Innovation

Solving Your Attitude Problem

Basic Direction Sensing with GPS

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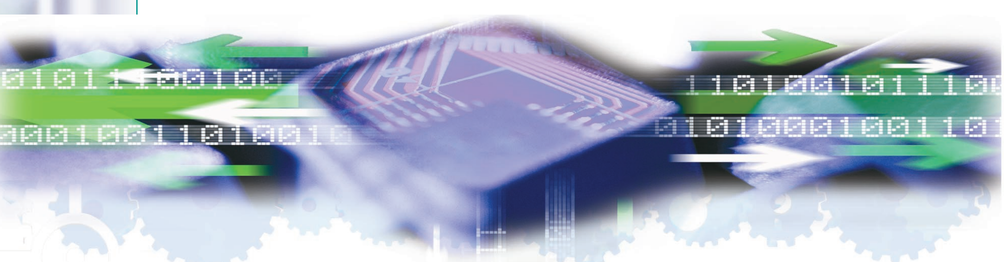
GPS is well known for its ability to determine a platform's position and velocity with high accuracy. Less well known is the ability of GPS also to provide the orientation *of the platform. Using three or more antennas feeding separate receivers, or separate channels in a single receiver, the baseline vectors connecting the antennas can be determined. The directions of these vectors determine the platform's three-dimensional orientation. Using the differences of the carrier phases simultaneously measured by the receiver channels, the baseline orientations can be determined to a fraction of a degree. If only two antennas are used, then only two angles or directions of the platform can be determined, such as the azimuth or heading of the platform and its elevation angle or pitch. In this month's column, Dr. Alessandro Caporali will introduce us to the basics of direction sensing with GPS and describe a prototype sensor he has built and tested on the canals of Venice.*

Dr. Caporali is a professor of solid Earth geophysics in the Centro Interdipartimentale di Studi e Attività Spaziali (CISAS) "G. Colombo" and the Dipartimento di Geologia, Paleontologia e Geofisica of the University of Padova, Italy. He received an M.Sc. in physics from the University of Padova in 1975 and a Ph.D. in physics from the Max Planck Institut für Physik und Astrophysik in Munich, Germany, in 1979. Dr. Caporali specializes in satellite geodesy and very long baseline interferometry. Currently he is responsible for the permanent GPS station UPAD, which is part of the IGS and EUREF scientific networks, and for the UPA Analysis Center which contributes to the regular maintenance of the European Permanent Network of GPS stations.



Using a pair of GPS receivers, we have developed a prototype attitude sensor suitable for navigation on the Earth's surface, or for platforms in low Earth orbit. The baseline joining a pair of antennas defines body-fixed angles, which are estimated in real time using a two-step procedure: first, a coarse estimation is made with the ambiguity resolution function (ARF) algorithm, and then a refined estimate is made by least squares. This approach yields an estimate of a pair of body-fixed angles epoch-wise, independent of the value they had at previous epochs. The estimated angles are unbiased and refer to the true geographic pole. Assuming a short baseline of 0.6 meters, the root mean square (rms) repeatability at 1 Hz is 0.1 degree for the horizontal angle (e.g. heading), and larger by a factor of 4 for the vertical angle, i.e. pitch or roll, depending on whether the baseline is parallel or orthogonal to the heading direction. Complementary use of the GLONASS or the proposed Galileo navigation satellites has the potential to improve epoch-wise on the geometry and, hence, on the rms figure. Alternatively, for greater accuracy a longer baseline may be used. In such a case one or more intermediate antennas may be used in a bootstrap mode, as the epoch-wise solution may be unstable, especially with few satellites in view. A possible application for a long baseline configuration (with a length around 10 meters) is to provide a reference for mapping magnetic declination, for cartographic use. The sensor has the capability to measure relatively small (down to 5-millimeter) changes in the baseline, simultaneously with the angles. As such, it can work as a strain gauge, to monitor large deformable structures in orbit, for example. Having no moving parts, the sensor can withstand shocks and is immune from thermal and mechanical drifts, but is sensitive to the occultation of the navigation satellites produced by nearby obstacles or structures.

