FAR-ZONE CONTRIBUTIONS TO TOPOGRAPHICAL EFFECTS IN THE STOKES-HELMERT METHOD OF THE GEOID DETERMINATION

R. TENZER[1], P. VANÍČEK[1], P. NOVÁK[2]

1 Department of Geodesy and Geomatics Engineering, University of New Brunswick, GPO Box 4400, Fredericton, Canada (rtenzer@unb.ca, vanicek@unb.ca)
2 Geodetic Institute, Stuttgart University, Stuttgart, Germany

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ABSTRACT

In the evaluation of the geoid done according to the Stokes-Helmert method, the following topographical effects have to be computed: the direct topographical effect, the primary indirect topographical effect and the secondary indirect topographical effect. These effects have to be computed through integration over the surface of the earth. The integration is usually split into integration over an area immediately adjacent to the point of interest, called the near zone, and the integration over the rest of the world, called the far zone. It has been shown in the papers by Martinec and Vaníček (1994), and by Novák et al. (1999) that the far-zone contributions to the topographical effects are, even for quite extensive near zones, not negligible.

Various numerical approaches can be applied to compute the far-zone contributions to topographical effects. A spectral form of solution was employed in the paper by Novák et al. (2001). In the paper by Smith (2002), the one-dimensional Fast Fourier Transform was introduced to solve the problem in the spatial domain. In this paper we use two-dimensional numerical integration. The expressions for the far-zone contributions to topographical effects on potential and on gravitational attraction are described, and numerical values encountered over the territory of Canada are shown in this paper.

Keywords: Far-zone contribution, geoid, topographical effect, topographical density

1. INTRODUCTION

Gravitational potential of topographical masses should be mathematically removed from the Earth's gravity potential in order to obtain the boundary-value problem in the form of Laplace equation. Since these effects are too large and cannot be precisely estimated, some compensation technique should be used (such as the first or second Helmert condensation). For example, the Stokes solution of the geodetic boundary-value problem can mathematically be obtained by "removing" the gravitational potential of all masses above the geoid from the earth's gravitational potential. To reduce its magnitude, the gravitational potential of a two-dimensional layer is introduced according to the second method of Helmert's condensation. Their difference is usually called a "*residual potential*" $\delta V(r,\Omega)$.

One can arrive at the expressions for the direct and secondary indirect effects on gravity by taking the harmonized disturbing potential

$$\forall\,\Omega\in\Omega_O: \qquad T^H(r_t,\Omega)=T(r_t,\Omega)-\delta V(r_t,\Omega), \tag{1}$$

by applying to it the Stokes boundary operator, which reads in the spherical approximation

$$\forall\,\Omega\in\Omega_O: \qquad \Delta g^H(r_t,\Omega)=-\left.\frac{\partial T^H(r,\Omega)}{\partial\, r(\Omega)}\right|_{r(\Omega)=r_t(\Omega)}-\frac{2}{r_t(\Omega)}T^H(r_t,\Omega), \tag{2}$$

where $r_t(\Omega)$ is the geocentric radius of the earth surface, and geocentric coordinates ϕ, λ are represented by the solid angle $\Omega=(\phi,\lambda)$ while Ω_O stands for the total solid angle $\left[\,\phi\in\langle-\pi/2,\pi/2\rangle,\lambda\in\langle 0,2\pi\rangle\right]$.

Substituting for $T^H(r_t,\Omega)$ from the first equation results in

$$\forall\,\Omega\in\Omega_O:$$

$$\begin{aligned}\Delta g^H(r_t,\Omega)\cong&-\left.\frac{\partial T(r,\Omega)}{\partial\, r(\Omega)}\right|_{r(\Omega)=r_t(\Omega)}+\left.\frac{\partial\,\delta V(r,\Omega)}{\partial\, r(\Omega)}\right|_{r(\Omega)=r_t(\Omega)}\\&+\frac{2}{r_t(\Omega)}T(r_t,\Omega)+\frac{2}{r_t(\Omega)}\delta V(r_t,\Omega)\end{aligned}. \tag{3}$$

After a solution for $T^H(r_t,\Omega)$ is obtained, the primary indirect effect on the geoidal height can be derived by applying the spherical Bruns' formula (*Bruns, 1878*) to the residual potential referred on the geoid

$$\forall\,\Omega\in\Omega_O: \qquad \delta N^H(\Omega)=\frac{\delta V(r_g,\Omega)}{\gamma_o(\phi)}, \tag{4}$$

where $\gamma_o(\phi)$ is the normal gravity on the mean geocentric reference ellipsoid (*Somigliana,1929*).

