Tropospheric Delay: Prediction for the WAAS User

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The weather — it affects us all. Sometimes disastrously with vicious storms; sometimes pleasantly with sunshine and warm breezes. It also affects GPS. But, whereas bad weather might disrupt our lives, causing us to curtail or postpone an activity, GPS continues to perform — it's an all-weather system. Rain, snow, fog, and clouds all have a negligible effect on GPS. However, unseen weather — temperature, pressure, and humidity variations throughout the atmosphere — does affect GPS observations. These parameters determine the propagation speed of radio waves, an important factor that must be accounted for when processing GPS or other radiometric observations. Because we cannot predict their exact values ahead of time, these invisible weather variables are a source of error in GPS positioning and navigation. In this month's column, we examine the atmosphere's effect on GPS and discuss how we've attempted to model it for the users of the forthcoming Wide Area Augmentation System.

I am joined this month by Paul Collins. Paul graduated from the University of East London in 1993 with a B.Sc. (Honors) degree in surveying and mapping sciences. He is currently enrolled in the M.Sc.E. degree program in the Department of Geodesy and Geomatics Engineering at the University of New Brunswick, where he is investigating the tropospheric effects on kinematic GPS positioning.

"Innovation" is a regular column featuring discussions about recent advances in GPS technology and its applications as well as the fundamentals of GPS positioning. The column is coordinated by Richard Langley of the Department of Geodesy and Geomatics Engineering at the University of New Brunswick, who appreciates receiving your comments as well as topic suggestions for future columns. To contact him, see the "Columnists" section on page 4 of this issue.

The ideal GPS measurement is the true range or distance between the receiver's antenna and the GPS satellite antenna. But as with most things in our imperfect world, we cannot achieve the ideal. A number of error sources bias GPS measurements, and processing software must account for these errors to obtain accurate position, velocity, or time information. One of these biases is the tropospheric propagation delay, for which all GPS receivers and postprocessing software employ a delay algorithm of some kind that attempts to model or predict, and thus minimize, its impact. Depending on the model's sophistication, however, some error or residual delay will remain.

Users often treat the residual tropospheric delay in GPS position estimation in a very offhand manner — they assume that the effect is small or that a simple estimation technique will resolve the problem. But what is the true impact? Just how much does the lower, electrically neutral, atmosphere vary in terms of refractivity and its effect on GPS applications? The aim of this article is to provide answers to these questions and offer a quantification of the neutral atmospheric effect for users of wide-area differential GPS systems, such as the Federal Aviation Administration's (FAA's) Wide Area Augmentation System (WAAS), who seek meter-level accuracy or better and operate within a position critical environment.

THE TROPOSPHERIC DELAY

Constituent gases affect an electromagnetic signal propagating through the neutral atmosphere. Their combined refractive index, slightly greater than unity (nominally 1.0003 at sea level), causes the signal's velocity to be lower than it would be in a vacuum and increases the time it takes a signal to reach a GPS receiver's antenna, thus extending the equivalent path length. Both these effects are often referred to as *delay*. Refraction also bends, and thereby lengthens, the raypath, further increasing the delay. Because the bulk of the delay occurs within the troposphere, the whole delay is often referred to solely as the *tropospheric delay*.

By assuming that the neutral atmosphere is both horizontally stratified and azimuthally symmetric, we can model the tropospheric delay in two parts: the delay experienced in the zenith direction and the scaling of that delay to the one experienced at the raypath's zenith angle (referred to as either the mapping function or obliquity factor). The common formulation of zenith delays and mapping functions seen in space geodetic and navigation literature derives from modeling tropospheric delay in this way. It can be described mathematically as

$$d_{trop} = d^z_{hyd} \cdot m_{hyd} + d^z_{wet} \cdot m_{wet}$$

in which total delay (d_{trop}) is a function of the delays in the zenith direction caused by the atmospheric gases in hydrostatic equilibrium and those that are not (d^z_{hyd} and d^z_{wet}, respectively) as well as their corresponding mapping functions (m_{hyd} and m_{wet}), which project the zenith delay into the line-of-sight delay. Gases not in hydrostatic equilibrium are primarily water vapor, and the mapping functions are usually described as functions of the satellite elevation angle — the complement of the zenith angle.