Interferometry

GPS is widely recognized for the precision measurement of the baseline vector between pairs of receiver antennas. By differencing the carrier phases simultaneously recorded by the receivers, the coordinates of one end of the baseline (the "remote" or "rover" site) can be established with respect to the other end (the "base" or "reference" site). Physicists refer to this technique of phase differencing as interferometry. If two beams of coherent monochromatic light waves with identical intensity shine on the same surface, they will interfere with each other producing a series of light and dark patterns called fringes. These interference fringes result from the increased intensity of the combined light when the phases of the waves are the same and the absence of light when they are out of phase by exactly 180 degrees. The term "interference" may be a bit confusing. In everyday speech, interference usually suggests opposition or hindrance, but as used in physics, interference can be constructive (when the waves are in phase) as well as destructive (when the waves are out of phase).

If the beams contain a spectrum of colors, such as white light, the fringes will be tinged with color. Such fringes can also be produced if a beam is reflected into our eyes from two closely spaced surfaces. We have all seen such fringes when looking at thin oil or gasoline slicks in puddles of water or at the surfaces of compact discs.

The interference phenomenon applies equally well to radio waves or any kind of wave for that matter — even water and seismic waves.

The phase of the fringe pattern is simply the difference of the phases of the interfering waves. In the case of GPS, the single differences of the carrier phases measured by two receivers are simply fringe phases.

GPS carrier-phase measurements are used mostly for positioning, with antenna coordinates determined either from postprocessing the collected data or in real time with the aid of a communication link. However, GPS interferometry can also provide information on the orientation or direction of the baseline connecting the antennas. If these antennas are rigidly mounted on a platform, then we can derive information on the platform's attitude. Following the work of others (see "Further Reading" sidebar), we have developed a simple prototype system to demonstrate this capability. In this article, I will describe our system and briefly discuss its operation and potential.

The minimum hardware for constructing a demonstration apparatus consists of a pair of conventional off-the-shelf single-frequency receivers each with its own antenna.

(Several companies market single-receiver GPS attitude systems whose receiver channels are fed by two, three, or four antennas. Our system can be replicated without the need for these special receivers.) Short baselines, typically of the order of a meter, are involved, and the length can be assumed very nearly constant and known. Thus the analysis of the fringe phase provides the direction of the baseline as given by the two baseline angles.

For short baselines of the order of meters or even tens of meters, the atmosphere has zero horizontal gradient and has no effect on the differential data. Assuming a resolution in the phase measurement of 10 degrees, or 5 millimeters at the L1 frequency (including the correct number of integer cycles found through a carrier-phase ambiguity search procedure), the orientation angle of a baseline of 1 meter length should be determinable to within about 0.3 degrees. Differential multipath noise will tend to degrade the accuracy, but probably the noise figure of 5 millimeters for the phase is conservative enough for most situations. Increasing the baseline length will scale the theoretical resolution proportionally, but will also tend to introduce more candidates in the ambiguity search, making the selection more uncertain, especially when only 4 or 5 satellites are simultaneously tracked.

Such an attitude/heading sensor has several interesting properties which make it useful in a number of applications. It has no moving parts, implying a high resistance to shocks. Its large mechanical and thermal inertia provides stable measurements.

The horizontal angle (azimuth or heading) is referenced to “true” geographic north, and is unaffected by local magnetic anomalies, unlike a compass. Finally, the sensor is capable of fast system initialization, of the order of one second of time. Typical applications include any kind of pointing and direction finders, closed-loop attitude control systems, calibration of a magnetic compass, initialization of an inertial platform or gyrocompass, and the control of the long-term drift of gyros, for example. If the length of the baseline is included in the estimation process, then the sensor can work as a strain gauge, with spatial resolution of 5 millimeters and sampling rates of the order of 1 Hz. Such a device is of interest for monitoring the low-frequency modes of vibration of large deformable structures whether on the Earth or in space.

