How Good Can It Get with New Signals?

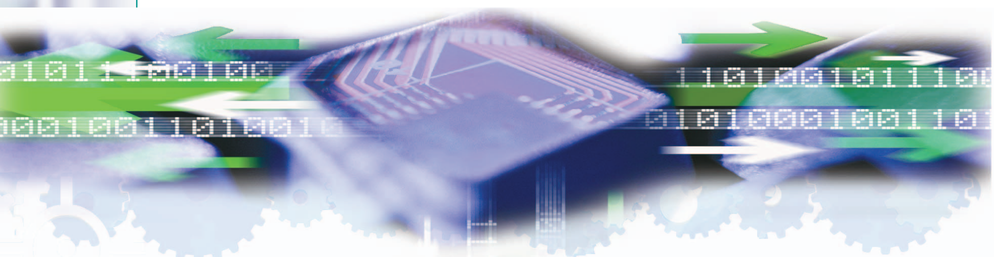
Multipath Mitigation

Lawrence R. Weill

Answer: The winner of the 1971 Nobel Prize for Physics. The question? Who defined the most important signal parameter for controlling GPS multipath? The British/Hungarian physicist Dennis Gabor won the prize for the invention of holography. However, Gabor was also one of a handful of mathematicians and scientists who developed communication theory during the 1940s.

Communication theory, also called *information theory*, is the branch of mathematics that deals with the efficient and accurate transmission of information-bearing signals from one place to another. Key to the theory is the concept of signal bandwidth. Now bandwidth can be defined in a number of different ways, but the particular bandwidth named after Gabor determines, in part, how accurately a GPS receiver can measure pseudorange or carrier phase in the presence of multipath. The Gabor bandwidth is determined by the particular shape of the signal's power spectral density function, and the larger the bandwidth the more resistant is the signal to multipath. The GPS receiver must have a processor that takes advantage of this resistance to provide measurements with minimal multipath contamination.

In this month's column, Dr. Lawrence Weill outlines an implementation of a multipath mitigation algorithm based on the statistical theory of maximum likelihood and describes its expected performance with the new GPS signals soon to be available — signals characterized by a higher Gabor bandwidth than those currently transmitted. — *R.B.L.*



Despite continuing improvements in GPS receivers, multipath signal propagation has remained a dominant cause of error in differential positioning. *Multipath* refers to the existence of signals reflected from objects in the vicinity of a receiver's antenna that corrupt the direct line-of-sight signals from the GPS satellites, thus degrading the accuracy of both code-based and carrier phase–based measurements. Particularly difficult is close-in multipath in which the reflected secondary signals arrive only slightly later (within about 100 nanoseconds) than does the direct-path signal, having been reflected from objects only a short distance from the receiver antenna.

For two fundamental reasons, close-in multipath is likely to cause more trouble than do signals arriving with greater relative delay. The first is that with few exceptions, signals reflected from nearby objects suffer less spreading loss, hence tend to be much stronger than secondary signals arriving from more-distant objects. In addition, it is more difficult to separate a close-in secondary signal from the desired signal because the combination of the two is almost indistinguishable from an uncontaminated signal.

The battle to reduce multipath errors takes place on a number of fronts. In some applications an effective solution is simply to site the antenna in an area free of nearby reflectors. When this is not possible, special antennas can be used that have low response in directions where reflectors are located. If the receiver is stationary and signals can be received for many hours or days, secondary-path signals can sometimes be isolated by observing cyclic patterns in signal-to-noise ratio (SNR) as the relative phase of secondary paths changes due to satellite motion. Receiver-based algorithms with various degrees of immunity to secondary-path interference have also been developed. Finally, the GPS signals themselves can be designed to provide inherent resistance to multipath errors.

The GPS modernization program has provided the opportunity to select new signal types with improved multipath immunity. The most important signal parameter in this regard is the Gabor bandwidth, which is the square root of the second moment of the normalized signal spectrum. (*Second moment* describes the spread of signal power within the signal bandwidth and is similar to the calculation of the variance of a statistical quantity.) Increasing the Gabor bandwidth improves inherent resistance to multipath by moving a greater percentage of total power farther from the signal center frequency. The new civil L5 and military M-coded signals will accomplish this for the respective civilian and military users. Higher transmitted power levels will also augment inherent multipath resistance of the modernized signals, but only when receiver-based multipath mitigation algorithms are designed to use the advantages of higher SNR. Unfortunately, almost all currently used algorithms do not fall into this category.

This article presents results of research conducted with my colleague, Dr. Benjamin Fisher, CEO of Comm Sciences Corporation, about the inherent multipath resistance of the complete suite of modernized signals. *Inherent* implies that the quoted results are obtained using theoretically optimum processing, which can be quite different from current mitigation methods. The article starts with a short history of mitigation techniques, followed by a discussion of theoretical error bounds for code-based pseudoranging, both with and without multipath. It will show that in the presence of multipath, the minimum mean-square error (MMSE) pseudorange estimator reaches a theoretical performance limit but requires far too much computation to be of practical value.

The article then describes a solution to this problem, which is a special implementation of a maximum-likelihood (ML) pseudorange estimator we developed in 1996. This estimator, which we call *multipath mitigation technology* (MMT), was found to have essentially the same performance as the MMSE estimator but is practical to implement in a GPS receiver.

Following a brief description of the characteristics of the modernized GPS signals, the multipath performance of each signal type using MMT is compared with the performance using the most popular current mitigation technique. Results for both code-based and carrier-phase measurements are presented.

Receiver-Based Mitigation

In 1973 Lawrence L. Hagerman of The Aerospace Corporation investigated the effect of multipath on L1 C/A-code measurements in receivers using standard code-tracking methods. At that time it was standard practice to limit the bandwidth of the received signal to approximately 2 MHz and to use early–late code spacing of 1 chip in the tracking correlators. Hagerman found that, depending on the relative delay, phase, and amplitude of the secondary path, pseudorange errors as large as 70–80 meters could occur, and that the errors are caused when the peak of the correlation function is displaced from its true position by the secondary path component(s) of the received signal.