Because this customary formulation of tropospheric delay assumes horizontal stratification and azimuthal symmetry, it precludes the existence of gradients in the atmosphere. Researchers have derived more sophisticated models to try to account for atmospheric gradients caused by pressure slopes and passing weather fronts, but the effects are typically at the centimeter level or less and are accordingly insignificant for most navigation applications.

DELAY MODELS

Researchers have developed many different algorithms over the years in an effort to empirically model the tropospheric delay with varying degrees of accuracy. When processing GPS observations, a receiver or post-processing software program predicts a value for the tropospheric delay from such a model using assumed or real-time values for the ambient temperature as well as total barometric and partial water-vapor pressures. Unfortunately, even with accurate real-time measurements, models can rarely predict the true total delay with a degree of accuracy much better than a few percent.

Although, we can determine the delay's hydrostatic component in the zenith direction at the millimeter level, the highly variable nature of atmospheric water vapor degrades the accuracy of the wet delay prediction to the centimeter or even decimeter level. This occurs because the hydrostatic delay in the zenith direction is a function of the total surface pressure only, which, under conditions of hydrostatic equilibrium, represents the total weight of the column of air above the user. Analogously, the zenith wet delay is a function of the total precipitable water — the amount of vapor present in the column of air above the user.

Tropospheric delay models express these two parameters in various ways. The most common method of describing the wet delay is through a combination of surface parameters (water-vapor pressure, or relative humidity, and temperature) and some kind of water-vapor lapse rate (commonly known as the lambda parameter). Not all models explicitly parameterize the wet delay as such, but they often do so implicitly.

The problem with modeling the wet delay in this way is that, unlike the hydrostatic delay, no simple physical law governs the distribution of water vapor in the lower atmosphere; hence, a precise definition and evaluation of the lambda parameter is not possible. As a consequence, the only way to accurately measure the lambda parameter is to employ some technique that attempts to sample the whole atmospheric column, such as a radiosonde or a radiometer. As these are impractical for real-time GPS users (and indeed for most other GPS users), the lambda parameter can only be represented empirically and will consequently always be associated with some error in the determination of the wet zenith delay.

We should point out that it is often possible to improve tropospheric delay modeling by estimating a zenith-delay correction from the GPS data itself. This procedure, however, is typically only carried out for such high-precision GPS applications as deformation monitoring.

DEVELOPING A NEW MODEL

Although the geodetic community has been acutely aware of the propagation delay problem and has striven over the years to improve the accuracy of tropospheric delay models, the navigation community has overlooked, for the most part, these developments and continued to use simpler, less-accurate models. The need for higher positioning accuracies, however, concurrent with the development of augmented GPS has necessitated a fresh look at tropospheric delay models for navigation applications. WAAS in particular requires a model, as aircraft employing WAAS-augmented receivers need the resulting increase in accuracy.

At the University of New Brunswick (UNB), we have developed a series of composite or hybrid models for GPS navigation, culminating in UNB3, which the FAA and Nav Canada have adopted for the WAAS user receiver. We based our original definition of the model on the zenith delay algorithms of Jouko Saastamoinen (whose work at the National Research Council of Canada in the early 1970s has withstood the test of time), the recent, high-accuracy mapping functions of Haystack Observatory's Arthur Niell, and a table of atmospheric parameter values derived from the U.S. 1966 Standard Atmosphere Supplements.

In the interest of computational simplicity, a subsequent proposal was made to replace the Niell mapping functions with the combined function of Harold Black and Arie Eisner, who both worked at the Johns Hopkins University Applied Physics Laboratory. This change has a negligible impact on most of the results presented in this article, because we deal mainly with the *zenith delay* residual error. (We did calculate our position error simulations described later in this article using the Black and Eisner mapping function; however, we do not expect the results to be significantly different when using the Niell functions.)