2. TOPOGRAPHICAL EFFECT ON THE POTENTIAL AND GRAVITATIONAL ATTRACTION

Topographical effect on the potential in the second method of Helmert's condensation (*Lambert, 1930*) is given by the residual topographical potential (*Helmert, 1984*)

$$\forall\,\Omega\in\Omega_O: \qquad \delta V^t(r,\Omega)=V^t(r,\Omega)-V^{ct}(r,\Omega), \tag{5}$$

where $V^t(r,\Omega)$ is the potential of topographical masses, $V^{ct}(r,\Omega)$ is the potential of condensed topographical masses.

Approximating the geoid by a sphere of radius R, i.e., considering the geocentric radius of the geoid $r_g(\Omega)$ to be equal to R, $\forall\, \Omega \in \Omega_O : r_g(\Omega) \approx R$, and the actual density of topographical masses $\rho(r,\Omega)$ by a laterally varying density $\rho(\Omega)$, the topographical potential $V^t(r,\Omega)$ can be written as (*Martinec, 1993*)

$$\forall\, \Omega \in \Omega_O :$$

$$V^t(r,\Omega) = G \iint_{\Omega'} \rho(\Omega') \int_{r(\Omega')=R}^{R+H(\Omega')} l^{-1}\left[r(\Omega),\psi,r(\Omega')\right] r^2(\Omega')\, \mathrm{d}r(\Omega')\, \mathrm{d}\Omega' , \tag{6}$$

where G is Newton's gravitational constant, $H(\Omega)$ is the orthometric height, and $l\left[r(\Omega),\psi,r(\Omega')\right]$ is the spatial distance between $r(\Omega)$ and $r'(\Omega)$ given by

$$\forall\, \Omega \in \Omega_O : \qquad l^2\left[r(\Omega),\psi,r(\Omega')\right] = r^2(\Omega) + r^2(\Omega') - 2r(\Omega)\, r(\Omega') \cos\psi . \tag{7}$$

The potential $V^{ct}(r,\Omega)$ of topographical masses condensed on the geoid is given by

$$\forall\, \Omega \in \Omega_O : \qquad V^{ct}(r,\Omega) = GR^2 \iint_{\Omega'} \sigma(\Omega')\, l^{-1}\left[r(\Omega),\psi,R\right] d\Omega' , \tag{8}$$

where $\sigma(\Omega)$ stands for the (surface) density of topographical masses condensed onto the geoid according to the principle of mass conservation condensation of topographical masses (*Wichiencharoen, 1982*)

$$\forall\, \Omega \in \Omega_O : \qquad \sigma(\Omega) = \rho(\Omega) \frac{r_t^3(\Omega) - R^3}{3R^2} . \tag{9}$$

Other condensation schemes exist but the mass conservation principle is what we have selected to use herein.

A radial derivative of the residual topographical potential $\delta V^t(r,\Omega)$ stipulated as being on the earth surface is called the "*direct topographical effect*" (*Vaníček et al., 1999*)

$$\forall\, \Omega \in \Omega_O :$$

$$\left.\frac{\partial\, \delta V^t(r,\Omega)}{\partial\, r(\Omega)}\right|_{r(\Omega)=R+H(\Omega)} = \left.\frac{\partial V^t(r,\Omega)}{\partial\, r(\Omega)}\right|_{r(\Omega)=R+H(\Omega)} - \left.\frac{\partial V^{ct}(r,\Omega)}{\partial\, r(\Omega)}\right|_{r(\Omega)=R+H(\Omega)} . \tag{10}$$

The first term on the right hand side in Eq.(10), i.e., the radial derivative of the topographical potential $V^t(r,\Omega)$, represents the gravitational effect of the whole topography, and is consequently known as the "*topographical effect*". It can be expressed as

$$\forall\, \Omega \in \Omega_O :$$

$$\left.\frac{\partial V^t(r,\Omega)}{\partial r(\Omega)}\right|_{r(\Omega)=R+H(\Omega)} = \tag{11}$$

$$= G \iint_{\Omega'} \rho(\Omega') \int_{r(\Omega')=R}^{R+H(\Omega')} \left.\frac{\partial\, l^{-1}[r(\Omega),\psi,r(\Omega')]}{\partial r(\Omega)}\right|_{r(\Omega)=R+H(\Omega)} r^2(\Omega')\, \mathrm{d}r(\Omega')\, \mathrm{d}\Omega' \,.$$

The second term in Eq.(10), i.e., the radial derivative of the condensed topographical potential $V^{ct}(r,\Omega)$, represents the gravitational effect of the condensed topography, and as is consequently known as the "*condensed topographical effect*". It is evaluated from the following expression:

$$\forall\, \Omega \in \Omega_O :$$

$$\left.\frac{\partial V^{ct}(r,\Omega)}{\partial r(\Omega)}\right|_{r(\Omega)=R+H(\Omega)} = GR^2 \iint_{\Omega'} \sigma(\Omega') \left.\frac{\partial\, l^{-1}[r(\Omega),\psi,R]}{\partial r(\Omega)}\right|_{r(\Omega)=R+H(\Omega)} \mathrm{d}\Omega' \,. \tag{12}$$