In the years that followed Hagerman's work, various methods of mitigating these effects were developed, with varying degrees of success. In one such method the receiver tracks the leading edge of the code's correlation function, which is multipath free, instead of its peak. Serious drawbacks to this method are that the SNR at the leading edge is much smaller than at the peak, and that some method must be used to make the tracking insensitive to the slope of the correlation function's leading edge, which varies with SNR.

A major breakthrough in multipath mitigation using the C/A code became widely known when "AJ" Van Dierendonck and his co-authors presented a paper about narrow-correlator technology at the 1992 Institute of Navigation (ION) National Technical Meeting. The authors demonstrated that significant reduction in multipath error can be achieved by using a receiver bandwidth much wider than was in current practice at that time, coupled with a significantly smaller spacing between the early and late correlator reference waveforms in the code-tracking loops. The large 70–80 meter errors described by Hagerman using the C/A code were reduced to about 8–10 meters under similar multipath conditions (usually one secondary path signal at one-half the amplitude of the direct path). However, for the C/A code this residual error persists out to about 300 meters of secondary-path relative delay.

In the 1993–1994 time frame, new methods attempted to cancel multipath errors by measuring the distortion of the correlation function caused by secondary-path signal components. Notable among these methods was Multipath Estimation Technology (MET) developed by Brian Townsend and Pat Fenton at NovAtel. In 1995 NovAtel introduced the first widely known and practical method of mitigation based on modern estimation theory, the Multipath Estimating Delay-Lock Loop (MEDLL). MEDLL, still in current use, is a maximum-likelihood estimation technique pioneered by Richard Van Nee at the Delft University of Technology. It improves the C/A-code narrow-correlator performance by confining the residual pseudorange error to a smaller region of secondary-path relative delay (out to approximately 30 meters). Within this range, the residual error is reduced to approximately 5 meters worst-case with a one-half amplitude secondary-path signal when the receiver bandwidth is 8 MHz. Although MEDLL requires a large number of correlators and quite a bit of algorithmic computation, it was an important evolutionary step in the receiver-based battle against multipath.

MCRW Technology. The strobe correlator, a very simple yet effective C/A-code receiver-based multipath mitigation technique, was introduced by Ashtech in a paper at the ION GPS 1996 conference. Although not quite as effective as MEDLL, the strobe correlator retains the advantages of MEDLL in that it completely eliminates C/A-code multipath beyond a path separation of 30 meters, and its simplicity is certainly appealing. Its debut spawned a variety of similar approaches that use a modified receiver correlator reference waveform that narrows the cross-correlation function in the code-tracking loop (see the "Further Reading" sidebar). Typical names given to this generic approach by various parties are *second derivative correlator, gated correlator,* and *pulse aperture correlator.* Some receiver manufacturers such as Trimble and Javad Navigation Systems may use variants of this method without making it explicit. Although there may be variations in the reference waveforms used, we will call this generic approach the *modified correlator reference waveform* (MCRW) method. It represents the current state of the art in many GPS receivers.

Unfortunately, the MCRW technique is not effective at small path separations (less than approximately 30 meters), where its performance still departs greatly from the achievable bounds described later in this article. Such small separations commonly occur when the GPS signal bounces from the ground below the receiver antenna. The ground bounce phenomenon is often a concern except in the case of antennas mounted flush with the ground. Furthermore, with the P/Y, civil L5, and military M codes, the MCRW method cannot provide significant improvement in multipath mitigation beyond what is already available, due to spectral efficiency of these codes.

Multipath Signal Model

In studying the effectiveness of multipath mitigation methods, the following 2-path signal model will be used:

$$r(t) = A_1 e^{j\phi_1} m(t-\tau_1) + A_2 e^{j\phi_2} m(t-\tau_2) + n(t) \qquad (1)$$

in which $r(t)$ is the complex-valued received signal at baseband, and $n(t)$ is an additive zero-mean complex Gaussian noise process with a flat power spectral density. The parameters A_1, ϕ_1, τ_1 are, respectively, the direct path signal amplitude, phase, and delay, and the parameters A_2, ϕ_2, and τ_2 are the corresponding parameters for the secondary path (by setting the parameter A_2 equal to zero, this model can also be used when no multipath exists). For convenience, we group the signal parameters into the vector

$$\bar{\theta} = [A_1, \phi_1, \tau_1, A_2, \phi_2, \tau_2] \qquad (2)$$

For simplicity, our model includes only one secondary path but can easily be extended to include additional secondary paths. It is assumed that the signal in Equation 1 has been frequency-shifted to baseband, Doppler-compensated, and stripped of navigation data modulation by well-known means.

Observation of the received signal is accomplished by sampling it at a sufficiently high rate during the time interval $[0,T]$ to produce a complex observed vector $\bar{r}$. The objective of pseudorange multipath mitigation is to accurately estimate the direct path delay τ_1 with minimum error resulting from multipath, according to an established error criterion.

Bounds on Multipath Error

GPS receiver thermal noise places a theoretical limit, called the *Cramer-Rao* (CR) bound, on pseudorange measurement error (see the "Further Reading" sidebar). This expression for the bound is

$$\sigma_{error} \geq \left[\frac{1}{2(E/N_0)(2\pi)^2 \int_{-\infty}^{\infty} f^2 S_m(f)\,df} \right]^{\frac{1}{2}} \qquad (3)$$

in which σ_{error} is the standard deviation of the pseudorange error in seconds (multiply by the vacuum speed of light to convert to meters). The right side of this inequality is the CR bound in which E/N_0 is the ratio of received signal energy to the noise power spectral density, f is frequency in Hertz, and $S_m(f)$ is the normalized power spectral density of the GPS pseudorandom code modulation (i.e., the integral of $S_m(f)$ is unity).

It is possible to design a GPS receiver whose raw pseudorange measurement error without multipath closely approaches the bound given earlier if the GPS signals are at least of moderate strength. Such a receiver would use an optimal 1-path maximum likelihood (ML) pseudorange estimator requiring expensive correlators whose reference waveforms closely match those of the received signal, including the effects of receiver filtering. In practice, the correlators of all GPS receivers (including those using the so-called narrow correlator) use simpler reference waveforms that are easily generated from digital logic, resulting in a noticeable departure from the theoretical performance limit given by the CR bound. Even so, the savings in cost and complexity is a price willingly paid.