UNB3. We designed the UNB3 model to improve on first-generation navigational-use tropospheric delay algorithms, such as the Altshuler model (developed by Edward Altshuler at the Air Force Cambridge Research

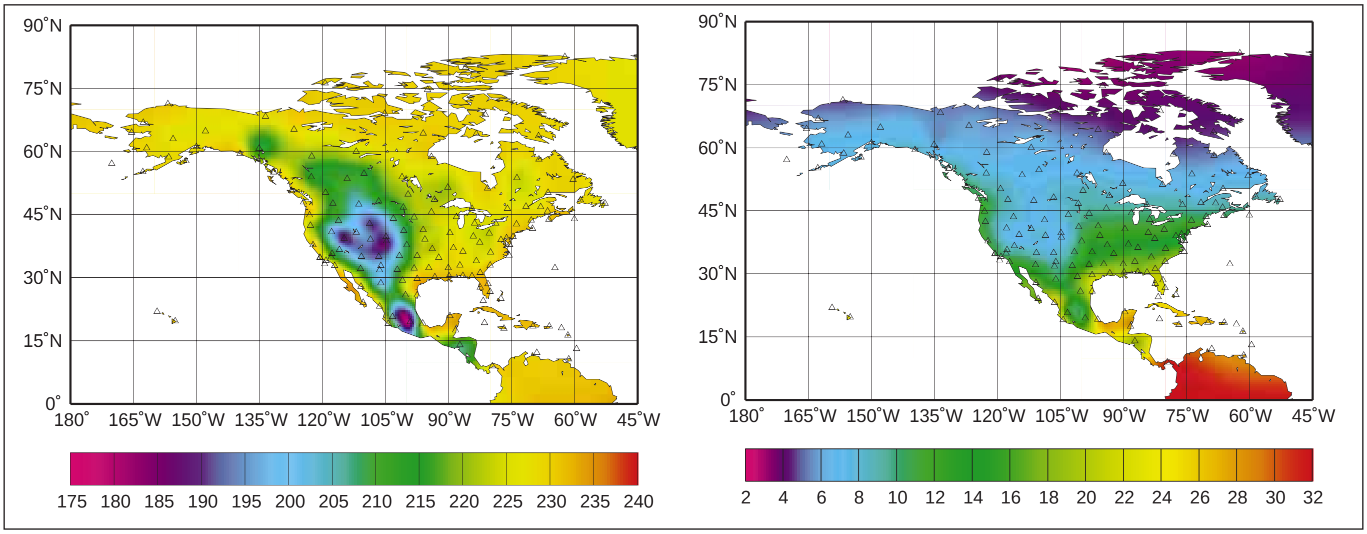

Figure 1. Mean hydrostatic (left) and wet (right) zenith delays in centimeters computed from radiosonde soundings made in 1992. Triangles indicate station locations.

Laboratories in the early 1970s), as well as the simple constant-value models attributed to NATO and Stanford Telecommunications, Inc. The latter two essentially use standard, predetermined parameter values at mean sea level and a constant atmospheric profile across the whole latitude range. They only account for the vertical variation to represent the change in a user's height (in an aircraft, for example).

In this article, our prototype model, UNB1, represents delay algorithms of this type. It uses U.S. Standard Atmosphere parameter values of 1013.25 millibars (total pressure), 288.15 kelvins (temperature), 11.7 millibars (water-vapor pressure), 6.5 kelvins per kilometer (temperature lapse rate), and 3 (lambda parameter). (We developed two other models — UNB2 in which we first tested latitudinal variation of the surface parameters and UNB4 which has slightly improved performance at high altitudes at the expense of reduced performance near the ground. For details on these other models, consult the report listed in the Further Reading sidebar.)

The kernel of the UNB3 model expands the representation of these five UNB1 atmospheric parameters into a look-up table that provides values that vary with respect to latitude and day of year. Linear interpolation is applied between latitudes, and a day-of-year sinusoidal function attempts to model the seasonal variation. Users employ the lapse rates to scale the sea-level pressures and temperature to their altitude.

METHODOLOGY

Because UNB3's look-up table is based on average climatological behavior, the model will not perform as well during anomalous weather conditions. We carried out a series of tests to gauge the model's accuracy. For comparison purposes, we also tested the Altshuler and UNB1 models.

To provide the benchmark data for our investigation of propagation delay errors, we ray-traced a portion of a four CD-ROM set of radiosonde data for North American sites that covers the years 1946–1996. The National Oceanic and Atmospheric Administration (NOAA) compiled the data set, which consists of radiosonde soundings at mandatory and significant pressure levels as high as 100 millibars (equivalent to a height of about 16 kilometers) from almost all the sites operating in the United States, Canada, Mexico, the Caribbean, and Central America. For our analyses, we used the last 10 full years of available data (1987–1996). This represents an average of 173 stations per year and approximately one million total soundings. Figure 1 shows a sample of mean hydrostatic and wet zenith delays across North America from this 10-year data set.