The analytical expression for the radial integral of the reciprocal distance $l^{-1}[r(\Omega),\psi,r(\Omega')]$ multiplied by $r^2(\Omega')$ is found in *Gradshteyn and Ryzhik (1980)* to be:

$$\forall\; r(\Omega) \neq r(\Omega') \wedge \psi \neq 0 :$$

$$\int_{r(\Omega')} l^{-1}[r(\Omega),\psi,r(\Omega')]\, r^2(\Omega')\, \mathrm{d}r(\Omega') = \frac{1}{2}\Big| \,[r(\Omega') + 3r(\Omega)\cos\psi]\; l[r(\Omega),\psi,r(\Omega')] \tag{13}$$

$$+ r^2(\Omega)\left(3\cos^2\psi - 1\right) \ln\left|\, r(\Omega') - r(\Omega)\cos\psi + l[r(\Omega),\psi,r(\Omega')] \,\right| \Big|_{r(\Omega')} \,.$$

Similarly, the radial derivative of reciprocal distance (*Martinec, 1993*)

$$\forall\; r(\Omega) \neq r(\Omega') \wedge \psi \neq 0 : \quad \left.\frac{\partial\, l^{-1}[r(\Omega),\psi,r(\Omega')]}{\partial r(\Omega)}\right|_{r(\Omega)} = -\frac{r(\Omega) - r(\Omega')\cos\psi}{l^3[r(\Omega),\psi,r(\Omega')]} \tag{14}$$

multiplied by $r^2(\Omega')$ and integrated with respect to $r(\Omega)$, can be expressed analytically as follows (*Martinec, 1993*)

$$\forall\, r(\Omega) \neq r(\Omega') \wedge \psi \neq 0 :$$

$$\int_{r(\Omega')} \frac{\partial\, l^{-1}[r(\Omega), \psi, r(\Omega')]}{\partial\, r(\Omega)}\Bigg|_{r(\Omega)} r^2(\Omega')\, \mathrm{d}r(\Omega')$$

$$= \Bigg| \frac{r^2(\Omega')\cos\psi + 3\, r^2(\Omega)\cos\psi + \left(1 - 6\cos^2\psi\right) r(\Omega)\, r(\Omega')}{l[r(\Omega), \psi, r(\Omega')]} \tag{15}$$

$$+\, r(\Omega)\left(3\cos^2\psi - 1\right)\ln\left|\, r(\Omega') - r(\Omega)\cos\psi + l[r(\Omega), \psi, r(\Omega')]\,\right| \;\Bigg|_{r(\Omega')} .$$

The "*primary indirect topographical effect*" on geoidal height (which is referred to the geoid), is given by applying Bruns's formula (*Bruns, 1878*) to the residual topographical potential. We get:

$$\forall\, \Omega \in \Omega_O :$$

$$\delta N^H(\Omega) = \frac{1}{\gamma_o(\phi)}\delta V^t(R, \Omega) = \frac{1}{\gamma_o(\phi)} V^t(R, \Omega) - \frac{1}{\gamma_o(\phi)} V^{ct}(R, \Omega)$$

$$= \frac{G}{\gamma_o(\phi)} \iint_{\Omega'} \rho(\Omega') \int_{r(\Omega')=R}^{R+H(\Omega')} l^{-1}[R, \psi, r(\Omega')]\; r^2(\Omega')\, \mathrm{d}r(\Omega')\, \mathrm{d}\Omega' \tag{16}$$

$$- \frac{G\rho_o}{\gamma_o(\phi)} \iint_{\Omega'} \rho(\Omega') \frac{r_t^3(\Omega) - R^3}{3}\; l^{-1}(R, \psi, R)\, \mathrm{d}\Omega' .$$

The "*secondary indirect topographical effect*" on gravity (which is reckoned on the earth surface) is given by

$$\forall\, \Omega \in \Omega_O :$$

$$\frac{2}{r_t(\Omega)}\delta V^t(r_t, \Omega) = \frac{2}{r_t(\Omega)} V^t(r_t, \Omega) - \frac{2}{r_t(\Omega)} V^{ct}(r_t, \Omega)$$

$$= \frac{2}{r_t(\Omega)} G \iint_{\Omega'} \rho(\Omega') \int_{r(\Omega')=R}^{R+H(\Omega')} l^{-1}[r_t(\Omega), \psi, r(\Omega')]\; r^2(\Omega')\, \mathrm{d}r(\Omega')\, \mathrm{d}\Omega' \tag{17}$$

$$- \frac{2}{r_t(\Omega)} G \iint_{\Omega'} \rho(\Omega') \frac{r_t^3(\Omega) - R^3}{3}\; l^{-1}[r_t(\Omega), \psi, R]\, \mathrm{d}\Omega' .$$