Although it is possible to determine the attitude of a short baseline of known length using the observations of only two satellites, the more satellites observed, the smaller the uncertainty in determining the attitude. So the nominal performance of a GPS attitude system can be expected to degrade if just a few satellites are tracked because, for example, the lines of sight to some satellites may be obstructed or the antennas may be inclined to the horizon, resulting in a degraded gain pattern. The complementary use of GLONASS and of the proposed Galileo satellites should help in minimizing the consequence of such a risk, increasing the number of satellites that can be tracked.

The System

To demonstrate the concept of an interferometric attitude sensor, we assembled a “breadboard” prototype (Figure 1) using two single-frequency receivers, each equipped with a standard antenna and 5-meter antenna cable. The receivers communicate with a notebook personal computer (PC) via serial links. The PC is equipped with a PCMCIA Type II serial card providing two serial ports (COM2 and COM4) in addition to the built-in COM1 port.

We wrote our own software in Fortran 90 for data logging and processing, complete with a graphical user interface. Our routines make use of specialized commercial scientific and communication software packages for certain tasks. Once the software opens the COM2/COM4 ports in polled mode for input/output processing, the software transfers data between the COM2/COM4 data segment and the receiver/sender buffer one character at a time. After port initialization, the software logs at 1 Hz three types of messages from the receivers: $RGEA, with code and phase data; $SATA, containing satellite elevation angle and azimuth and rejection flags; and $POSA, with the receivers' latitude, longitude and height determinations.

The data processing is based on the first order model of the between-receiver single differences for each satellite tracked:

$$\Delta\phi^A = \frac{\vec{b}\cdot\hat{s}^A}{\lambda} + \frac{c\Delta t}{\lambda} + N^A + \varepsilon^A \qquad (1)$$

where

$\vec{\mathbf{b}}$ is the baseline vector,

$\hat{\mathbf{s}}$ is the line of sight unit vector to satellite A,

c is the speed of light,

λ is the L1 wavelength,

Δt is the instantaneous offset of the clocks in the two receivers,

N^A is the single difference integer ambiguity, and

ε_A is the noise term, comprising a random measurement error and a quasi-systematic component (multipath).

As mentioned earlier, we assume for such a short baseline that the atmosphere (both ionosphere and troposphere) affects the signals recorded by both receivers identically so that there is no contribution to the single difference. Furthermore, we assume that the signal wavefront is planar on the scale of the baseline so that the direction of a satellite is the same as viewed by both antennas. The first term on the right-hand side of equation 1 is the scalar or dot product of the baseline vector and the unit vector in the direction of the satellite. It is equal to the length of the baseline (in cycles) multiplied by the cosine of the angle between the baseline and the satellite direction.

After detecting the satellites common to both receivers at each epoch, the software selects the satellite with the highest elevation angle as the reference or "hub" satellite. It then forms double differences of the generic A satellite with respect to this hub satellite, labeled H:

$$\nabla\Delta\phi^{AH} = \Delta\phi^A - \Delta\phi^H = \frac{\vec{b}\cdot\left(\hat{s}^A - \hat{s}^H\right)}{\lambda} + N^{AH} + \varepsilon^{AH} \qquad (2)$$

where $N^{AH} = N^A - N^H$, and $\varepsilon^{AH} = \varepsilon^A - \varepsilon^H$.

Note that in forming the double differences, the clock term, originating from the lack of synchronization of the clocks in the two receivers, is automatically removed.

Our software estimates the values of the unknown parameters on the right-hand side of equation 2. These include the two angles of the baseline in the first term of the right-hand side (the spatial angle between the baseline vector and the difference of the satellite unit vectors can be decomposed into two orthogonal angles, say in the horizontal and vertical planes, to give the azimuth and elevation angle of the baseline), and $n - 1$ ambiguities, where n is the number of common satellites observed by both receivers. Apart from the noise term, the remaining quantities on the right-hand side of the equation are the unit vectors describing the directions to the satellites which can be computed from their known azimuths and elevation angles.