The fundamental quantities that appear in the CR bound are E/N_0 and the integral in the denominator of Equation 3, which is the second moment of the code's normalized power spectral density. The square root of the integral is the Gabor bandwidth, defined earlier. Both E/N_0 and the Gabor bandwidth should be as large as practicable to achieve the minimum theoretically achievable pseudorange error. Because of the weighting by f^2 in the integral, the Gabor bandwidth can be increased by using a pseudorandom code whose power spectrum has most of its power concentrated at the outer edges of its frequency band.

Error Bound with Multipath. Experimental evidence based on simulations supports the hypothesis that for a given value of E/N_0, the inherent ability of a GPS signal to resist errors from multipath is also determined by the Gabor bandwidth of its pseudorandom code. A code with a large Gabor bandwidth generally has a narrower autocorrelation function with a sharper peak whose location is less affected by multipath components. In general many possible codes exist with essentially the same Gabor bandwidth, and the detailed structure of the chipping sequences for these codes has negligible bearing on inherent resistance to multipath errors.

Unfortunately, when multipath is present the 1-path ML pseudorange estimator can no longer be claimed optimal in the sense of reaching the CR bound, because the estimates will no longer be unbiased. Therefore, the CR bound, which is valid only for unbiased estimators, cannot be used. Can we find a useful error bound when multipath is present?

MMSE Estimator. In 1995 I developed such a bound, which is provided by the MMSE estimator. The MMSE estimator of direct path delay τ_1 is a function $\hat{\tau}_1 = f(\bar{r})$, which is optimal in the sense that it minimizes the conditional expectation, given the observation $(\bar{r})$, of the squared error $(\hat{\tau}_1 - \tau_1)^2$ with respect to the joint probability density of the six multipath parameters. It can be shown that the MMSE estimator of direct path delay τ_1 is simply the conditional expectation of τ_1, given $\bar{r}$ (see the "Further Reading" sidebar):

$$\begin{aligned}\hat{\tau}_1 &= E\{\tau_1 | \bar{r}\} \\ &= \int_{\bar{\theta}} \tau_1\, p(\bar{\theta}|\bar{r})\, d(\bar{\theta}) \\ &= \frac{1}{p(\bar{r})} \int_{\bar{\theta}} \tau_1\, p(\bar{r}|\bar{\theta})\, p(\bar{\theta})\, d(\bar{\theta}),\end{aligned} \qquad (4)$$

in which

$$p(\bar{r}) = \int_{\bar{\theta}} p(\bar{r}|\bar{\theta})\, p(\bar{\theta})\, d\bar{\theta} \qquad (5)$$

These equations use a single integral sign to denote what is actually an integration over the six-dimensional space of multipath parameters. It should be noted that replacing τ_1 by ϕ_1 in Equation 4 will yield the MMSE estimate $\hat{\phi}_1$ of direct path carrier phase ϕ_1.

Unfortunately, it is seldom the case that the multipath parameters can justifiably be modeled as random variables with a known a priori probability density p($\bar{\theta}$). The MMSE estimator may still be used when the multipath parameters are nonrandom by assuming a density function p($\bar{\theta}$) (usually a uniform joint density gives excellent results), but the aforementioned optimality property can then no longer be guaranteed. However, in this case the MMSE estimator has another optimality property that is independent of the assumed p($\bar{\theta}$) and the unknown true multipath parameter values, namely, no other estimator has a uniformly smaller root-mean-square (r.m.s.) error. **Figure 1** shows an example of the performance of the MMSE estimator for C/A-code measurements.

Computational Problems. In view of the foregoing optimality property, why not just use the MMSE estimator in a GPS receiver? Unfortunately, the six-fold integration required to estimate τ_1 (or any of the other parameters) is computationally intractable. A rough calculation indicates the magnitude of the problem. Suppose that each of the six parameters in Equation 2 requires 100 samples to span its possible range of values with adequate resolution. Then the integrals in Equations 4 and 5 would each require summation over $(100)^6 = 10^{12}$ sample values, clearly not feasible in real time.

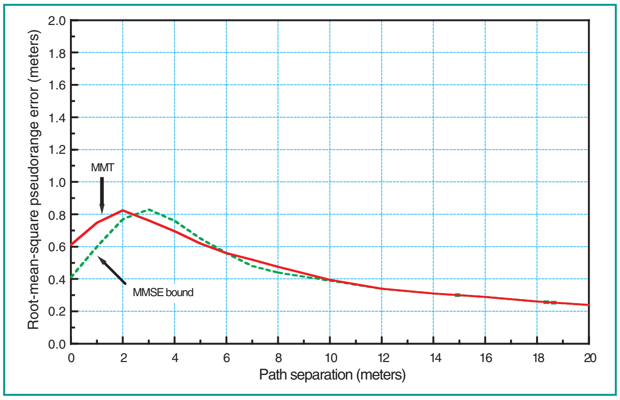


FIGURE 1 Comparison of the MMSE and MMT algorithms in mitigating multipath errors for pseudoranging with the C/A-code. RF signal bandwidth is limited to 20 MHz, E/N_0 is 45 dB-Hz-s, and the secondary path amplitude is 6 dB below the direct path. The r.m.s. error is averaged over random secondary-path relative phases.

Two-Path ML Estimator

In view of the computational intractability of the MMSE estimator, my colleague and

I focused our research efforts from 1995 to 1998 on finding an algorithm that would perform as well, or nearly as well, but with much less computation. Realizing that a 1-path ML estimator is essentially optimum when there is no multipath, I investigated the possibility of using an ML estimator designed to handle multiple paths. This choice was motivated by the fact that in general ML estimators have excellent performance, and also because the ML estimate involves only maximization of a likelihood function over a parameter space instead of the difficult multidimensional integration required by the MMSE estimator. Simulations of a 2-path ML estimator were gratifying, as can be seen from the similar performance of this estimator and the MMSE estimator using C/A-coded signals as shown in Figure 1.

