Making a Difference with GPS

Time Differences for Kinematic Positioning with Low-Cost Receivers

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LET'S REVIEW. Most radio signals consist of a carrier wave that is modulated in some way. This includes the GPS satellite signals. The pseudorandom-noise ranging codes and the navigation message are modulated onto the L-band carriers using binary biphase modulation. A GPS receiver uses the ranging codes to determine its distance from multiple satellites and then, through the process of multilateration, its position. But what about the carrier phase? Is it just a means to convey the ranging codes and navigation message? Definitely not.

A GPS receiver determines its velocity as well as its position and it does this not by differencing sequential code-based positions, which would not be very accurate, but rather by measuring the Doppler shift of the received carrier.

INNOVATION INSIGHTS with Richard Langley

If you time-difference phase measurements, the ambiguity disappears.

But the carrier can be used in other ways too. In fact, it can be used for determining positions, just like the code, but with much higher precision. Over 20 years ago, surveyors and geodesists devised ways to make use of recorded measurements of the phase of the received carriers to determine accurate relative positions between a roving receiver and a base or reference receiver at a known location. The technique was enhanced over the years, evolving into an approach known as RTK or real-time kinematic positioning. As its name suggests, RTK is usually employed in real time using auxiliary radio communications (often cell-phone-based) between the base and rover receivers. However, RTK-style positioning can also be used to postprocess collected data, achieving the same high-accuracy standards. But one of the difficulties with the RTK approach is resolving the so-called carrier-phase ambiguities. One cycle of the carrier looks just like the next, so how can you determine the exact number of cycles in the carrier between the satellite's antenna and the receiver's antenna? Well, it can be done, but even with increasingly sophisticated techniques, there is a limit to how far away a rover can be from the base station.

Isn't there a way to get rid of the integer ambiguity problem? There is. If you time-difference sequential carrier-phase measurements, the ambiguity actually disappears! As we'll see in this month's column, you can determine accurate relative positions using time-differenced carrier-phase measurements. But there are some caveats. Read on.

"Innovation" is a regular column that features discussions about recent advances in GPS technology and its applications as well as the fundamentals of GPS positioning. The column is coordinated by Richard Langley of the Department of Geodesy and Geomatics Engineering at the University of New Brunswick, who welcomes your comments and topic ideas. To contact him, see the "Contributing Editors" section on page 8.

GPS is rapidly maturing into a system that provides positioning information to wide range of civil users. This phenomenon is driving both the rapid growth of location-based applications and the development of receiver technology targeting the needs of a true mass market. These receivers, designed to operate reliably under adverse conditions and in different environments, are typically based on miniaturized, low-cost modules. They are commonly restricted to modest accuracy (meter range) and low navigation update rates (1 hertz, or a few Hz at most).

Using primarily L1 code ranges for onboard navigation, some receivers also provide raw measurement data sampled at up to 10 Hz and additionally containing L1 carrier-phase ranges. The quality of the latter is usually sufficient to successfully apply relative phase-based processing techniques such as real-time kinematic (RTK) positioning. With such an approach, centimeter-level accuracy, as required for certain scientific applications, becomes possible with mass-market receivers. However, RTK requires the use of a base or reference receiver, typically within a range of 10 to 20 kilometers. Moreover, some kind of static or dynamic initialization is required in order to solve for phase ambiguities.

Instantaneous ambiguity resolution can only be accomplished under exceptionally good conditions. Besides complicating the measurement set-up and the data processing, these constraints can prohibit the application of phase-based differential techniques in certain situations. In response to these challenges, we have investigated an approach for achieving high-accuracy positions based on measurement time differences from a single low-cost receiver.

The idea of time differences is to expand the concept of differential GPS to the time domain. The use of time differences overcomes the need for a base receiver or any initialization process, while still providing decimeter and even sub-decimeter precision when postprocessing data from low-cost receivers. Solely using phase measurements, the method does not use any smoothing or filtering, which could cause hard-to-resolve effects in the resulting solution. However, these advantages do not come for free as the accuracy of a time-difference-based solution inevitably degrades with time (that is, the length of the data-collection time span). Nevertheless, the use of external correction data does allow for precise processing of parts of a platform's trajectory for up to several minutes at a time.

