnetic signals traversing a closed path will take different amounts of time to complete the circuit, depending on whether the direction of propagation is in the general direction of, or opposite to, the rotation. This has been observed in the laboratory and is known as the *Sagnac effect.*

One way to understand this effect is in terms of the breakdown of the concept of simultaneity. In Figure 3, imagine a train moving with speed v, and suppose lightning strikes the engine and the caboose of the train simultaneously when viewed from the ground. After the simultaneous events, the light from the front of the train travels backward with speed c, while light from the back of the train travels forward with speed c. The two light signals intersect at the midpoint of the original position of the train, before it moved. Thus a criterion for simultaneity of two events is that signals originating from the two events, traveling with the same speed c, must meet at some instant at the midpoint between the two events. This is equivalent to Einstein synchronization.

But now view these events from a different reference frame: that of a passenger (a moving observer) sitting exactly in the middle of the train. The passenger at the midpoint travels forward with speed v. Light from the front of the train travels backward with speed c, while light from the back of the train travels forward with speed c. (Remember, the speed of light is c in all directions relative to the train, as well as relative to the ground.) If these signals intersect at the midpoint of the original position of the train, then to the passenger they cannot possibly have started out simultaneously from the front and the back.

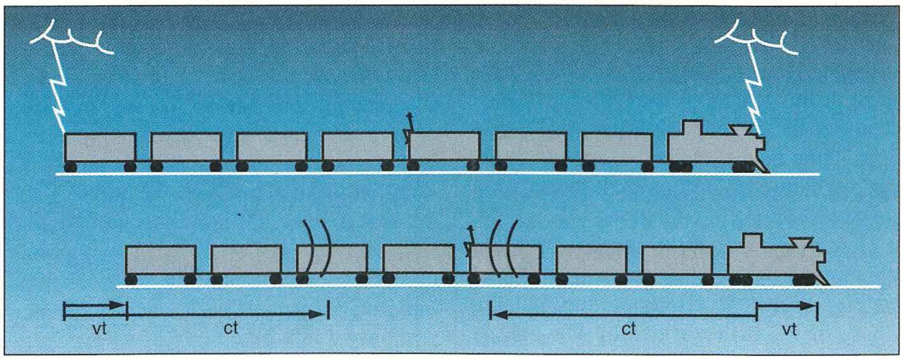

Figure 3. Thought experiment illustrating the breakdown of simultaneity.

In fact, the passenger will run into the signal from the front of the train first. Thus, it is necessary to conclude that the concept of simultaneity does not mean the same thing in the two different reference frames. Two events that are simultaneous in one inertial frame may not be simultaneous in another inertial frame. The discrepancy in synchronization is proportional to the length of the train and is increased as the relative velocity increases.

To see how this affects clock synchronization on the rotating earth, imagine viewing signal propagation from the ECI frame, in which the speed of light is a constant. Imagine two clocks fixed a small east–west distance x apart on the earth's equator; then viewed from the ECI frame, they will be moving with approximately equal speeds $v = \omega r$, where ω is the angular rotation rate of the earth, and r is the distance of the clocks from the rotation axis. If a clock synchronization process involving electromagnetic signals is carried out by earth-fixed observers who ignore the earth's rotation, then the two clocks will not be synchronous when viewed from the nonrotating frame. The magnitude of the discrepancy can be shown to be approximately $vx/c^2 = (2\omega/c^2)(rx/2)$. In this expression, $rx/2$ can be interpreted as the area of the triangle swept out by a vector from the rotation axis to the light pulse. In general, such discrepancies depend on the path along which the light signals travel relative to the rotation axis. For example, if a sequence of clocks were synchronized proceeding eastward around the globe, then upon arriving back at the starting point, a discrepancy of about 207 nanoseconds would be observed. Consequently, it is impossible to establish a self-consistent network of synchronized clocks using Einstein synchronization in the rotating frame. The property of transitivity is lost: If clock B is synchronized with A, and C is synchronized with B, then C is not necessarily synchronized with A. An acceptable way to avoid this problem is to synchronize the clocks in the nonrotating frame. This is what is done with GPS.