Unfortunately, we found that although the 2-path ML estimator has a smaller computational load than does the MMSE estimator, the amount of computation is still a daunting task. A brute-force search over the 6-dimensional multipath parameter space to find the maximum of the likelihood function takes too long to be of practical value. Reliable gradient-based or hill-climbing methods are also too slow to be useful. Finding the maximum using differential calculus is also difficult because of the nonlinearity of the resulting equations and the possibility of local likelihood function maxima, which may not be global maxima. Iterative solution techniques are often difficult to analyze and may not converge to the correct solution in a timely manner, if they converge at all.

MMT Algorithm

Late in 1998, several major breakthroughs resulted in a practical approach for implementing the 2-path ML estimator, MMT. MMT solves the previously discussed computational problems by using a nonlinear transformation on the multipath parameter space to permit rapid computation of a log-likelihood function that has been partially maximized with respect to four of the six multipath parameters. The final maximization requires a search in only two dimensions, aided by acceleration techniques. To further increase computational efficiency, MMT operates on a data vector of small dimensionality, obtained by a proprietary method for lossless compression of the raw signal observation data.

Baseband Signal Samples. In developing the MMT algorithm, we can benefit from separating the complex baseband signal $r(t)$ given in Equation 1 into its real component $x(t)$ and imaginary component $y(t)$:

$$\begin{aligned} x(t) &= A_1 \cos\phi_1 m(t-\tau_1) \\ &\quad + A_2 \cos\phi_2 m(t-\tau_2) + n_x(t), \\ y(t) &= A_1 \sin\phi_1 m(t-\tau_1) \\ &\quad + A_2 \sin\phi_2 m(t-\tau_2) + n_y(t), \end{aligned} \tag{6}$$

in which $n_x(t)$ and $n_y(t)$ are independent, real-valued, zero-mean Gaussian noise processes with flat power spectral density. Both $x(t)$ and $y(t)$ are synchronously sampled on the time interval $[0,T]$ somewhat above the Nyquist rate $2W$, corresponding to the low-pass baseband bandwidth W, to produce the vectors $\bar{x}$ and $\bar{y}$ in which the noise components of distinct samples are essentially uncorrelated (hence independent, because they are jointly Gaussian).

Log-Likelihood Function. The ML estimates of the six parameters in the vector $\bar{\theta}$ given by Equation 2 are obtained by maximizing the likelihood function, or equivalently, the log-likelihood function, with respect to these parameters. For MMT the likelihood function is the joint probability density $p(\bar{x}, \bar{y}|\bar{\theta})$ of the sample vectors $\bar{x}$ and $\bar{y}$ with $\bar{\theta}$ as a functional parameter vector. The ML estimate of the six multipath parameters is obtained by finding the vector $\bar{\theta}$ which maximizes $p(\bar{x}, \bar{y}|\bar{\theta})$ using the fixed values of vectors $\bar{x}$ and $\bar{y}$ based on observation of the baseband GPS signal. The maximization of the likelihood function is equivalent to maximization of the log-likelihood function which, in turn, is equivalent to minimization of

$$\begin{aligned} \Gamma &= \int_0^T \begin{bmatrix} x(t) - A_1 \cos\phi_1 m(t-\tau_1) \\ -A_2 \cos\phi_2 m(t-\tau_2) \end{bmatrix}^2 dt \\ &\quad + \int_0^T \begin{bmatrix} y(t) - A_1 \sin\phi_1 m(t-\tau_1) \\ -A_2 \sin\phi_2 m(t-\tau_2) \end{bmatrix}^2 dt \end{aligned} \tag{7}$$

with respect to the six multipath parameters. This is a highly coupled, nonlinear minimization problem on the six-dimensional space spanned by the parameters A_1, ϕ_1, τ_1, A_2, ϕ_2, and τ_2, and as previously discussed, is very difficult to solve.

However, a major breakthrough results by using an invertible transformation to reduce the number of unknowns to four, represented by a, b, c, and d. Taking the partial derivatives with respect to these new parameters results in a linear system of four equations whose coefficients are determined by correlation functions described below.

For each pair of values τ_1 and τ_2, this linear system can be explicitly solved for the minimizing values of a, b, c, and d. Thus the space to be searched for a minimum is now 2-dimensional instead of 6-dimensional. The minimization procedure is as follows: Search the (τ_1, τ_2) domain. At each point (τ_1, τ_2) compute the values of the correlation functions in the linear system and then solve the system to find the values of a, b, c, and d which minimize Γ at that point. Identify the point $(\hat{\tau}_1, \hat{\tau}_2)_{ML}$ where the smallest of all such minima is obtained, as well as the associated minimizing values of a, b, c, and d. Transform these values of a, b, c, and d back to the estimates $\hat{A}_{1ML}, \hat{A}_{2ML}, \hat{\phi}_{1ML}, \hat{\phi}_{2ML}$ by using the inverse of the parameter transformation.

Computation of Correlation Functions. The minimization procedure involves the correlation functions $R_{xm}(\tau)$, $R_{ym}(\tau)$, and $R_{mm}(\tau)$, which are computed very rapidly by first applying a proprietary signal compression process in which the large number of signal samples (on the order of 10^8–10^9) that would normally be involved is reduced to only 4 to 27 samples, depending on which type of modernized signal is being processed. This process is easily done in real time by a field programmable gate array (FPGA) or application-specific integrated circuit (ASIC) and permits the correlation functions to be computed by the receiver's general-purpose microprocessor instead of requiring multiple correlators in the FPGA or ASIC.

In addition to providing an output of extremely low dimensionality, signal compression preserves the fidelity of the signal waveform so that the correlation functions can use a reference waveform that is truly matched to the incoming signal, as modified by the combined bandpass characteristics of filters used in the satellites and the receiver. This is strikingly different from the conventional correlation processing used in today's GPS receivers, in which nonfiltered correlator reference waveforms cause pseudorange bias errors. Such errors are normally not a problem in a multipath-free environment, because the bias is identical for all satellite channels and can thereby be included in the navigation solution for receiver clock bias. However, elimination of this bias is critical in optimal multipath processing because it will vary across channels having different degrees of multipath contamination.