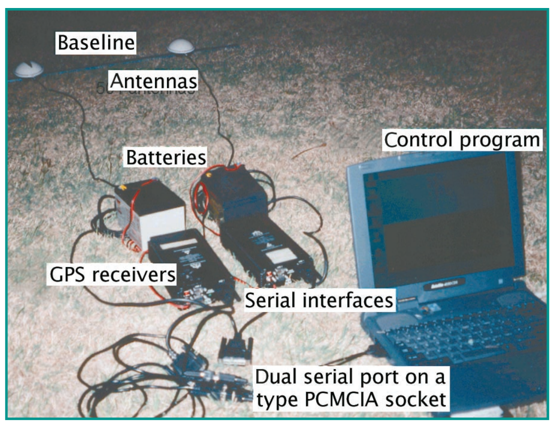


FIGURE 1 The prototype attitude and heading system consists of a pair of antennas mounted on a rigid support at a known spacing, each feeding a separate single-frequency GPS receiver. The receivers are interfaced to a portable computer through two serial ports.

If one used a least-squares or Kalman filter at this stage to estimate angles and ambiguities, some integration time would be required in static mode to enable the angles to decouple from the ambiguities, which is undesirable and impractical. Therefore, we have used a combination of the ambiguity resolution function (ARF) and least-squares algorithms to provide epochwise estimates of the angles and the ambiguities. The ARF method is an ambiguity-independent algorithm originally introduced in 1981 by Prof. Charles Counselman III and Dr. Sergei Gourevitch — two pioneers of the GPS research community. It tests trial values of the azimuth and elevation angle of the baseline and attempts to maximize the ambiguity resolution function:

$$ARF\left(az,el\right) = \cos\left[\nabla\Delta\phi^{AH}_{obs} - \nabla\Delta\phi^{AH}_{trial\left(az,el\right)}\right] \qquad (3)$$

The pair of (az, el) values of the baseline which maximizes the sum of the squares is the "most likely" set, although not in a rigorous least-squares sense. Theoretically, the