Almost all atmospheric water vapor is found well below the 100-millibar level and thus the CD-ROM data sets were sufficient for our purposes in this respect. To ray-trace hydrostatic delay, however, the temperature profile must be extended above this height. We accomplished this by using a suitable temperature profile *model*, namely the Committee on Space Research (COSPAR) International Reference Atmosphere (CIRA) model, which provides monthly mean temperatures at 5-kilometer intervals for altitudes as high as 120 kilometers for every 10 degrees of latitude. By computing a weighted average of the four CIRA profiles surrounding a radiosonde launch site and epoch (two in latitude and two in time) and then offsetting it to match the radiosonde temperature value at the truncation height, we obtained the required time and location profiles.

We subtracted the zenith delay values computed by the various models from the zenith ray-trace values to obtain the residual tropospheric delay and hence the zenith model errors. The results are presented in three parts: the average statistical performance of the tropospheric delay models; the extreme delay errors; and the impact of those extremes on position computations simulated with real GPS ephemerides.

AVERAGE MODEL PERFORMANCE

To test the distributions of the residuals, we can use Gaussian plots to compare them to a standardized or normal distribution (see Figures 2 and 3). Figure 2 indicates the degree to which some tropospheric delay models can be biased and skewed toward large residuals. The mean of the Altshuler model residuals is 15.9 centimeters, with a standard deviation (σ) of 8.1 centimeters. Carefully choosing parameter values, as was done for UNB1, can

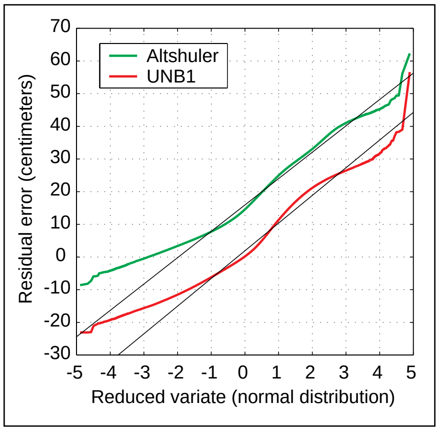

Figure 2. Gaussian plot of zenith delay residuals for the Altshuler model and UNB1. Thin lines represent best-fit normal distributions.

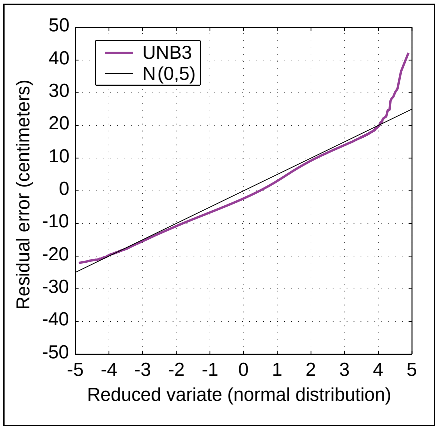

Figure 3. Gaussian plot of zenith delay residuals for UNB3. The thin line shows the zero-mean, 5-centimeter standard deviation normal distribution.

provide a near-zero-mean model, but the distribution is highly skewed and has a large standard deviation. UNB1 has a 1.9-centimeter mean error, with 8.5-centimeter standard deviation.

Figure 3 shows the distributions of UNB3 model residuals and a theoretical, zero-mean distribution with 5-centimeter standard deviation. This normal distribution characterizes the residuals of the UNB3 model quite well up to approximately ±4σ where the value of the residuals is almost exactly ±20 centimeters. Beyond the 4σ level, the lower bounds for UNB3 become progressively conservative because the magnitude of the negative residuals appears to level off. A normal distribution also quite drastically underestimates the residuals beyond the upper bound.

These plots indicate that characterizing tropospheric delay errors using a normal distribution beyond the 4σ level cannot be recommended, especially with simpler models because the true distribution will be underestimated. The probability level equivalent to 4σ in a normal distribution is 99.994 percent. Safety-critical systems, however, may demand even higher levels.