The time-difference GPS postprocessing technique is currently implemented at the Institute of Flight System Dynamics at the Technical University of Munich, where it is used for the precise evaluation of flight-path sections and individual maneuvers such as take-off, flare-out, landing, and dynamic soaring cycles, to gain insight into the flight-mechanics characteristics of the observed aircraft. Because our research is also focused on the flight characteristics of miniaturized aerial vehicles and even birds, the sensor used must be fully self-contained, small, and lightweight. The maximum size should be 150 × 100 × 20 millimeters and the weight must not exceed 100 grams, including the data logger and power supply, which should last for 70 hours. These specifications can be met using one of the recently developed single-frequency GPS receiver modules. As the nature of the intended applications requires high precision without having to worry about any kind of initialization pattern or operating a second nearby receiver, time differences are used. By postprocessing the kinematic phase measurements in time-difference mode, we can achieve the desired results. To validate our approach, we use RTK solutions generated by the Department of Earth Observation and Space Systems at Delft University of Technology (TU Delft). An RTK solution uses both L1 phase and code measurements and is based on resolving the integer ambiguities by means of the LAMBDA method.

The Concept

The basic problem when working with GPS phase data is the unknown cycle ambiguity in all range measurements. The benefit of any time-difference-based approach is the possibility of simply canceling these parameters while obviating the need for a second receiver and dedicated statistical methods for ambiguity estimation.

The Observable. An ideal phase range measurement can be modeled as follows:

$$\Phi(t) = \rho(t) + c\delta^R(t) + \lambda N'$$

with the (true) geometric range, ρ, between receiver and satellite (in meters); the vacuum speed of light, c (in meters per second); the receiver clock offset with respect to GPS System Time, δ^R (in seconds); the L1 carrier wavelength (λ = 0.1903 meters); and the (non-integer) ambiguity N' (in cycles). Note that N' is not a function of time, t, assuming that phase lock is maintained to the observed satellite; it has a constant value since signal lock-on. Differencing across two epochs, t_b and t_i, yields

$$\Phi(t_i) - \Phi(t_b) = \left[\rho(t_i) + c\delta^R(t_i) + \lambda N'\right] - \left[\rho(t_b) + c\delta^R(t_b) + \lambda N'\right].$$

With ∇ denoting the time-difference operator, one obtains

$$^{bi}\nabla\Phi = {}^{bi}\nabla\rho - c\,{}^{bi}\nabla\delta^R$$

where the ambiguity has been canceled!

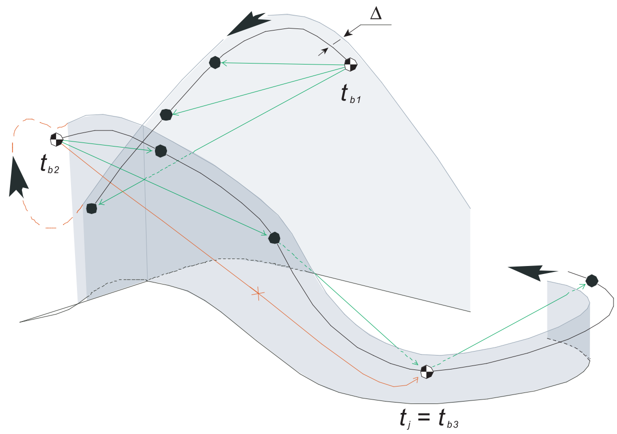
Applications. The advantage of canceling ambiguities is the basis of various applications using time differences in different ways. For example, time-differenced double differences (so called triple differences across two receivers, two satellites, and two observation epochs) can be used for carrier-phase cycle-slip detection. Such observables can also be used for computing a precise baseline between a base or reference receiver and a roving receiver, provided that there are at least seven satellites in view (if only phase data is used). In tightly coupled GPS/INS systems, triple differences can support the dynamics estimation for attitude computation. In a similar context, carrier phases directly differenced between sequential epochs can be used instead of the noisier delta-range measurements to improve attitude information without the need for a base station. Time differences have been used in stand-alone GPS applications to process static data for gun-laying. This approach, enhanced by a loop-misclosure procedure, can also be applied to static measurements from civil receivers.