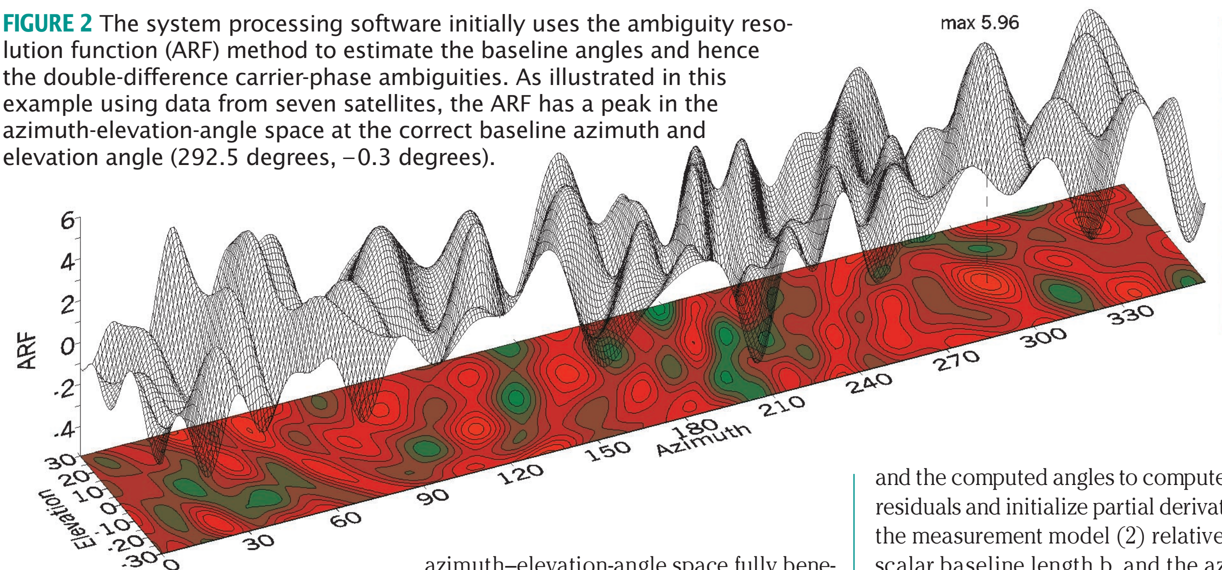


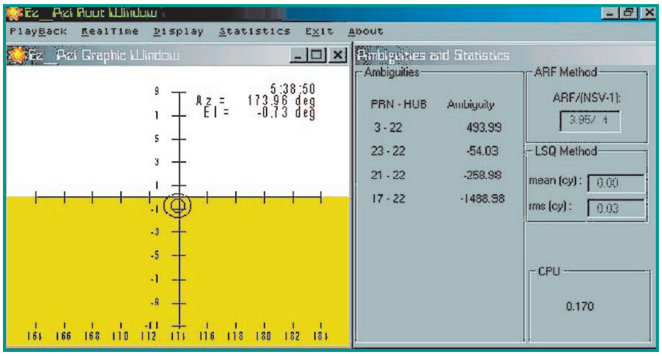
FIGURE 2 The system processing software initially uses the ambiguity resolution function (ARF) method to estimate the baseline angles and hence the double-difference carrier-phase ambiguities. As illustrated in this example using data from seven satellites, the ARF has a peak in the azimuth-elevation-angle space at the correct baseline azimuth and elevation angle (292.5 degrees, −0.3 degrees).

maximum value should coincide with the number of common satellites minus one (i.e. the number of independent combinations of double differences), but phase noise, multipath and quantization error in the ambiguity search will prevent the ARF from achieving the theoretical maximum. The search in azimuth–elevation-angle space fully benefits from the knowledge of the baseline length. In our algorithm, the azimuth search range is a full 360 degrees, while the elevation-angle search range is programmable and was constrained to ±30 degrees to the horizontal (Figure 2).

After maximization of the ARF, the software uses the values of the baseline length and the computed angles to compute pre-fit residuals and initialize partial derivatives of the measurement model (2) relative to the scalar baseline length b, and the azimuth and elevation angle. The ambiguities may be assumed known, after a successful scan in the azimuth–elevation-angle space of the ARF. The 3 3 3 normal equation system must be complemented with an a priori variance-covariance matrix that accounts for the baseline scalar length b being known with higher confidence than the baseline azimuth and elevation angle. The algebraic system is solved and the results are stored in a file and displayed via a graphical user interface (Figure 3).

As shown in Figure 3, the status of our system can be checked with a "child window" displaying an artificial horizon on a grid. Dialog boxes provide time, satellite and position information, and the values of the ambiguities and of the statistics of the solution, both in ARF and least-squares mode. Targets or waypoints can be indicated on the artificial horizon display, helping in open-loop guidance and steering applications such as "blind docking." Off-course angles provide numeric input for closed-loop control. Further system enhancements may include superimposing a bitmap image on the display, grabbed from frames streaming from a digital video camera attached to the antenna support.

FIGURE 3 The system software below features a graphical user interface (GUI) that displays system status including satellites tracked, ambiguity solution, platform position and attitude. The GUI includes an artificial horizon and heading display showing the platform's azimuth and elevation angle.



Testing the System

The present version of the software computes the baseline angles at each epoch, independently of the values they had at previous epochs. This "zero memory" filter implementation ensures the maximum achievable dynamic range, in the sense that it supports random changes in baseline orientation. For more predictable situations, a smoother can help in reducing the epoch-by-epoch measurement noise. The epoch-wise solution mode is suitable for stability tests. We carried out a number of such tests with the system stationary. We found that the rms repeatability of the azimuth is on average 0.11 degrees, and is smaller than for the elevation angle, which has an average rms of 0.43 degrees. Most importantly, we tested the stability of the estimated angles in static mode, measuring the drift in azimuth and elevation angles over intervals ranging from tens of seconds to hours, and with different GPS constellation geometries. A regression analysis showed that there is negligible drift in the values of either angle even for a short lapse of time and when only a few satellites are used. The rms dispersion of the baseline estimates is 5.8 millimeters, which can be considered nominal. A kinematic test on a motorboat in the lagoon and canals of Venice demonstrated the capability of the system in an operational environment (Figure 4).