The mean and standard deviation statistics for UNB3 based on the entire 10-year data set are –1.9 and ±4.9 centimeters respectively. These statistics are very consistent from year to year, indicating that it is sufficient to use one year's worth of data (with wide geographical coverage) to quantify the average, or typical, tropospheric delay model performance.

EXTREME DELAY ERRORS

We will no longer consider the Altshuler and UNB1 models in this article because their performance with regard to large, or extreme, residuals is poor. Comparing Figures 2 and 3 reveals that the UNB3 model provides a 56-percent improvement over the Altshuler model at the upper 4σ level.

It is convenient to use a nonextreme cutoff range of ±20 centimeters for the UNB3 model, based on the results shown in Figure 3. A zenith delay error of this magnitude, however, could lead to a potential 2-meter bias in height (see later discussion). In total, 76 residuals from 1,011,651 profiles in the 10-year data set exceed the ±20-centimeter range for the UNB3 model. This is equivalent to approximately 0.00751 percent. Correspondingly, 99.99249 percent of the residuals are within the nonextreme range.

We checked all the initially detected extreme profiles, rejecting several as unlikely, supposing radiosonde or other errors. Unfortunately, without detailed knowledge of each station's climatic conditions, we cannot be completely sure about the remaining extremes. We believe, however, that our results are a good representation of the true errors.

Extreme Locations. Figure 4 displays the locations of stations having extreme residuals. Those with at least one positive extreme (a residual of more than +20 centimeters zenith delay error) are marked with a triangle and labeled with a station identification number to the right. Stations marked with an inverted triangle and labeled to the left have at least one negative extreme (a residual magnitude

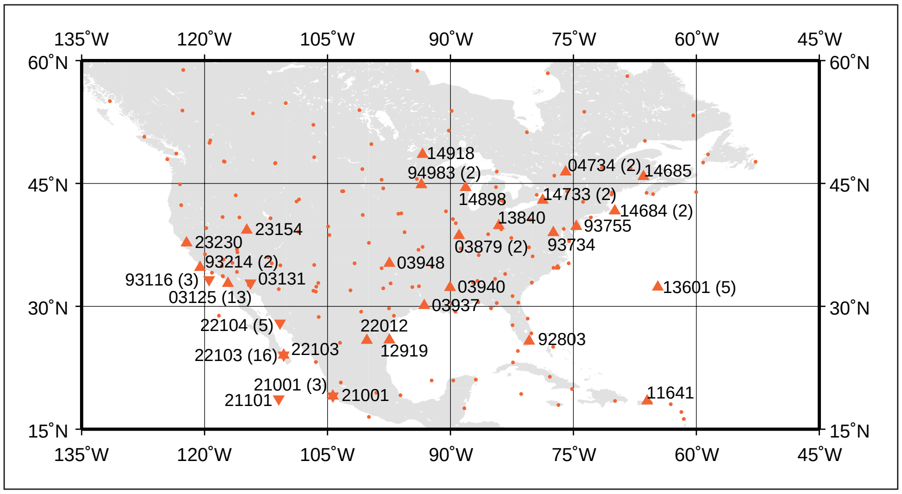

Figure 4. Location of radiosonde stations providing data between 1987 and 1996. In addition to their identification numbers, stations with positive extremes from model UNB3 are labeled ▲ and stations with negative extremes are labeled ▼. Parenthetical digits indicate the number of extremes for sites having more than one.

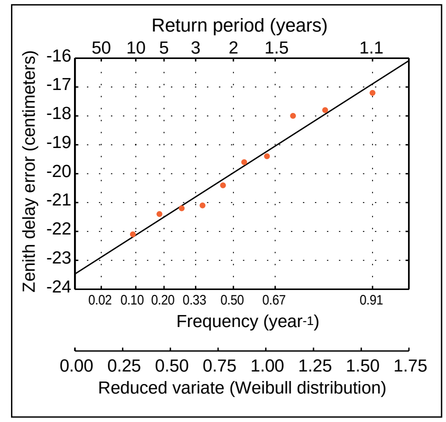

Figure 5. Extreme value cumulative probability plot for negative extremes

greater than –20 centimeters zenith delay error). Parenthetical numbers indicate more than one extreme during the 10-year period.