Kinematic Time Differences. Using time differences for processing kinematic data is an unconventional approach, which emerged from the need for a high-quality but low-effort navigation solution. The model of time-differenced phase ranges, $^{bi}\nabla\hat{\Phi}$, is a function of receiver position and clock bias at the base epoch, t_b, and the current time, t_i. Assuming the position and time bias at the base epoch to be known, one can solve $^{bi}\nabla\hat{\Phi} = {}^{bi}\nabla\tilde{\Phi}$ if measurements, $^{bi}\nabla\tilde{\Phi}$, to at least four satellites continuously tracked between t_b and t_i are available. These equations are linearized and solved iteratively via a least-squares estimator in a similar manner as that for standard single-point positioning. The resulting solution is the baseline vector pointing from the position of the receiver at the base epoch, t_b, to its location at the current time, t_i.

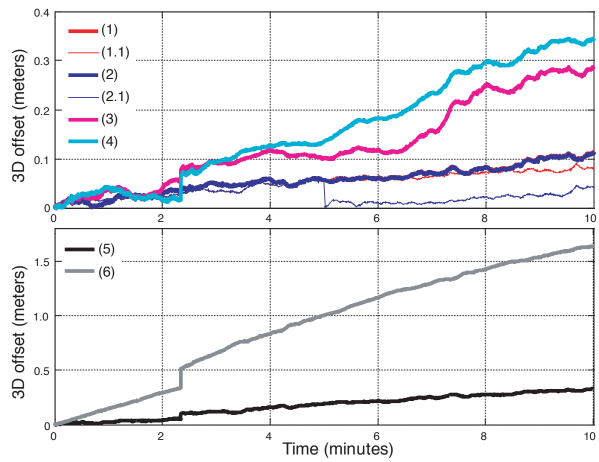
For transforming this basic approach into a tool for successful data processing, several real-world effects have to be accounted for. The measured phase ranges are inevitably afflicted with non-modeled, remaining errors caused by the "usual suspects" in GPS navigation: receiver-independent atmospheric delays, satellite clock and ephemeris errors, and receiver-dependent multipath and measurement noise. The receiver-independent errors cancel completely in the very first moment of processing but start to grow with increasing time spans, $t_i - t_b$. This slow drift directly degrades the solution and is the limiting factor when working with time differences. Increasing the temporal correlation of the non-modeled errors is the only way to compensate for this effect.

In addition, a "geometric" error affecting the quality of the relative solution is caused by an offset of the base position at t_b from the true location. To keep this error acceptably small, the base position has to be determined within an absolute accuracy of a few meters. For these reasons, a way to monitor the quality of the final solution is required for aborting processing when the expected precision exceeds a user-defined threshold. Ambiguities are assumed to be time-invariant during calculations. However, this is only true provided there is constant phase lock during t_i - t_b. Cycle slips are discontinuities in the phase measurements caused by a temporary loss-of-lock in the carrier-tracking loop. Hence, the detection and exclusion of such slips and other outliers has to be ensured when working with time differences.

FIGURE 1 illustrates the algorithm chosen to meet the above-mentioned concerns. At the beginning of the trajectory section to be analyzed, a starting point at t_{b1} is determined via code-based single-point processing. The position of this point is accurate only to within a few meters and therefore offset by Δ from the true track (grey line). All subsequent epochs, t_i, are processed with respect to t_{b1} using time differences and the resulting track (black line) is precise relative to this starting point. If maneuvers causing loss-of-lock on too many satellites are performed (like the Immelmann loop aerobatic maneuver shown by the dashed red line), processing has to be aborted. A new base position at t_{b2} can be imported from the single-point solution right after the maneuver (no re-initialization) and processing can be continued relative to the new base epoch. Such an event will inevitably cause a gap in the resulting trajectory. In the example, the solution fails again between the base epoch, t_{b2}, and the current time, t_j. However, this time there are enough healthy satellites observed at t_{j-1} and t_j to calculate the baseline between these two points (referred to as an inter-epoch solution later in this article). A base-epoch handover preventing a gap in the solution can be realized and processing is hereupon continued to t_{b3}.



▲ **FIGURE 1** Basic principle of time differences.



▲ **FIGURE 2** Static test 3D error: (1) best solution, (1.1) uncertainty estimate of best solution, (2) best solution with handover, (2.1) uncertainty estimate of best solution with handover, (3) no high-rate clocks, (4) broadcast ephemeris, (5) no ionospheric correction, (6) no tropospheric correction.