Conclusion

A pair of standard commercial, single frequency GPS receivers providing both code and carrier-phase measurements can be configured as an accurate sensor providing attitude and heading information, at low cost. The prototype system described in this article shows an rms stability of 0.1 degrees in the horizontal plane (the azimuth or heading) and 0.4 degrees in the vertical plane (the elevation angle of the baseline corresponding to the pitch or roll angle, depending on antenna placement), with a 0.6 meter baseline, at data sampling at a rate of 1 Hz. Using a 300 MHz PC operating in a Windows 98 environment, the duty cycle of the software for data logging and processing is generally 0.2–0.3 seconds, so that without any hardware or software changes one could double the data sampling rate.

The estimation process is sensitive to the number of tracked satellites. If only four GPS satellites are visible, the algorithm can occasionally converge to a wrong result, especially if the baseline length is solved for as an unconstrained parameter. However, as noted, the algorithm has "zero memory" and recovers as soon as the coverage becomes nominal. Future work will extend the solution to include single-differenced phase data from the GLONASS satellites.

FIGURE 4 The antennas were mounted on the bow of a motor boat for tests on Venice's Canal Grande.

Acknowledgment
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Manufacturers
The receivers used for our prototype sensor were **NovAtel** (Calgary, Alberta, Canada) single-frequency *GPS-3051* receivers, each equipped with a standard NovAtel *GPS-501* antenna.

The software was written using the **Lahey Computer Systems** (Incline Village, Nevada) *Lahey Fortran v.4.5* implementation of the ANSI and ISO Fortran 90 standards, complemented with the **Fujitsu** (Kanegawa, Japan) Scientific Software Library and the *SciComm Communication Library* by **MicroGlyph Systems** (Lexington, Massachusetts).

The computer used was a **Toshiba** Satellite 4030 Notebook PC equipped with a dual channel RS232 PCMCIA Asynchronous Adapter DSP-100 manufactured by **Quatech** Inc.

Further Reading
For an introduction to attitude specification and measurement using GPS, see

- "Mathematics of Attitude Determination with GPS," by A. Kleusberg, in *GPS World,* Vol. 6, No. 9, September 1995, pp. 72–78.

For a thorough analysis of GPS attitude determination, see

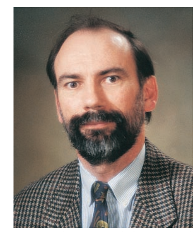
- "Attitude Determination" by C.E. Cohen, Chapter 19 in *Global Positioning System: Theory and Applications, Vol. II,* published by the American Institute of Aeronautics and Astronautics, Inc., Washington, D.C., 1996.

For discussions of example applications of GPS-based attitude determination, see

- "GPS Interferometric Attitude and Heading Determination: Initial Flight Results," by F. van Graas and M. Braasch, *Navigation,* Vol. 38, No. 4, 1991, pp. 297–316.
- "Results of Testing on a GPS-based Compass," by J. Spalding and M. Lunday, in the *Proceedings* of ION GPS-95, the 8th International Technical Meeting of The Institute of Navigation, Palm Springs, CA, 12–15 September 1995, pp. 941–948.
- *Use of GPS for a Berthing Guidance System,* a Ph.D. dissertation by M. Ueno, Département des sciences géomatiques, Université Laval, Québec, 1999.

For details on the ambiguity resolution function, see

- "Miniature Interferometer Terminals for Earth Surveying: Ambiguity and Multipath with the Global Positioning System," by C.C. Counselman and S.A. Gourevitch, published in the *IEEE Transactions on Geoscience and Remote Sensing,* Vol. GE-19, No. 4, 1981, pp. 244–252.
- "Improving the Computational Efficiency of the Ambiguity Function Algorithm," by S. Han and C. Rizos, published in *Journal of Geodesy,* Vol. 70, No. 6, 1996, pp. 330–341.



"Innovation" is a regular column featuring discussions about recent advances in GPS technology and its applications as well as the fundamentals of GPS positioning. The column is coordinated by Richard Langley of the Department of Geodesy and Geomatics Engineering at the University of New Brunswick, who appreciates receiving your comments as well as topic suggestions for future columns. To contact him, see the "Columnists" section on page 4 of this issue.