GPS TIME

With so many significant relativistic effects occurring on earth-fixed and earth-orbiting clocks, the problem of synchronization of the clocks becomes an acute one. Rates are affected by motion and gravitational effects; synchronization on the spinning earth is inconsistent if Einstein synchronization is used. How is it possible then to synchronize a network of distributed, rapidly moving clocks so that the navigational system will work? What works extremely well for GPS is the use of the time in a hypothetical underlying local inertial frame, with origin attached to the earth but not spinning (the ECI frame) as the measure of time. This time is not time on any clock orbiting the earth; instead, one makes use of relativity theory to correct the readings of such clocks so they will agree with hypothetical clocks at rest in the ECI frame.

Thus, in the ECI frame, one imagines a fictitious set of clocks available anywhere, all synchronized through the Einstein procedure, and running at agreed-upon rates such that synchronization is maintained. Call the clocks *coordinate clocks,* and call the resulting time scale *coordinate time.* Now introduce a set of clocks distributed around the surface of the rotating earth or orbiting the earth in space. To each one of these clocks a set of systematic corrections may be applied, so that at each instant the clock as corrected agrees with the time on the fictitious coordinate clocks that are at rest in the ECI frame, with which it instantaneously coincides. This set of clocks will therefore all be keeping

The Special and General Theories

The special theory of relativity [published by Einstein in 1905] . . . solved completely a certain definite problem: to account for the experimental fact that, when two bodies are in uniform relative motion, all the laws of physics, both those of ordinary dynamics and those connected with electricity and magnetism, are exactly the same for the two bodies. "Uniform" motion, here, means motion in a straight line with constant velocity. But although one problem was solved by the special theory, another was immediately suggested: what if the motion of the two bodies is not uniform? Suppose, for instance, that one is the earth while the other is a falling stone. The stone has an accelerated motion: it is continually falling faster and faster. Nothing in the special theory enables us to say that the laws of physical phenomena will be the same for an observer on the stone as for one on the earth. . . . The general theory of relativity [published by Einstein in 1915] removes this restriction, and allows the observer to be moving in any way, straight or crooked, uniformly or with an acceleration. —*ABC of Relativity* by Bertrand Russell

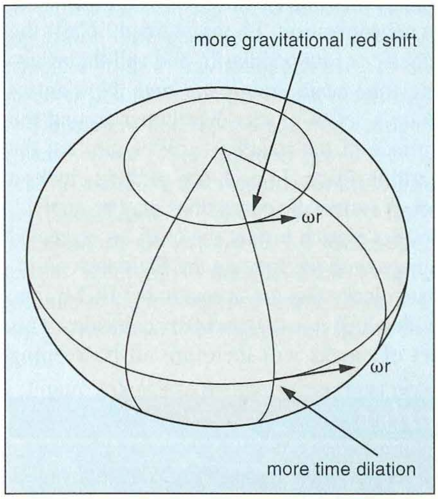

Figure 4. On the geoid of the oblate spinning earth, changes in gravitational frequency shift are precisely compensated by second-order Doppler shifts.

nate clocks that are at rest in the ECI frame, with which it instantaneously coincides. This set of clocks will therefore all be keeping coordinate time. In other words, coordinate time is equivalent to time measured by clocks in the local inertial frame.

This picture needs to be slightly modified to account for gravitational effects due to the earth. In general relativity, space and time are mapped out in terms of four coordinates, including coordinate time. This coordinate time variable theoretically exists over all of space, although it is not realized with a clock at every point for practical reasons. This time corresponds closely to coordinate time in the local freely falling frame, the ECI frame, but as modified by gravitational effects. In a static gravitational field, a single clock at rest is designated as the master clock; transmitted signals of definite frequency are then used to synchronize other clocks without any relativity corrections. After synchronization, or initialization, the system of clocks will then keep coordinate time.

Because it is a combination of the satellite clocks and an ensemble of ground-based clocks that provides the reference rate for GPS time, relativistic effects on ground-based clocks must be considered also. A clock fixed at the surface of the earth suffers a frequency shift due to the earth's gravitational field, compared with clocks a great distance away. This effect is more pronounced the closer the clock is to the center of the earth. Also, the earth's surface is approximately an oblate ellipsoid — a legacy of the rotation of the primordial earth. As a result, the earth's field is not purely radial but has a small contribution that can be described as a quadrupole potential (see the October 1993 "Innovation" column in *GPS World* for a discussion of how the earth's gravity field is described). There are, in fact, many higher multipole contributions to the earth's potential, but they are too small to be significant in understanding time in GPS.