Secondary Path Amplitude Constraint. In the majority of multipath scenarios, the amplitudes of secondary-path signals are smaller than those of the direct path. The multipath mitigation performance of MMT can be significantly improved by minimizing Γ subject to the constraint

$$\frac{A_2}{A_1} \le \alpha \tag{8}$$

in which α is a positive constant (a typical value is 0.7). The constraint in terms of the transformed parameters a, b, c, and d is

$$b^2 + d^2 \le \alpha^2\left(a^2 + c^2\right) \tag{9}$$

The constrained minimization of Γ uses the method of Lagrange multipliers.

Modernized Signal Structures

Although five types of pseudorandom codes will be used in the modernized signals, for purposes of multipath performance, the signals can be grouped into only three classes according to their gross spectral characteristics. In describing these classes, I will omit detailed signal characteristics, which are not relevant to multipath performance (the interested reader can refer to references in the "Further Reading" sidebar for more information about the modernized signals).

L1 C/A- Coded and L2 Civil Signals (Class I). *The C/A-Coded Signal:* The GPS modernization program retains the C/A code at the L1 carrier frequency (1575.42 MHz) for legacy purposes, mostly for civilian users. These codes are maximal-length direct-sequence Gold codes, each consisting of a 1023-chip sequence transmitted at 1.023×10^6 chips per second that repeats every 1 millisecond.

The L2 Civil Signal. Originally the modernization plan also called for the C/A code at the L2 carrier frequency (1227.60 MHz) to provide the civilian community with ionospheric correction capability as well as additional flexibility and robustness. However, late in the planning process participants saw that additional advantages could be obtained by replacing the planned L2 C/A-code signal with a new L2 civil signal (L2CS). The decision was made to use this new signal, and its structure was made public in early 2001. Both the L2CS and the new military M-coded signal, described later in this article, will appear on the L2 in-phase (I) channel, with the P/Y-coded signal on the quadrature (Q) channel.

Like the C/A code, the L2CS code appears to be a 1.023×10^6 chips per second sequence. However, it is generated by 2:1 time-division multiplexing of two independent subcodes, each having half the chipping rate, namely 511.5 $\times 10^3$ chips per second. Each of these subcodes is made available to the receiver by demultiplexing.

P/Y-Coded and L5 Civil Signals (Class II). *P/Y-Coded Signal.* For legacy purposes, GPS modernization will retain the P/Y-code on both the L1 and L2 frequencies. This code will be in phase quadrature with the C/A-code and the military M-code at the L1 frequency and at L2 will be in quadrature with the new L2 civil signal and the M-code. The P/Y-code is transmitted at 10.23×10^6 chips per second in either unencrypted (P-code) or encrypted (Y-code) form. The P-code

Further Reading

For a more complete recent history of GPS multipath mitigation, see

- *Global Positioning Systems, Inertial Navigation, and Integration*, by M.S. Grewal, L.R. Weill, and A.P. Andrews, Wiley and Sons, New York, 2001, pp. 71–76.
- "Conquering Multipath: The GPS Accuracy Battle," by L.R. Weill, *GPS World*, Vol. 8, No. 4, April 1997, pp. 59–66.

Theoretical limits for receiver-based multipath mitigation are developed in

- "Achieving Theoretical Accuracy Limits for Pseudoranging in the Presence of Multipath," by L. Weill, published in the *Proceedings of ION GPS-95, the 8th International Technical Meeting of the Satellite Division of The Institute of Navigation,* held in Palm Springs, California, September 12–15, 1995, pp. 1521–1530.

Properties of maximum-likelihood estimators and the Cramer-Rao bound on estimation error are described in

- *Introduction to Probability and Mathematical Statistics*, by L. Bain and M. Engelhardt, PWS Publishers, Boston Massachusetts, 1987, pp. 264–296.

Minimum mean square error (MMSE) estimation, a special case of Bayes estimation, is treated in

- *Detection, Estimation, and Modulation Theory, Part I*, by H. Van Trees, Wiley & Sons, New York, 1968, pp. 54–56.

For a description of multipath mitigation using the narrow correlator, see

- "Theory and Performance of Narrow Correlator Spacing in a GPS Receiver," by A.J. Van Dierendonck, P. Fenton, and T. Ford, published in the *Proceedings of the Institute of Navigation National Technical Meeting*, held in San Diego, California, January 27–29, 1992, pp.115–124.

Details about the Multipath Estimating Delay-Lock Loop (MEDLL) can be found in

- "Performance Evaluation of the Multipath Estimating Delay-Lock Loop," by B. Townsend, D.J.R. van Nee, P. Fenton, and K. Van Dierendonck, published in the *Proceedings of The Institute of Navigation National Technical Meeting*, held in Anaheim, California, January 18–20, 1995, pp. 277–283.
- "L1 Carrier Phase Multipath Error Reduction Using MEDLL Technology," by B. Townsend, P. Fenton, K. Van Dierendonck, and R. van Nee, published in the *Proceedings of ION GPS-95, the 8th International Technical Meeting of the Satellite Division of The Institute of Navigation,* held in Palm Springs, California, September 12–15, 1995, pp. 1539–1544.

Descriptions of multipath mitigation using modified correlator reference waveforms (MCRW) appear in the following references:

- "Strobe & Edge Correlator Multipath Mitigation for Code," by L. Garin, F. van Diggelen, and J. Rousseau, published in the *Proceedings of ION GPS-96, the 9th International Technical Meeting of the Satellite Division of The Institute of Navigation,* held in Kansas City, Missouri, September 17–20, 1996, pp. 657–664.
- "GPS Multipath Mitigation by Means of Correlator Reference Waveform Design," by L. Weill, published in the *Proceedings of The Institute of Navigation National Technical Meeting*, held in Santa Monica, California, January 14–16, 1997, pp. 197–206.
- "GNSS Multipath Mitigation Using Gated and High-Resolution Correlator Concepts," by G. McGraw and M. Braasch, published in the *Proceedings of The Institute of Navigation National Technical Meeting*, held in San Diego, California, January 25–27, 1999, pp. 333–342.