Quality of Results. Integrity and precision monitoring is realized through residual analysis. The (unbiased) residuals, ε, are the differences between the observed time-differenced phase ranges and the estimated ones; that is, the ranges dropping out from the last iteration step of the non-linear least-squares procedure. For more than four used satellites, the variance of the measurements, $^{bi}\nabla\tilde{\Phi}$, can be estimated from an analysis of the residuals according to

$$\sigma_{\nabla\tilde{\Phi}} = \sqrt{\frac{\sum_{k=1}^{m} \varepsilon_m^2}{m-4}}$$

where m is the number of observed satellites. The Jacobian needed for linearizing the navigation equations is constructed using the line-of-sight unit vectors as in the single-point processing procedure. Therefore, the known concept of position dilution of precision (PDOP) can be applied to estimate the precision of the resulting baseline between t_b and t_i:

$$\sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2} = \mathrm{PDOP} \cdot \sigma_{\nabla\tilde{\Phi}}$$

Using this metric, the expected precision of the solution can be monitored at any time. The quality estimation is made less transparent in the case of a base-epoch handover, as the residual level drops down close to zero for such events. For this reason, handovers should be handled with care! Cycle slips or outliers increase the size of the residuals — which can, in principle, be used as a flag for detecting such bad measurements. As error drift raises the residual level over time, outliers run the risk of being buried by systematic errors. However, the residual level for a solution between subsequent epochs is very low as error drift has virtually no effect over small time intervals. The time-difference approach permits us to use the inter-epoch residual time history for outlier detection and classification. We are currently developing an algorithm to automate this process.

Practical Validation

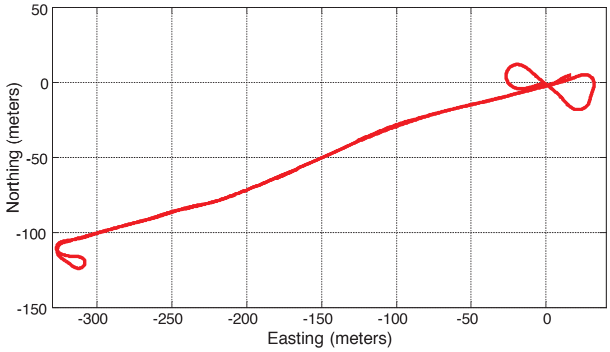
We will illustrate both the potential and the limitations of the time-difference approach using data gathered by a single-frequency receiver module during one static and two dynamic experiments. The receiver module has a footprint of 25 × 25 millimeters and a mass of 3 grams. For the tests, we used an evaluation kit offered by the manufacturer with an active 25 × 25 millimeter patch antenna. An RTK solution generated by TU Delft served as a reference for the dynamic tests.

Static Test. A static test was performed on July 17, 2007, at the airfield of Oberpfaffenhofen, Germany. To attenuate the effect of drifting errors, we applied various corrections when processing the data: signal propagation delays caused by the troposphere were ameliorated using the UNB3 model developed at the University of New Brunswick. The model features three surface parameters (temperature, total pressure, and water vapor pressure) in conjunction with expressions to describe their change with altitude. No external meteorological data are required for modeling. We accounted for ionospheric phase advances by a thin layer model. Here, the total electron content (TEC) is extracted from ionospheric correction maps provided by the International GNSS Service (IGS) with a latency of approximately eight days. For obtaining improved satellite positions and clock corrections, we used final ephemeris products in conjunction with 30-second sampled satellite clocks — both provided by the Center for Orbit Determination in Europe (CODE), also provided with a latency of approximately eight days. More detailed information concerning the correction methods can be found in our research paper listed in “Further Reading.”

At the beginning of the test, eight satellites were used yielding a PDOP of 1.4. After 140 seconds, PRN27 drops below the elevation mask angle of 10 degrees causing the PDOP to increase to 1.6. **FIGURE 2** illustrates the 3D offset of the resulting solution from the reference trajectory. Line (1) shows the best solution calculated using all corrections mentioned above. The drift of the remaining errors is slow and the error stays within 11 centimeters during 10 minutes of processed data. This result agrees well with the estimate of the 3D error shown by line (1.1). Line (2) represents the same solution with a base-epoch handover artificially triggered after 5 minutes. While the final result is virtually unaffected by this event, the estimate of the error, line (2.1), is. The estimate is reset to zero when changing the base epoch. Omitting the 30-second sampled clock corrections, line (3), or using broadcast instead of precise ephemerides, line (4), significantly degrades the solution. Here, the use of clock corrections is even more important than the application of precise orbits. Omitting ionospheric, line (5), or tropospheric, line (6), models while using all other corrections causes a strongly increased error drift which significantly reduces the time span with good precision. The dropout of PRN27 causes steps in all solutions not using high-rate clock solutions. Even though this effect demands further investigation, it is clear that using good satellite-clock corrections when performing time-relative data processing is very important.