3. FAR-ZONE CONTRIBUTIONS TO THE DIRECT TOPOGRAPHICAL EFFECT, THE PRIMARY INDIRECT TOPOGRAPHICAL EFFECT AND THE SECONDARY INDIRECT TOPOGRAPHICAL EFFECT

The evaluation of any of the topographical effects introduced above, is done by integrating over the surface of the whole earth. The surface of the earth, i.e., integration domain Ω_{O}, can be divided into two sub-domains, the near-zone Ω_{NZ} and far-zone Ω_{FZ}.

The far-zone contribution - which tends be of a low-frequency character - to the direct topographical effect can be written as

$$\forall\, \Omega \in \Omega_{\mathrm{FZ}} :$$

$$\begin{aligned}
&\left.\frac{\partial\, \delta V^{t}_{\Omega_{\mathrm{FZ}}}(r,\Omega)}{\partial\, r(\Omega)}\right|_{r(\Omega)=R+H(\Omega)} \\
&= \left.\frac{\partial V^{t}_{\Omega_{\mathrm{FZ}}}(r,\Omega)}{\partial\, r(\Omega)}\right|_{r(\Omega)=R+H(\Omega)} - \left.\frac{\partial V^{ct}_{\Omega_{\mathrm{FZ}}}(r,\Omega)}{\partial\, r(\Omega)}\right|_{r(\Omega)=R+H(\Omega)} \\
&= G\rho_o \iint_{\Omega'_{\mathrm{FZ}}} \int_{r(\Omega')=R}^{R+H(\Omega')} \left.\frac{\partial\, l^{-1}[r(\Omega),\psi,r(\Omega')]}{\partial\, r(\Omega)}\right|_{r(\Omega)=R+H(\Omega)} r^{2}(\Omega')\, \mathrm{d}r(\Omega')\, \mathrm{d}\Omega' \\
&- G\rho_o \iint_{\Omega'_{\mathrm{FZ}}} \frac{r_t^{3}(\Omega)-R^{3}}{3} \left.\frac{\partial\, l^{-1}[r(\Omega),\psi,R]}{\partial\, r(\Omega)}\right|_{r(\Omega)=R+H(\Omega)} \mathrm{d}\Omega' .
\end{aligned} \tag{18}$$

To a sufficient accuracy, the laterally varying topographical density $\rho(\Omega)$ can be approximated by the mean value $\rho_o = 2.67\ \mathrm{gcm}^{-3}$ in a computation of the far-zone contributions to the all topographical effects.

According the Eq.(16), the far-zone contribution to the primary indirect topographical effect on geoidal height is

$$\forall\, \Omega \in \Omega_{\mathrm{FZ}} :$$

$$\begin{aligned}
\delta N^{\mathrm{H}}_{\Omega_{\mathrm{FZ}}}(\Omega) &= \frac{1}{\gamma_o(\phi)}\delta V^{t}_{\Omega_{\mathrm{FZ}}}(R,\Omega) = \frac{1}{\gamma_o(\phi)} V^{t}_{\Omega_{\mathrm{FZ}}}(R,\Omega) - \frac{1}{\gamma_o(\phi)} V^{ct}_{\Omega_{\mathrm{FZ}}}(R,\Omega) \\
&= \frac{G\rho_o}{\gamma_o(\phi)} \iint_{\Omega'_{\mathrm{FZ}}} \int_{r(\Omega')=R}^{R+H(\Omega')} l^{-1}[R,\psi,r(\Omega')]\ r^{2}(\Omega')\, \mathrm{d}r(\Omega')\, \mathrm{d}\Omega' \\
&- \frac{G\rho_o}{\gamma_o(\phi)} \iint_{\Omega'_{\mathrm{FZ}}} \frac{r_t^{3}(\Omega)-R^{3}}{3}\ l^{-1}(R,\psi,R)\, \mathrm{d}\Omega' .
\end{aligned} \tag{19}$$