The negative extremes are confined to the Baja California, Sonora, and Sinaloa regions of Mexico as well as the southern tips of California and Arizona. The number and location of the positive extremes are geographically more scattered than the negative extremes, yet concentrations do occur. Bermuda (station number 13601), for example, seems particularly prone to extreme conditions, a possible consequence of its mid-Atlantic location.

Forecasting Extremes. We examined the extremes as a function of day of year to understand, in particular, the pattern of negative extremes. We found that most negative extremes occur during late spring. Examining the residual time series for stations in the west of Mexico indicates that the climate is relatively constant through this time of year. Unfortunately, UNB3 has a mean bias that, when combined with the increasing sinusoidal seasonal variation, can result in an error for these stations that exceeds the ±20-centimeter error limit.

The positive extremes are more likely in the summer — although the second and third largest extremes occurred in the winter. Unlike the majority of negative extremes, the positive extremes are outliers in the overall time series, suggesting the influence of short-period, transient weather systems.

Given that Figure 3 indicates different tail distributions from that of the Gaussian curve, we can examine the extreme residuals separately using Gumbel, Frechet, and Weibull probability distributions. These functions represent the limiting forms that most common distributions take when only considering the largest or smallest values in a sample set. It is not necessary to know the underlying distribution to consider the distribution of the extreme values.

An extreme value probability plot has the potential to provide an abundance of useful information. As an example, Figure 5 shows the extreme value probability plot for the largest yearly negative extremes. Of the three distributions, the positive extremes fit a Frechet distribution best, and the negative extremes agree most closely with a Weibull distribution.

The power in these plots lies in the ability to extrapolate and compute the return periods of future extremes. The return period merely denotes the inverse of the occurrence probability. By using the largest value in each year, we can ascertain the return periods in years. It should be pointed out that the use of extreme value statistics usually requires a set of at least 20 samples. With only 10 years of data in our sample, the confidence in these results is not high. We can use our data, however, as a good example of the application of extreme value statistics.

If the positive extremes do follow the Frechet distribution, then an average return period of 25 years is forecast for an extreme zenith delay value of at least 55 centimeters (with a 63-percent probability). Both the Frechet and Weibull distributions have threshold values beyond which the expected return period is infinite. The Frechet threshold for positive extremes is approximately 11 centimeters, indicating that an error at least this large will occur every year.

If we assume no error in the hydrostatic delay, the maximum wet zenith delay value of the UNB3 model (approximately 27 centimeters) limits the magnitude of the negative extremes. This error would occur with a dry, or nearly dry, atmosphere in the tropics. Thus, we could specify this value as the threshold in the Weibull distribution; however, we tried to use the data itself to identify the value. This approach appears to work because the Weibull distribution best fit specifies a cut-off value of –23.5 centimeters.

The slight difference between this value and the theoretical one suggests either insufficient data or indicates that the maximum zenith wet delay error will never be reached because some water vapor (and, therefore, a few centimeters' worth of wet zenith delay) is always present in the atmosphere. The forecast return period for an extreme zenith delay error less (that is, greater in absolute value) than –23 centimeters is 50 years.

Look-up Table. The UNB3 model uses a look-up table to obtain values of the meteorological parameters because most WAAS aircraft receivers will not have access to real-time meteorological data. We did investigate replacing the look-up table with real data extracted from the radiosonde data set. We discovered that, although the mean error of the model (at sea level) is reduced to 0.2 ± 3.4 centimeters, there is no apparent benefit, with regard to extreme errors, in replacing a good meteorological look-up table with real-time values. The small improvement in overall bias is negligible in comparison to other potential error sources, such as multipath and the ionosphere.

POSITION DETERMINATION IMPACT

Elevation-angle dependence complicates the impact of an unmodeled tropospheric range delay on GPS positions. The elevation angle will not be the same for all the satellites in view, and thus users cannot rely on simple vertical dilution of precision (VDOP) calculations to model these errors. The only reliable way to study the impact of troposphere model errors is to undertake position simulations, replacing the GPS range with the unmodeled delay. In this way, we are able to predict how the error manifests itself in position coordinates.

We computed position solution biases for all the stations with extreme residuals, using broadcast ephemerides from 1997 to represent satellite constellations for six hours around the time of each radiosonde launch. We assumed that the extreme residuals could occur at any time of year because performing position simulations for each day of the year would be extremely time consuming. We also presumed that the tropospheric error remains constant for several hours. In any case, we were able to derive a general relationship between receiver-satellite geometry, tropospheric delay error, and the resulting position bias.