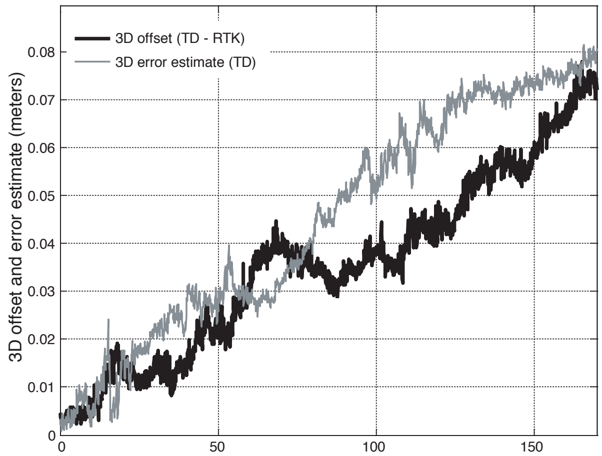
The evaluation of a single test can only provide a first impression. It is not sufficient for making general statements concerning the precision of the time-difference approach or the effect of error-correction models. A strict statistical evaluation is missing. However, the coincidence of the estimate of the error and the true offset of the solution helps us to judge the quality of future tests in the absence of a reference trajectory.

Vehicle Test. The trajectory of a vehicle test performed through open fields on February 14, 2007, with a measurement rate of 10 Hz is illustrated in **FIGURE 3**. For validation purposes



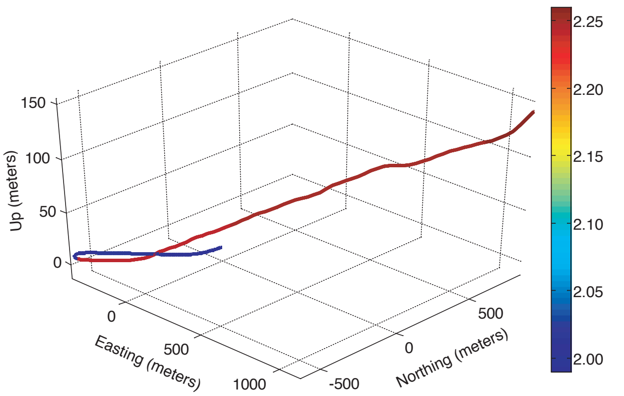
▲ **FIGURE 3** Vehicle test: 2D trajectory in local topocentric coordinates.

▲ **PHOTO** Research aircraft of the Institute of Flight System Dynamics.



▲ **FIGURE 4** Vehicle test: 3D offset of the time-difference solution from the RTK solution with error estimation.

▲ **FIGURE 5** Flight test: Overall trajectory (visualized using Google Earth).



▲ **FIGURE 6** Flight test/take-off: 3D trajectory color-coded with PDOP.

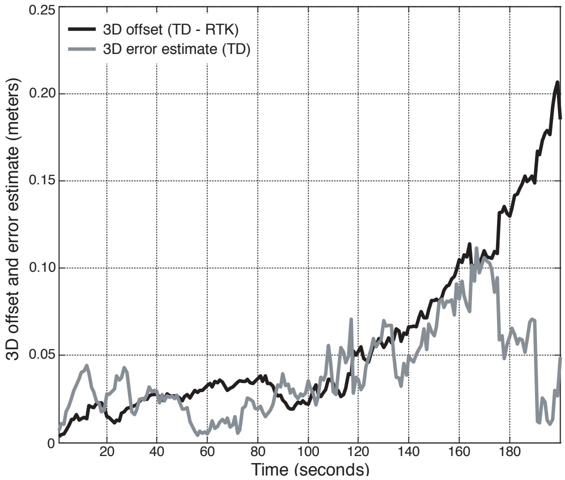
only, we installed a second receiver nearby to serve as a base station, and a static initialization period of about 15 minutes was provided to enable RTK processing. Seven satellites above an elevation mask angle of 10 degrees with a PDOP of 2.1 were tracked during 170 seconds of driving. The time-difference solution was obtained using all of the above-mentioned corrections. Its offset from the reference RTK solution stays below 8 centimeters during the whole processing interval (see the black line in **FIGURE 4**). As with the static test, this result coincides well with the estimate of the 3D error (see the grey line in the figure).