The far-zone contribution to the secondary indirect topographical effect on gravity follows from the Eq.(17):

$$\forall\, \Omega \in \Omega_{\mathrm{FZ}} :$$

$$\begin{aligned}
\frac{2}{r_t(\Omega)} \delta\ V^{t}_{\Omega_{\mathrm{FZ}}}(r_t,\Omega) &= \frac{2}{r_t(\Omega)} V^{t}_{\Omega_{\mathrm{FZ}}}(r_t,\Omega) - \frac{2}{r_t(\Omega)} V^{ct}_{\Omega_{\mathrm{FZ}}}(r_t,\Omega) \\
&= \frac{2}{r_t(\Omega)} G\rho_o \iint_{\Omega'_{\mathrm{FZ}}} \int_{r(\Omega')=R}^{R+H(\Omega')} l^{-1}[r_t(\Omega),\psi,r(\Omega')]\ r^2(\Omega')\ \mathrm{d}r(\Omega')\ \mathrm{d}\Omega' \\
&- \frac{2}{r_t(\Omega)} G\rho_o \iint_{\Omega'_{\mathrm{FZ}}} \frac{r_t^3(\Omega) - R^3}{3}\ l^{-1}[r_t(\Omega),\psi,R]\ \mathrm{d}\Omega'\ .
\end{aligned} \tag{20}$$

4. RESULTS

The far-zone contributions to the topographical effects computed for the territory of Canada are shown in Figs. 1–8. One-degree step has been used in the surface numerical integration in the far-zone integration sub-domain Ω_{FZ}, chosen as $2.5^\circ \le |\phi' - \phi|$, $2.5^\circ \le |\lambda - \lambda'|$.

Minimum, average, and maximum values of the topographical effects on the potential and on the gravitational attraction are summarized in Tables 1 to 4. Minimum, average, and maximum values of the far-zone contributions to the primary indirect topographical effect on the geoidal height and the secondary indirect topographical effect on gravity at the earth surface are summarized in Table 5.

We note that the third rows in Tables 1 and 2, which represent the differences between the first and second rows, looks a bit misleading. The extreme values are, of course, located at different locations from the extreme values referred to in the first two rows.

The same observation as offered to Tables 1 and 2 is valid here. Comparing the numerical results in Table 1 and 2, and 3 and 4, the far-zone contributions to the topographical effects on the potential and gravity do not seem to depend on the height of the computation point. We may also note that the contribution of the far zone to the topographical effect and to the condensed topographical effect, are very similar. Consequently, the contribution of the far zone to the direct topographical effect is significantly smaller than the other two effects.

The far-zone contributions to the topographical potential, the condensed topographical potential and the residual topographical potential are shown in Figs. 1, 2, and 3. We no longer specify if these effects are taken at the earth surface or at the geoid, as these are practically the same.

The far-zone contributions to the topographical, the condensed topographical and the direct topographical effects on gravity are shown in Figs. 4, 5 and 6. Once again, we do not distinguish between the contributions taken at the earth surface and at the geoid, as these two are practically the same.

Table 1. Minimum, average, and maximum values of the far-zone contributions to the topographical effects on the potential taken at the earth surface [$m^2\ s^{-2}$].

Topographical effects on the potential referred on the earth surface	Minimum	Average	Maximum
Topographical Potential	2999.47	3794.48	4622.44
Condensed Topographical Potential	2999.68	3794.73	4622.81
Residual Topographical Potential	–0.375	–0.25	–0.21

Table 2. Minimum, average, and maximum values of the far-zone contributions to the topographical effects on the potential taken at the geoid [$m^2\ s^{-2}$]].

Topographical effects on the potential referred on the geoid	Minimum	Average	Maximum
Topographical Potential	2999.47	3794.48	4622.44
Condensed Topographical Potential	2999.68	3794.73	4622.81
Residual Topographical Potential	–0.375	–0.25	–0.21

Table 3. Minimum, average, and maximum values of the far-zone contributions to the topographical effects on gravity taken at the earth surface [mGal =10^{-5} ms^{-2}].

Topographical effects on the gravitational attraction referred on the earth surface	Minimum	Average	Maximum
Topographical Effect	–33.96	–29.62	–23.53
Condensed Topographical Effect	–36.28	–29.78	–23.54
Direct Topographical Effect	0.009	0.165	3.54

Table 4. Minimum, average, and maximum values of the far-zone contributions to the topographical effects on the gravity taken at the geoid [mGal =10^{-5} ms^{-2}].

Topographical effects on the gravitational attraction referred on the geoid	Minimum	Average	Maximum
Topographical Effect	–33.96	–29.62	–23.53
Condensed Topographical Effect	–36.28	–29.78	–23.54
Direct Topographical Effect	0.009	0.165	3.54

The far-zone contribution to the primary indirect topographical effect on geoidal height [cm] is shown in Fig. 7, while Fig. 8 shows the far-zone contribution to the secondary indirect topographical effect on gravity [mGal = 10^{-5} ms^{-2}].

Table 5. Minimum, average, and maximum values of the far-zone contributions to the primary indirect topographical effect on the geoidal height [cm] and the secondary indirect topographical effect on gravity at the earth surface [mGal $=10^{-5}$ ms^{-2}].

