New look at spherical Bouger anomaly

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New concepts concerning gravity anomaly contained in this poster:

First: We identify a distinct difference between a surface [2-D] gravity anomaly (the difference between actual gravity on one surface and normal gravity on another surface) and a solid [3-D] gravity anomaly defined by the fundamental gravimetric equation.

Second: We introduce the "no topography" gravity anomaly (later shown to be identical to complete spherical Bouguer anomaly) as a means to generate a quantity that is smooth, thus suitable for griding, and harmonic, thus suitable for downward continuation.

Third: We show that the planar Bouguer anomaly is not harmonic, and thus cannot be downward continued.

Basic concepts, principles and definitions:

Vertical separation $Z(r,\Omega)$ between normal and disturbed actual equipotential surfaces (defined by the same value of potential) is very close to the disturbing potential $T(r,\Omega)$:

$$\forall \Omega \in \Omega_0 : N(\Omega) = Z[r_g(\Omega)] \qquad (1)$$

In particular:

$$\forall r \geq r_0(\phi), \Omega \in \Omega_0 : \quad Z(r,\Omega) \approx \frac{T(r,\Omega)}{\gamma(r,\phi)} \qquad (2)$$

Two parallel definitions of gravity anomaly

Surface gravity anomaly: form in general use in practice

$$\forall r \geq r_0(\phi), \Omega \in \Omega_0 : \Delta g(r,\Omega) = g(r,\Omega) - \gamma[r - Z(r,\Omega), \phi] \qquad (3)$$

Consequences of accepting this particular definition of gravity anomaly: it does present a problem if we are interested in evaluating gravity anomaly, say, on the geoid, beneath the topographical masses (e.g., for geoid determination by Stokes's formula). The definition by Eq. (3) is mute as far as gravity anomaly values at points where gravity $g(r,\Omega)$ is not known; it does not help us to determine/define the gravity anomaly at points where actual gravity is not already known/observed.

Solid gravity anomaly (from the "fundamental gravimetric equation") :

$$\forall r \geq r_0(\phi), \Omega \in \Omega_0 : \Delta g(r,\Omega) = -\frac{\partial T(r,\Omega)}{\partial n} + \gamma[r - Z(r,\Omega), \phi]^{-1} \frac{\partial \gamma(r,\phi)}{\partial n} T(r,\Omega) \qquad (4)$$

The definition of the gravity anomaly in Eq. (4) intrinsically requires some knowledge of the behaviour of $T(r,\Omega)$ with depth/height, and would be mostly used when we work with a 3-D mathematical model of the disturbing potential $T(r,\Omega)$. This important contrast between Eqs. (3) and (4) makes the conceptual situation somewhat analogous with the distinction between surface and solid spherical harmonics. As such, we call the gravity anomaly in Eq. (4) the **solid gravity anomaly**.

Compatibility between definitions of surface and solid gravity anomaly

Surface gravity anomaly (Eq. 3) can be used as a 3-D quantity, i.e., as the solid gravity anomaly (Eq. 4) provided the "actual gravity" $g(r, \Omega)$ is known in the 3-D region of interest. We were not able to derive Eq. (4) from Eq. (3) for an arbitrarily varying gravity. It appears that surface gravity anomaly is, in some sense, more restricted than the solid variety. Hence, the use of Eqs. (3) and (4) side-by-side as two equivalent alternatives, as has been the custom in geodesy, should be questioned.

If a gravity anomaly is of a solid kind, it is automatically also of a surface kind, but not the other way round (that is, Eq. (3) can be generally derived from Eq. (4) but not vice versa). To enforce the "correspondence" (going in the "opposite direction"): start with the description of a physically meaningful disturbing potential $T(r, \Omega)$ of desired properties which would generate the solid gravity anomaly through Eq. (4). So generated, solid gravity anomaly must then equal to the desired surface gravity anomaly at the defining surface.

To make sure that such disturbing potential exists, we introduce the appropriate model gravity field consisting of the density distribution, gravity potential, disturbing potential, gravity, gravity anomaly, gravity disturbance, co-geoid height, orthometric height, etc., as being defined in a separate geometrical space. This cannot be done for the free-air, planar Bouguer, Faye anomalies.

Such a new space, different from the space where the actual gravity field is studied, must allow us to apply the laws of Newtonian physics needed for the study, to avoid operations such as "removing" some part of gravity field, or "restoring" some other part of gravity field: operations that do not make much physical sense.

"No Topography" space (NT-space): a reinterpretation of Bouguer's aim

The solid Earth without topography is nothing else but the geoid, taken as a solid body with the actual (real) distribution of density within it. The removal of topographical and atmospherical masses significantly changes the potential of the Earth and hence the geoid computed from it, i.e., the NT co-geoid. The corresponding indirect effect of removing the topography is thus very large and must be accounted for. Accordingly, the effect of topography must be dealt with in some appropriate way, if the computation of the real geoid is envisaged The main motivations for using the "no topography" space is to generate a harmonic field that is also smooth and is thus suited for gridding and interpolation.

Gravity and gravity anomaly on the surface of the Earth in the NT-space:

$$\forall r > r_g(\Omega), \Omega \in \Omega_0 : g^{NT}(r,\Omega) = g(r,\Omega) + \frac{\partial V^B(r,\Omega)}{\partial H} + \frac{\partial V^R(r,\Omega)}{\partial H}$$

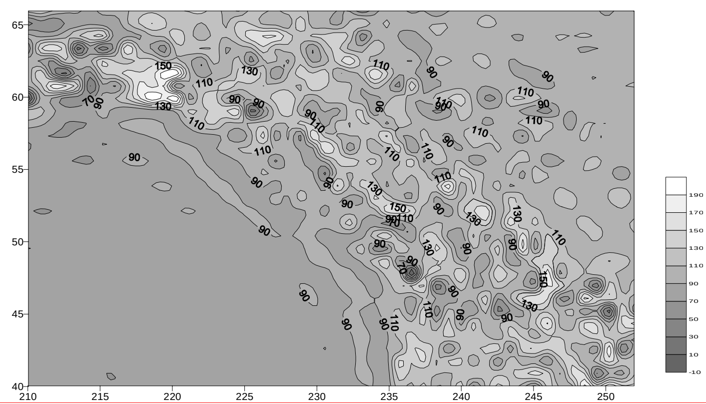
$$\forall \Omega \in \Omega_0 : \Delta g^{NT;S}[H(\Omega)] = g^{NT;S}[H(\Omega)] - \gamma\{H(\Omega) - Z^{NT}[H(\Omega)], \phi\}$$

NT-anomaly is nothing else than the spherical complete Bouger anomaly

Difference between spherical and planar complete Bouger

$$\forall \Omega \in \Omega_0 \ : g^{CB;P}[H(\Omega)] - g^{CB;S}[H(\Omega)] \approx 2\pi G \rho_0 H(\Omega) -$$
$$- TopoC^B[\delta\rho; H(\Omega)] + TC^P[H(\Omega)] - TC^S[\rho_0; H(\Omega)]$$

Difference between the complete spherical and planar Bouguer anomalies at the Earth's surface for an area in the Canadian Rocky Mountains (Max: 314.29, min: -151.20; mean: 99.13; st.dev: 27.36 [mGal])



Conclusions

1. Eqs. (3) and (4) are not equivalent: a solid gravity anomaly is automatically also a surface anomaly but not vice-versa.

2. Spherical and planar complete Bouguer anomalies are different: the spherical one contains the secondary indirect topographical effect (which is very large as compared to Helmert's anomaly)

3. The NT-anomaly is nothing else than the spherical Bouguer anomaly.

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