Flight test. We performed a flight test on July 17, 2007, starting from the airfield in Oberpfaffenhofen, Germany. The GPS antenna was mounted on the top of the cockpit of the Institute of Flight System Dynamics' research aircraft (see **PHOTO**).

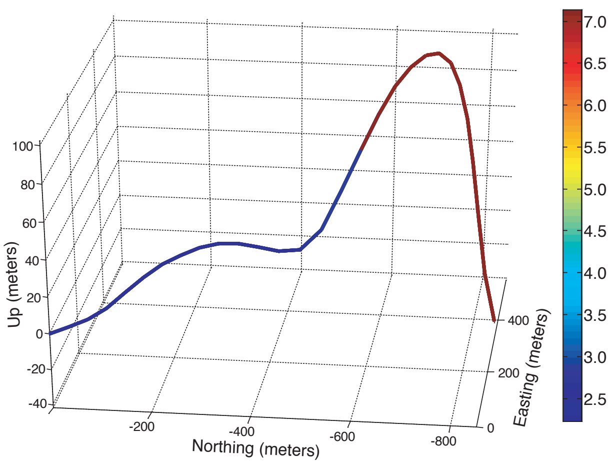
As for the vehicle test, a base receiver was mounted nearby (the one used for the static test previously described) and a static initialization period was provided for generating a reference trajectory. Several dynamic maneuvers with bank angles of up to 70 degrees were performed during the 47-minute flight (see **FIGURE 5**).

Because of the high dynamics, the recorded phase measurements are afflicted with numerous outliers and cycle slips. These errors were detected, identified, and adapted at TU Delft through successful calculation of an RTK solution for the flight. Here, final precise orbits, Saastamoinen *a priori* tropospheric corrections, and Kalman filtering with constant ambiguities (assuming no cycle slips occur) were used to process the data with an elevation cutoff angle of 15 degrees. (The 4-Hz sampled data were processed at 1 Hz for reasons beyond the scope of this article.) Correct ambiguities were obtained after 9 minutes of static initialization. A standard deviation of 1 centimeter was assumed for the undifferenced phase measurements (compared to 1 meter for the code ranges) taken by the low-cost receiver.

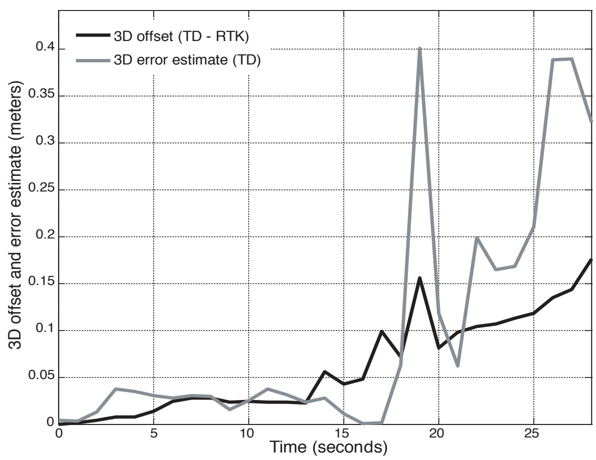
Two representative sections of the flight path were processed with the time-difference method (marked in magenta in Figure 5). Again, high-rate clock corrections in conjunction with



▲ **FIGURE 7** Flight test/take-off: 3D offset of the time-difference solution from the RTK solution with error estimation.



▲ **FIGURE 8** Flight test/dynamic curve: 3D trajectory color-coded with PDOP.



▲ **FIGURE 9** Flight test/take-off: 3D offset of the time-difference solution from the RTK solution with error estimation.

precise ephemerides and tropospheric and ionospheric corrections were used to process the cycle-slip-and-outlier-corrected data. **FIGURE 6** shows the resulting 3D trajectory for taxi and take-off in local topocentric coordinates, color-coded with PDOP values. With the number of used satellites dropping from six to five after 116 seconds, PDOP rises from 2.0 to 2.25. This moderate increase only marginally affects precision itself but has a negative impact on its estimation.