	Minimum	Average	Maximum
Primary Indirect Topographical Effect to the geoidal height [cm]	−3.83	−2.59	−2.12
Secondary Indirect topographical Effect [mGal]	-1.18×10^{-7}	-7.97×10^{-8}	-6.54×10^{-8}

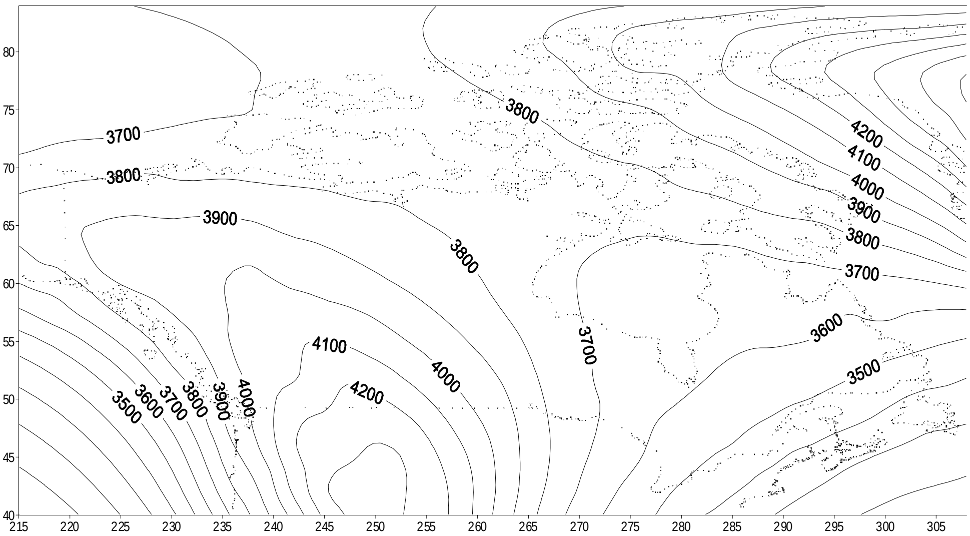

Fig. 1. The far-zone contribution to the topographical potential [m^{2}s^{-2}].

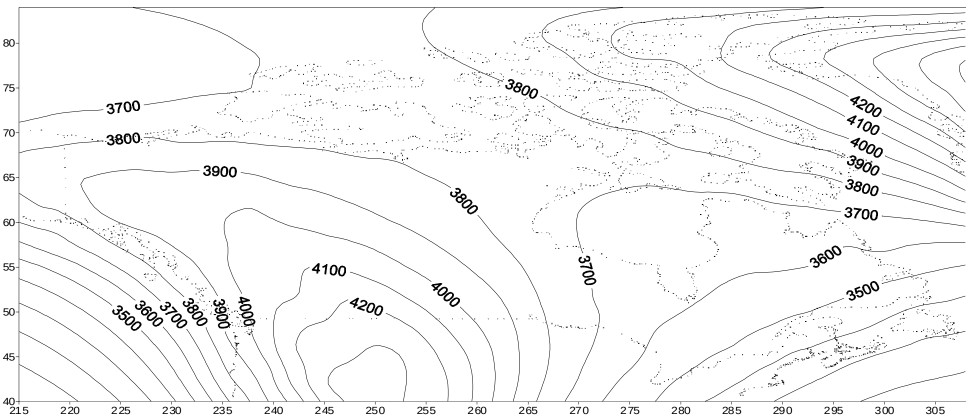

Fig. 2. The far-zone contribution to the condensed topographical potential [m^{2}s^{-2}].