Now consider, in Figure 4, clocks at different locations on a meridian, viewed from the ECI frame. A clock near the pole is lower down in the gravitational potential, so it will suffer a gravitational red shift compared with a clock near the equator. This effect by itself would amount to about 200 nanoseconds per day. However, a clock near the equator moves more rapidly than one at the pole because of the earth's spin and suffers a greater second-order Doppler shift. These effects, when combined with comparable effects from the quadrupole potential due to the earth's oblate shape, conspire to cancel out! The reason is that very early in its history, the earth assumed a shape whose surface is very close to that of a hydrostatic equipotential surface in the rotating frame. The result is that clocks on this surface (the geoid) all beat at the same rate; the definition of the Système International (SI) second is based on this rate.

Clocks not on the geoid still suffer gravitational frequency shifts with respect to those on the geoid. For example, if a clock in Colorado Springs (at about 1,830 meters above the geoid) were chosen as a reference (master) clock for the distribution of GPS time, it would be blueshifted with respect to clocks on the geoid by about six parts in 10^{12}, or about 16 nanoseconds per day. In principle, a rate correction could be applied to bring the rate of a reference clock at such an altitude into correspondence with Universal Coordinated Time as maintained by the U.S. Naval Observatory. However, in practice, this effect is simply lumped together with other effects that cause small frequency offsets, and it is treated as a systematic error.

In summation, the three principal relativistic effects that must be accounted for in GPS are time dilation of moving clocks, gravitational frequency shifts, and the Sagnac effect.

RELATIVITY IN GPS

The gravitational frequency shifts on satellite clocks and reference clocks on the geoid and the second-order Doppler shifts due to orbital motion and the earth's spin produce a combined effect. To an observer on the geoid, a clock in a GPS satellite in a nominally circular orbit would appear to be blueshifted in frequency by 4.465 parts in 10^{10}, or about 39,000 nanoseconds per day. (A previously documented value of 4.45 parts in 10^{10} resulted from the omission of a term in the expression for the earth's gravitational potential.) To compensate for this, system designers decided to adjust the 10.23 MHz reference frequency of orbiting clocks downward to 10.229 999 995 453 MHz. This adjustment is accomplished on the ground before the satellites are launched. When placed in orbit, the resulting blue shift will make the reference frequency precisely equal to 10.23 MHz.

At the time of launch of the NTS-2 GPS-precursor satellite (June 23, 1977), which contained the first cesium atomic clocks to be placed in orbit, there were some who doubted that relativistic effects were real effects that had to be accounted for. A frequency synthe-

The Doppler Effect

If an emitter of electromagnetic waves (either light or radio waves) and a receiver of those waves are in relative motion, the frequency of the received signals will be shifted from the frequency that would be measured if the emitter were at rest with respect to the receiver (the "rest" frequency). This is essentially the same phenomenon, albeit for sound waves, that gives rise to the change in pitch of a locomotive's whistle as a train passes in front of you at a level crossing.

The effect is named after Christian Doppler, an Austrian physicist, who propounded the principle in 1842, first in relation to sound and then to light. If the emitter and receiver are approaching each other, the number of waves or cycles received per second (the frequency) is greater than that which would be observed if the emitter and receiver were not in relative motion. If the emitter is receding from the receiver, the number is smaller.

As a result of the Doppler effect, the spectral lines in the light emitted by stars are shifted toward the red end of the spectrum (a lower frequency) if the star is receding from the earth and toward the blue end of the spectrum (a higher frequency) if the star is approaching the earth. The terms *red shift* and *blue shift* can also be used to describe the Doppler shift of radio waves. Although the effect can be partially explained using classical physics, a full and proper explanation of the motional Doppler shift — which incorporates time dilation — requires the special theory of relativity. As discussed in the article, the frequency of electromagnetic waves or the oscillator of a clock can also be shifted by a change of gravitational potential. Such gravitational red shifts and blue shifts are predicted by the general theory of relativity. — R.B.L.

sizer was built into the satellite clock system so that after launch, if in fact the rate of a clock in its final orbit were that predicted by general relativity, then the synthesizer could be turned on, bringing the clock to the coordinate rate necessary for operation. After one of the cesium atomic clocks was turned on in NTS-2, it was operated for about 20 days to measure its clock rate before turning on the synthesizer. The frequency measured during that period was +442.4 parts in 10^9 compared to the ground, while relativity theory predicted +446.47 parts in 10^9. The discrepancy was only about four parts in 10^9, well within the accuracy capabilities of the orbiting clock. This then gave about a 1 percent validation of the combined second-order Doppler and gravitational red-shift effects for a clock at 4.2 earth radii.