We performed two kinds of position solution simulations: a regular, unweighted least-squares solution and a weighted least-squares solution using the squared inverse of the mapping function to down-weight any low-elevation-angle errors. The position biases were computed every two minutes. For almost all of the position solutions, the weighted vertical biases were one-third to two-thirds less than the unweighted biases. In general, the weighted solution reduced the extreme vertical bias to the meter level or less. The horizontal biases for both solutions were always much smaller — at or below the decimeter level.

Maximum Bias. We discovered that for one particular time period at one station, the satellite constellation was dominated by low-elevation-angle satellites. During this interval (approximately 10 minutes), the weighted and unweighted position solutions converged toward the same value (~1.5 meters). Figure 6 illustrates this phenomenon and shows the correlation between the unweighted vertical position bias and the maximum tropospheric delay error. Figure 6 also clearly demonstrates that any function of simple VDOP will not correctly model this kind of error. The residual zenith error for this station was approximately 21 centimeters, with the unweighted vertical bias approaching 2 meters during one time period.

This analysis indicates that the amount of vertical position bias resulting from unmodeled tropospheric range delay is approximately equal to the maximum residual tropospheric delay present in the solution. Because of the tropospheric delay's elevation-angle dependence, this position bias will essentially be the delay error from the lowest-elevation-angle satellite. Given the expected zenith delay error, an approximate mapping function value will give the correct result. At a 5-degree elevation angle, the mapping function value is roughly 10. With this figure, the maximum possible vertical height error value can be easily calculated.

The size of the maximum bias in the computed position will be limited in one direction because of the lower limit of the UNB3 model. A negative error indicates that the tropospheric delay model prediction was too large. By effectively shortening the range, the computed position will be higher than the true position. An aircraft flying below its intended height can therefore only ever be approximately 3 meters too low because of UNB3 tropospheric delay mismodeling. For an aircraft using wide-area differential GPS, flying above its intended height, and having an unfavorable satellite constellation in unusual weather conditions, 4-meter vertical position biases are possible solely from mismodeled tropospheric delays. The extreme value theory predicts errors as high as 5 meters.

CONCLUSIONS

Our results indicate that the maximum size of the residual tropospheric delay is not too large, as long as a good model is used. For the UNB3 model, only seven or eight in 100,000 predictions resulted in residual zenith delay errors outside the range of ±20 centimeters. A zero-mean normal distribution, with 5-centimeter standard deviation up to the ±4σ point can adequately represent the distribution of the majority of the errors. Beyond that point, the normal distribution is too conservative for the positive extreme residuals.

Of the extreme values beyond ±20 centimeters, there are slightly more negative extremes than positive. The negative extremes appear more consistent with prevailing weather conditions and are limited in magnitude by the maximum wet zenith delay value of UNB3 (approximately 27 centimeters). There is more potential for greater positive extremes, but our small sample size (10 values) limits us from providing a reliable forecast. More data would be required to improve the confidence of future predictions.

Mismodeled tropospheric range delays primarily impact a computed position's height component. VDOP values are not a good indicator of vertical bias in this case because they only model the position error resulting from receiver-satellite geometry, assuming a constant average range error for all satellites in view. A better indication is provided by the maximum residual error — that is, the unmodeled range on the lowest elevation satellite. This error maps almost directly into the vertical position component.

WAAS will augment GPS to provide the required accuracy improvement for precision aircraft approaches to airports, as well as navigation integrity, continuity, and availability for all phases of flight. Although tropospheric delay is only one component of the WAAS error model, it is an important one that demands an accurate, reliable user model to ensure that WAAS achieves its operational objectives. As we've described in this article, the UNB3 composite model fulfills these requirements. Other satellite-based wide-area augmentation systems such as the European Geostationary Navigation Overlay System or Japan's Multifunctional Transportation Satellite system could also make use of UNB3, although further testing with radiosonde data covering the intended service regions would be needed.