Descriptions of the modernized GPS signals can be found in

- "The Modernized L2 Civil Signal," by R.D. Fontana, W. Cheung, and T. Stansell, *GPS World*, Vol. 12, No. 9, September 2001, pp. 28–32.
- "The New L5 Civil Signal," by A.J. Van Dierendonck and C. Hegarty, *GPS World*, Vol. 11, No. 9, September 2000, pp. 64–70.
- "Overview of the GPS M-Code Signal," by B.C. Barker, et al., published in the *Proceedings of The Institute of Navigation National Technical Meeting* held in Anaheim, California, January 26–28, 2000, pp. 542–549.

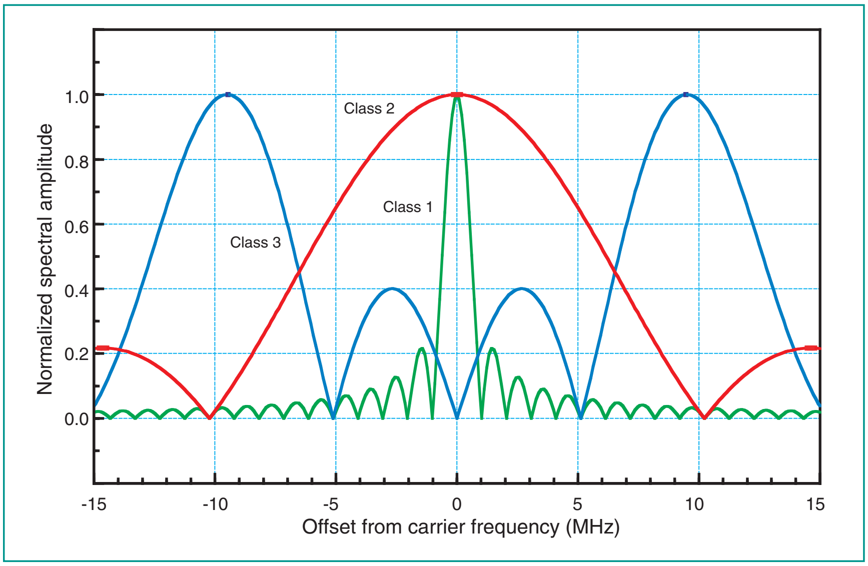


FIGURE 2 Normalized power spectra of the three GPS signal classes: Class I (L1 C/A and L2 Civil), Class II (L1 or L2 P/Y, and L5 Civil), and Class III (L1 and L2 military BOC). The L1, L2, and L5 carrier frequencies at the center of each spectrum are, respectively, 1575.42 MHz, 1227.6 MHz, and 1176.45 MHz.

sequence assigned to each satellite is publicly known and has a very long period of one week. The Y-code is formed by modulating the P- code with a slower sequence of encrypting chips, called a *W-code,* generated at 511.5×10^3 chips per second.

L5 Civil Signal. GPS modernization calls for a completely new civil signal at a carrier frequency of 1176.45 MHz, with the total received signal power divided equally between in-phase (I) and quadrature (Q) components. Each component is modulated with a different, but synchronized, 10,230-chip direct sequence L5 code transmitted at 10.23×10^6 chips per second, the same rate as the P/Y-code but with a 1-millisecond period (the same as the C/A-code period). Compared with the C/A code, the 10-times larger chip sequence of the I- and Q-channel civil L5-codes provides lower autocorrelation sidelobes, and the 10-times higher chipping rate substantially improves ranging accuracy, provides better interference protection, and substantially reduces multipath errors at longer path separations (far multipath).

The M-Coded Signal (Class III). New military M-coded signals will be transmitted on the L1 and L2 carriers, with the capability of using different codes on the two frequencies. The received L1 M-code will appear in the I-channel additively superimposed on the C/A code, and the L2 M-code will appear in the I-channel superimposed on the civil L2 code. The M-code is a binary offset modulation code with a 10.23 MHz square wave subcarrier modulated by a 5.115×10^6 chips per second spreading sequence (BOC[10,5]). Each spreading chip subtends exactly two cycles of the subcarrier, with the rising edge of the first subcarrier cycle coincident with initiation of the spreading chip. The spectrum of the BOC(10,5) code has considerably more relative power near the edges of the signal bandwidth than do any of the C/A-coded, L2 Civil, L5 Civil, or P/Y-coded signals. As a consequence, the M-coded signal not only offers the best pseudoranging accuracy and resistance to multipath, but it also has minimal spectral overlap with the other GPS transmitted signals, which permits transmission at higher power levels without mutual interference.

Spectra of the Three Signal Classes. **Figure 2** shows the code spectrum for each of the three signal classes. Although these spectra are confined within a bandwidth of no less than ±12 MHz at the satellite, the distribution of power within this bandwidth is markedly different for each class. The power of the Class I signals (L1 C/A and L2 Civil codes) is mostly concentrated within a ±1 MHz band, resulting in a comparatively small Gabor bandwidth. Therefore, Class I signals have the least inherent resistance to multipath error. The power in Class II signals (P/Y and L5 Civil codes) is more highly spread, with most of the power in a ±10 MHz band, thus offering significantly better multipath performance. The military M-coded signal (Class III) concentrates even more power near the ±12 MHz band edges and should therefore provide the best multipath performance.

Simulation Results

Figures 3a and 3b, respectively, show the multipath mitigation performance of the MMT algorithm for direct path code-based pseudoranging and carrier-phase estimation for the three classes of modernized GPS signals, which is compared with typical performance of the MCRW technique for Class I and II signals (data are not available about application of the MCRW technique to Class III signals). Both figures show direct-path r.m.s. errors averaged over random secondary-path relative phases where the secondary-path amplitude is one-half that of the direct path, the RF signal bandwidth is limited to 24 MHz by an 8-pole Butterworth filter, and E/N_0 is 45 dB-Hz-s. As expected, both figures show decreasing error for Class I, II, and III signals in correspondence with their increasing Gabor bandwidth. In all cases there is a striking improvement in performance compared with that of the MCRW technique. Not only are the average MMT errors almost an order of magnitude smaller, but the range of secondary-path separation where these errors assume worst-case values is substantially smaller. In addition, MMT clearly takes advantage of the increased Gabor bandwidth of Class II signals compared with the Class I signals, whereas the MCRW technique cannot.