The 3D offset between the time-difference solution and the RTK solution stays within 2 decimeters during 3 minutes and 20 seconds of processing (see black line in **FIGURE 7**). During the first 170 seconds, the estimate of the error (grey line) agrees well with the observed offset. This is not true for the last 30 seconds. Here, the residual level drops to low values, which directly affects the precision estimate. Even though the true trajectory is not known, this effect is most likely caused by the low number of used satellites, which impedes the proper estimation of the measurement variance in this case.

A second section, as depicted in **FIGURE 8**, was processed with time differences. Here, a dynamic soaring-like maneuver was flown with a bank angle of approximately 50 degrees in an upward curve. This trajectory caused the number of used satellites to drop from six to five after 12 seconds and a base-epoch handover after 16 seconds. PDOP values rose from 2.1 to 2.85 after 12 seconds. Following the base handover, the geometry of the visible constellation remained poor with only five satellites available (PDOP of 7.1).

Due to the adverse visibility conditions, measuring the described maneuver is difficult and the time span which can be processed with satisfactory precision is limited to half a minute. The low number of used satellites and the base-epoch handover after 16 seconds (error estimate drops to zero) reduce the reliability of the error estimation. This is confirmed by the grey line in **FIGURE 9**. In this case, the error estimate is very unsteady and too pessimistic — a result which can only be confirmed by comparison with a reference. During the considered time-span, the RTK and time-difference solutions coincide with less than a 2 decimeter offset (see black line in Figure 9).

Conclusions

The time-difference approach is a true L1-only phase-observation processing method for measuring kinematic trajectories. Only one, low-cost, single-frequency GPS receiver is required to obtain decimeter or sub-decimeter precision relative to a starting point for trajectory sections lasting up to several minutes. The temporal limitation represents the main downside of the approach. Further, one has to be aware that phase-data observables are very sensitive to signal obstruction and antenna tilting. This is a disadvantage faced by every type of phase-based processing. The user must always be aware of what he or she is actually doing with the receiver. However, no additional effort such as initialization, implementing a nearby base station, surveying a reference point, or making use of a satellite- or ground-based augmentation system is required for the time-difference method.

The technique permits trajectory determination virtually

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anywhere and anytime. For successful application, the use of error-correction data is the key. A static test showed that besides atmospheric compensation models, the use of precise, 30-second-sampled satellite-clock-correction products is very helpful. The required data are available free of charge but only after a latency of several days, which currently prevents real-time application of the technique. The validation of data from a vehicle test under good GPS conditions with an RTK solution showed that the precision obtained with static data can be maintained when working with kinematic data. Furthermore, the possibility for monitoring the varying precision of the solution has been pointed out. The time-difference method has been developed for flight-data processing. The validation of two flight-path sections with an RTK reference solution with successful ambiguity resolution showed that precision expectations were achieved.

New software and hardware developments have spawned a wide variety of GPS applications unthought of only a few years ago. Consideration of the time-difference method might trigger the development of even more applications, adding to the panoply of GPS uses.

Manufacturer

The tests used evaluation kits incorporating TIM-LL and TIM-LP GPS receiver modules manufactured by **u-blox** (*www.u-blox.com*).

JOHANNES TRAUGOTT received his Dipl.-Ing. degree in mechanical engineering (specialization in aerospace technology) at the Technical University of Munich (TUM) in 2004. Subsequently, he started to work on his doctoral thesis with the Institute of Flight Mechanics and Flight Control at TUM where his focus is on satellite navigation and flight path reconstruction.

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Further Reading

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FURTHER READING

• Time-differences for Stand-alone Navigation

"Two approaches to Precise Kinematic GPS Positioning with Miniaturized L1 Receivers" by D. Odijk, J. Traugott, G. Sachs, O. Montenbruck, and C.C.J.M. Tiberius in *Proceedings of ION GNSS 2007*, the 20th International Technical Meeting of the Satellite Division of The Institute of Navigation, Fort Worth, Texas, September 25-28, 2007, pp. 827-838.

• Triple-differenced Carrier Phases

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• Correction Models and External Correction Data

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