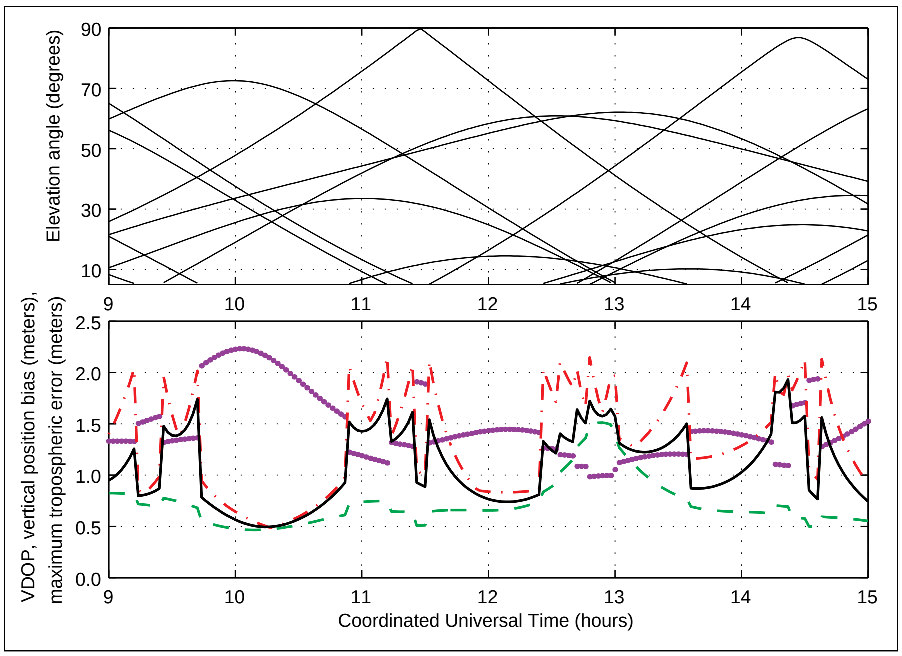

Figure 6. Satellite constellation at station 12919, Brownsville, Texas, on July 18, 1997, and simulated vertical position biases from a 21-centimeter zenith delay error. The solid and dashed lines show the unweighted and weighted solution biases respectively. The dashed and dotted line denotes maximum tropospheric error (the error at lowest elevation satellite), and the dotted line displays the vertical dilution of precision (VDOP).

ACKNOWLEDGMENTS

Our work was undertaken on behalf of Nav Canada in conjunction with the U.S. Federal Aviation Administration, who are cooperatively developing WAAS for North America. This article is based on the paper "The Residual Tropospheric Propagation Delay: How Bad Can It Get?" presented by the authors at ION GPS '98, in Nashville, Tennessee, September 1998. ■

FURTHER READING

For a thorough discussion of tropospheric propagation delay and the models used to account for it in GPS and other space-based radiometric data, see

■ *Modeling the Neutral-atmosphere Propagation Delay in Radiometric Space Techniques,* by V. de Brito Mendes, a Ph.D. dissertation published as Department of Geodesy and Geomatics Engineering Technical Report No. 199, University of New Brunswick, Fredericton, New Brunswick, Canada, April 1999. Electronic supplement: <http://mat.fc.ul.pt/eg/lattex/PhD_e_sup.html>.

For further details about UNB3, see

■ "Limiting Factors in Tropospheric Propagation Delay Error Modelling for GPS Airborne Navigation," by P. Collins, R. Langley, and J. LaMance, published in the *Proceedings of The Institute of Navigation 52nd Annual Meeting,* Cambridge, Massachusetts, June 19–21, 1996, pp. 519–528.

■ *Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment,* RTCA/DO-229A, prepared by Special Committee 159, RTCA, Inc., Washington, D.C., June 1998.

■ *A Tropospheric Delay Model for the User of the Wide Area Augmentation System,* by J.P. Collins and R.B. Langley, Final contract report for Nav Canada Satellite Navigation Program Office and published as Department of Geodesy and Geomatics Engineering Technical Report No. 187, University of New Brunswick, Fredericton, New Brunswick, Canada, September 1997.

For previous articles about tropospheric delay modeling in *GPS World*, see

■ "Effect of the Troposphere on GPS measurements," by F.K. Brunner and W.M. Welsch, in *GPS World,* Vol. 4, No. 1, January 1993, pp. 42–51.

■ "The Effect of Weather Fronts on GPS Measurements," by T. Gregorius and G. Blewitt, in *GPS World,* Vol. 9, No. 5, May 1998, pp. 52–60.