Figure 4 shows the multipath error for code-based pseudoranging of Class II signals under the same conditions of Figure 3, except that the curves are parameterized by E/N_0 The curves demonstrate an important characteristic of ML estimation: errors are reduced by increasing E/N_0 (although not shown, a similar property holds for Class I and Class III signals). Because E/N_0 is the product of C/N_0 and the signal observation time, reduction in multipath-induced error using MMT can be achieved simply by observing the signal for a longer period. This action is especially important in more-demanding applications and/or under weak-signal conditions. The MCRW technique does not have this advantage because its errors are in the form of a bias that cannot be reduced by averaging.

Laboratory Tests

In 1998 my colleague and I conducted

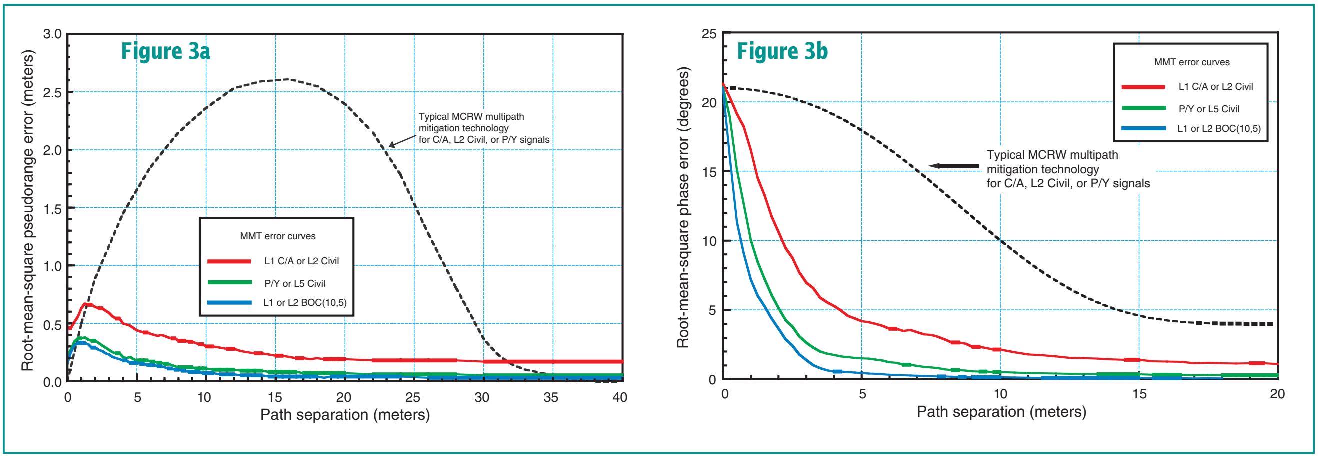


FIGURE 3 Multipath mitigation performance of the MMT algorithm for the three classes of signals compared with MCRW technology. RF signal bandwidth is limited to 24 MHz, E/N_0 is 45 dB-Hz-s, and the secondary-path amplitude is 6 dB below the direct path. The r.m.s. error is averaged over random secondary path relative phases. (a) Direct path code-based pseudorange error. (b) Direct path carrier-phase estimation error.

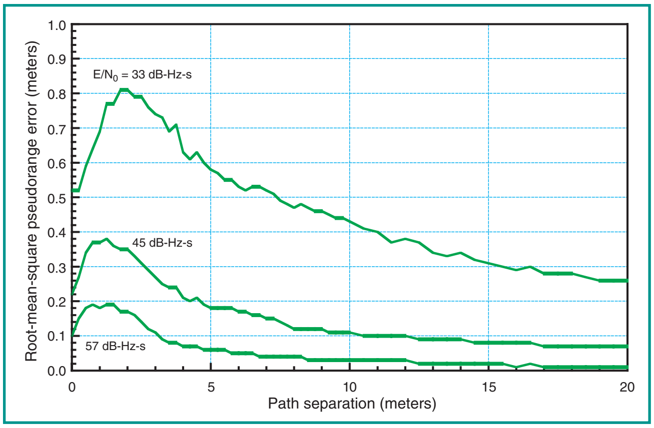


FIGURE 4 Dependence of residual multipath error on E/N_0 for code-based pseudoranging using the MMT algorithm with Class II GPS signals (L1 or L2 P/Y, and L5 Civil). Conditions are the same as in Figure 3, except that the curves are parameterized by E/N_0. Note that E/N_0 can be increased by increasing the signal observation time, thereby improving performance.

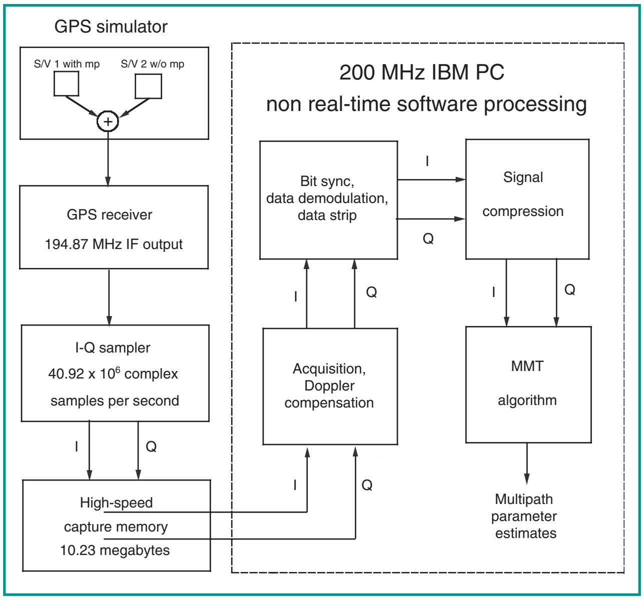


FIGURE 5 Laboratory test setup for evaluating the MMT algorithm with a commercially available receiver. See text for details.

tests of the MMT algorithm with a GPS receiver provided by a major manufacturer. A GPS signal generator was configured to simultaneously generate a reference signal without multipath and another signal with the same direct path delay, but contaminated by a secondary path signal. The two signals used different C/A codes so that they could be separately observed. Errors in the MMT algorithm were determined by comparing accurate measurements of the reference signal with those produced by MMT in observing the contaminated signal. **Figure 5** shows the test setup. Both observed signals were obtained by sampling the receiver's final IF frequency at 40.23×10^6 complex samples per second and capturing one second of data in a high-speed digital capture memory. All subsequent processing was performed in non–real time by software on a 200 MHz PC as shown in the figure. Although the results are the property of the receiver manufacturer, it can be said that they agree closely with the Class I signal results presented in this article.