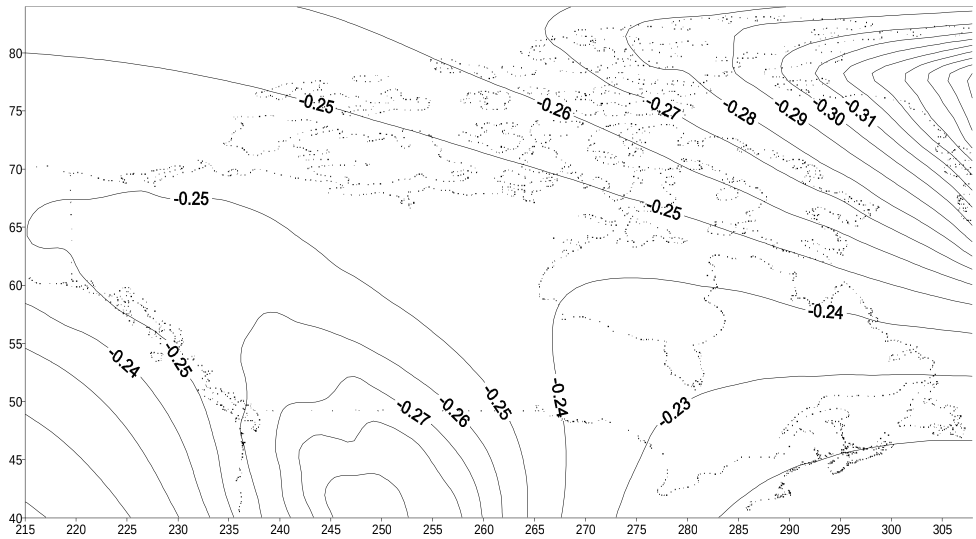

Fig. 3. The far-zone contribution to the residual topographical potential [m^2s^{-2}].

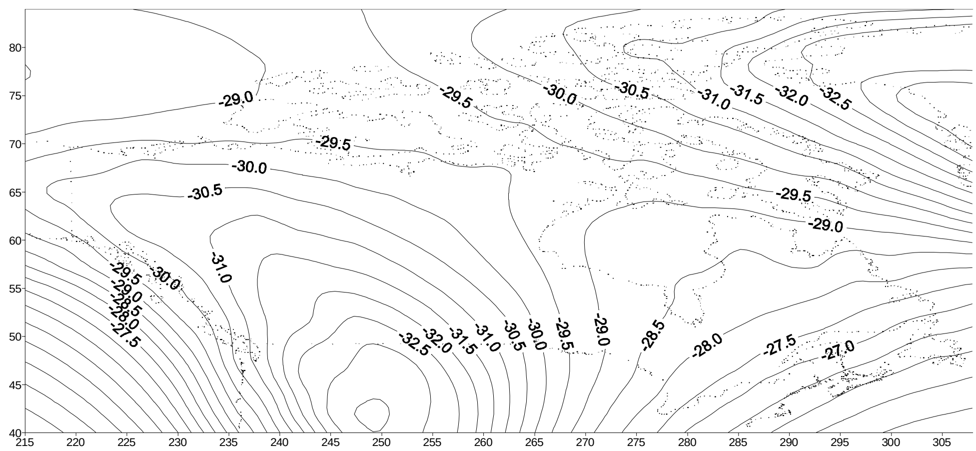

Fig. 4. The far-zone contribution to the topographical effect on gravity [mGal = 10^{-5} ms^{-2}].

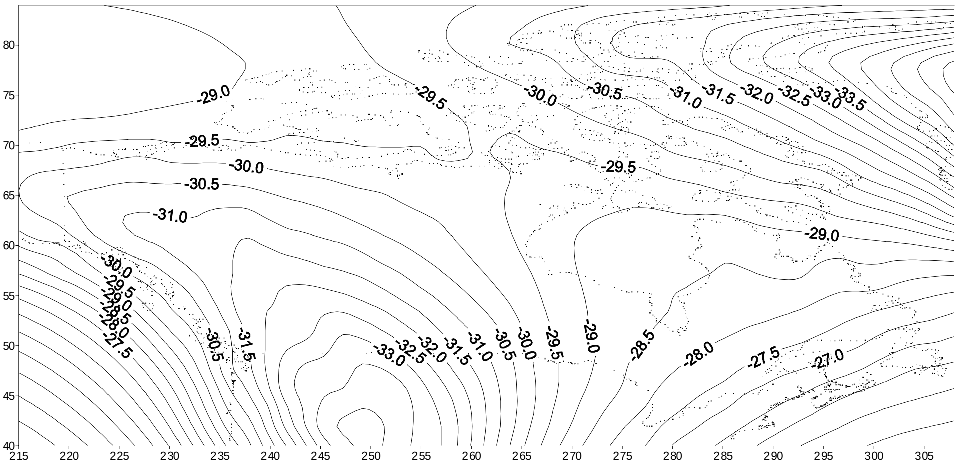

Fig. 5. The far-zone contribution to the condensed topographical effect on gravity [mGal = 10^{-5} ms^{-2}].