Additional relativistic effects arise when the satellite orbits are eccentric, as they usually are. As a satellite comes closer to earth, it speeds up in order to conserve its angular momentum; the clock will then slow down a little due to time dilation. Also, as the clock comes closer to the earth, its rate will slow as it suffers more gravitational red shift. The net effect of this "yo-yo motion" of the clock due to the eccentricity of the orbit is that its rate changes periodically. The magnitude of the shift is proportional to the orbit eccentricity. For a GPS satellite of eccentricity $e = .01$, the maximum size of the effect is about 23 nanoseconds. System designers decided to leave it to the user to apply a correction to the time signals received in order to compensate for this effect. Transmission of satellite ephemerides with the timing signals make it possible to do this in the receiver software. It is interesting that in the GLONASS system, which has many similarities to GPS, orbit eccentricities are typically less than .001, so this effect is rather small.

GPS system design also requires the user to correct the time signals for the Sagnac effect. This can be done in software in several ways. If one describes the propagation of the signal in the ECI frame, then the Sagnac correction is obtained by accounting for motion of the receiver from the instant of transmission to the instant of reception. This motion is automatically accounted for in the iterative solution of the propagation delay equations. From the point of view of observers in the rotating, earth-fixed frame, the correction to the coordinate time received is of magnitude $2\omega A/c^2$, where A is the projected area on the earth's equatorial plane, swept out by a vector whose tail is at the center of the earth and whose head is at the position of the signal pulse. Information contained in the transmitted navigation messages determines the position of the satellite at the instant of transmission; calculation of the receiver position then allows one to calculate the correction to be applied to the received coordinate time.

Rough estimates of the gravitational potentials due to the sun, moon, or other planets show that it might be necessary to consider gravitational frequency shifts due to such solar system bodies. For example, when a satellite is in the earth's shadow, its clock should be blueshifted because of the sun's gravitational potential, when compared with a satellite between the sun and earth. For such a configuration, the fractional frequency shift between the clocks in the two satellites is about three parts in 10^{12}, which in one hour would cause a 12-nanosecond timing error to build up. Fortunately, we do not have to worry about this at all. This effect is canceled to high precision by second-order Doppler shifts, because the satellite in the earth's shadow is actually falling around the sun faster than the other one and is subject to a greater motional red shift. This is to be expected because of the principle of equivalence: Because the earth and its satellites are in free fall around the sun, the gravitational effects of the sun are canceled out or transformed away in the ECI frame.

Thus, the application of relativity to GPS, albeit sufficiently complicated that much confusion can arise, is made simpler by delicate cancellations of two pairs of fairly large effects. Fortunately, the remaining relativistic effects are not noise; they are large but systematic and well-understood, and they can be accounted for to a high accuracy by applying calculated corrections to the data. For the sake of completeness, it should be mentioned that there are also relativistic effects on the orbits of the GPS satellites. However, these effects are quite small and are only barely significant in high-precision GPS applications requiring satellite positions at the decimeter level.

CONCLUSION

GPS works as well as it does in part because relativistic effects have been carefully taken into account. All in all, there are about a dozen separate relativistic effects that must be considered. It is interesting to think that this marvelous technology actually provides a laboratory for relativity. The satellites quietly orbit the earth, falling freely with the earth around the sun and providing continual confirmation of the principle of equivalence. Each position measurement that a user makes that is accurate to 100 meters or better, or each time measurement made that is accurate

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E. Total Distribution	35,514	34,173
F. Copies Not Distributed:		
1. Office use, left over, unaccounted, spoiled after printing	1,079	1,036
2. Return from News Agents	None	None
G. TOTAL	36,593	35,209

I certify that the statements made by me above are correct and complete

gned: Richard Wilkes
Richard Wilkes, Circulation Director