Implementation Issues

The MMT estimator is practical to implement with a firmware/software combination consisting of either an FPGA or ASIC in combination with software operating in a general-purpose microprocessor. We have estimated that a moderate-size FPGA (approximately 50,000 gates) would handle the previously described compression process for 8 parallel GPS channels in addition to the other low-level, high-speed processes required for conventional GPS receiver signal processing. The MMT algorithm itself can be written in any high-level language and compiled into assembly code for execution in the microprocessor. Although it is not the dominant language in use today, we have found that PowerBASIC has significantly faster execution than just about any other compiled language, including various versions of C or FORTRAN. Once the receiver is tracking a GPS satellite signal, an MMT estimate of signal amplitude, delay, and carrier phase for each propagation path can be obtained with approximately 10–80 milliseconds of

Maximum Likelihood Estimation

Maximum likelihood (ML) estimation is a conceptually simple yet powerful technique for estimating nonrandom parameters (such as the multipath parameters of a received GPS signal) based on observation of a noisy random variable or vector (such as the samples of the received GPS signal in thermal noise). This method is popular for a variety of reasons. ML estimators often are computationally efficient and have desirable asymptotic properties. *Asymptotic* refers to the behavior of the estimator as the estimation error is reduced by observing more samples (such as increasing the number of received GPS signal samples, thereby increasing the E/N_0 of the signal). For ML estimators these asymptotic properties include the following: 1. The estimate converges in probability to the true parameter value; 2. The estimate is asymptotically efficient, that is, the error variance approaches the minimum possible value; 3. The estimation error is asymptotically Gaussian.

As a simple illustration of ML estimation, consider a random variable X that is normally distributed with known variance σ^2 but whose mean μ is unknown. The probability density function of X is

$$p(x,\mu)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

in which the notation $p(x,\mu)$ emphasizes that the probability density function depends on the unknown parameter μ. Suppose we make N independent observations of X and arrange them in an N-dimensional vector $\bar{x} = (x_1, x_2, \ldots, x_N)$. Because the observations are independent, the joint probability density of the X_i, denoted by $p(\bar{x}, \mu)$ is the product of their individual densities:

$$\begin{aligned}p(\bar{x},\mu)&=\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^N\prod_{i=1}^{N}\exp\left[-\frac{(x_i-\mu)^2}{2\sigma^2}\right]\\&=\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^N\exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}(x_i-\mu)^2\right].\end{aligned}$$

Once the observations have been made, their values x_i are fixed. The ML estimate of μ is made by finding the value of μ that maximizes $p(\bar{x})$, which is the probability that the values $x_1, x_2, \ldots, x_N$ would have been obtained. Because the natural logarithm function is an increasing function of its argument, maximization of $p(\bar{x})$ is equivalent to maximization of the log-likelihood function, which is

$$\begin{aligned}L(\bar{x},\mu)&=\ln[p(\bar{x},\mu)]\\&=N\ln\left(\frac{1}{\sqrt{2\pi}\sigma}\right)-\frac{1}{2\sigma^2}\sum_{i=1}^{N}(x_i-\mu)^2\end{aligned}$$

This function is quadratic in μ, so it is guaranteed to have a unique global maximum that can be found by ordinary calculus. Taking the derivative of $L(\bar{x}, \mu)$ with respect to μ, setting it to zero, and solving for μ yields the ML estimate $\hat{\mu}$ of the mean:

$$\hat{\mu}=\frac{1}{N}\sum_{i=1}^{N}x_i$$

This estimate is just the average of the observed values x_i, which certainly is intuitively satisfying. It can be shown that this estimator is unbiased (i.e., the average error in the estimate is zero) and that it has the minimum error variance of any unbiased estimator of μ.

computation, depending on the speed of the microprocessor.

Concluding Remarks

Those familiar with our work have often asked about the ability of MMT (or any ML estimation technique, for that matter) to handle a multipath scenario more complex than that presented here. For example, how does it respond to interference, jamming, or the existence of more than one secondary path? We are currently studying such scenarios. Preliminary results indicate that with an adequate dynamic range in the signal-sampling process, the presence of interference or jamming simply appears as a decrease in signal-to-noise ratio and can be moderated by using longer signal observation times. The MMT algorithm appears to be quite robust in the presence of additional secondary paths, but it can be designed to model additional secondary paths if desired. Theory states that such an accommodation will improve performance when the number of paths in the model is the same as the number of significant paths that actually exist, but that an increase in error will result if there is a mismatch. Our contention is that MMT should model at most two secondary paths, for two reasons: First, the algorithm can be made to be inherently insensitive to almost all secondary paths delayed by more than approximately 30 meters when using Class I signals and to all such secondary paths when using Class II and III signals. In addition, our belief is that within the range of 0–30 meters of path separation, it is unlikely that more than two secondary paths will exist that have sufficient strength to require mitigation.

Acknowledgments

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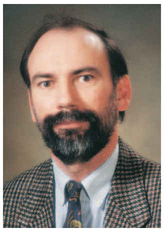
Manufacturers

A **Nortel GPS Simulators** — now **Spirent Communications** (Paignton, Devon, United Kingdom) — STR2760 GPS signal simulator was used to test the MMT algorithm. We implemented the algorithm using the *BASIC* compiler developed by **PowerBASIC, Inc.** (Carmel, California)

Lawrence R. Weill is currently professor of mathematics emeritus at California State University, Fullerton, and a well-known GPS industry consultant. He is one of the three technical founders of Magellan Systems Corporation (now a division of Thales Navigation).



"Innovation" is a regular column featuring discussions about recent advances in GPS technology and its applications as well as the fundamentals of GPS positioning. The column is coordinated by Richard Langley of the Department of Geodesy and Geomatics Engineering at the University of New Brunswick, who appreciates receiving your comments and topic suggestions. To contact him, see the "Columnists" section on page 2 of this issue.