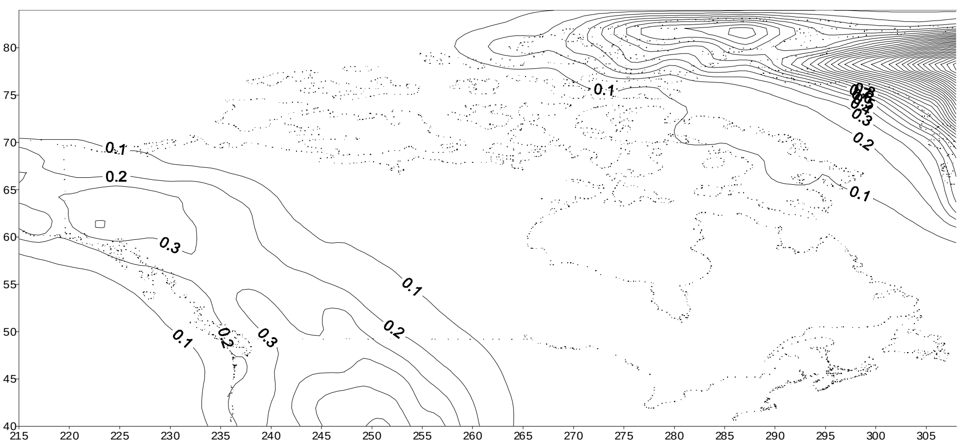

Fig. 6. The far-zone contribution to the direct topographical effect on gravity [mGal = 10^{-5} ms^{-2}].

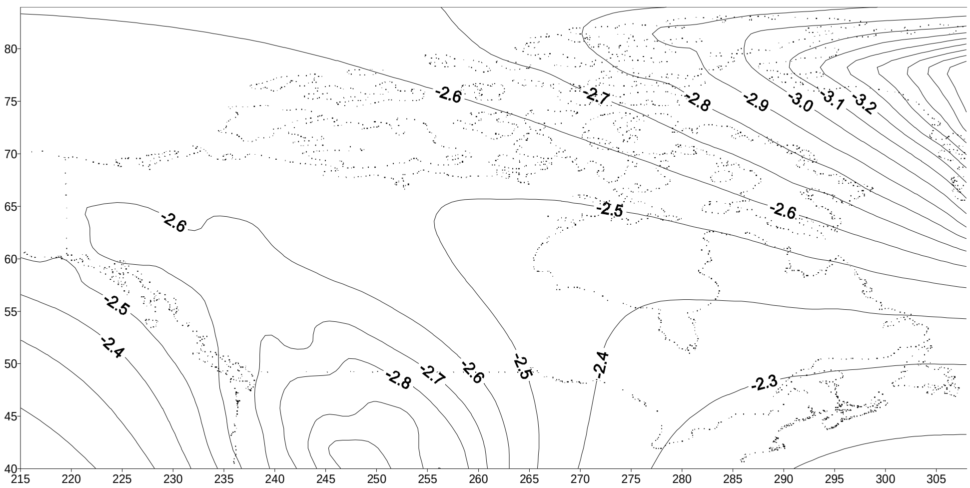

Fig. 7. The far-zone contribution to the primary indirect topographical effect on geoidal height [cm].

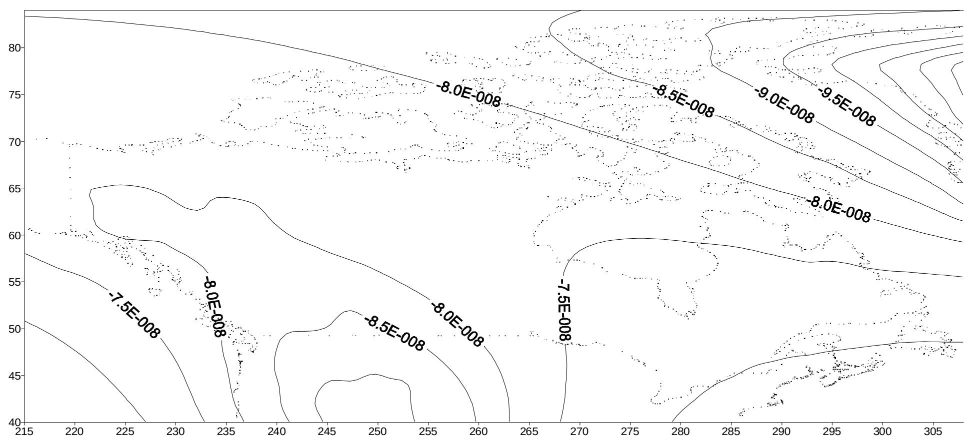

Fig. 8. The far-zone contribution to the secondary indirect topographical effect [mGal = 10^{-5} ms^{-2}].

5. CONCLUSIONS

Comparing the numerical results in Table 1 and 2, and 3 and 4, the far-zone contributions to the topographical effects on the potential and the gravity do not seem to depend on the height of the computation point. We may also note that the contribution of the far-zone to the topographical effect and to the condensed topographical effect, are very similar. Consequently, the contribution of the far-zone to the direct topographical effect is significantly smaller than the other two contributions.

We no longer specify if these far-zone effects are taken at the earth surface or at the geoid, as these are practically the same.

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COMMENTS OF THE EDITORS

The methodologies like that by the authors are currently used in the applied geophysics. However, in geodesy more accurate methodologies have been used since 1945.