Computational Methods for the Discrete Downward Continuation of the Earth Gravity

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Introduction: The downward continuation (DC) can be considered as a 'projection' in which the gravity anomalies observed at a surface above the geoid are 'mapped' onto the geoid. There are primarily two DC methods: the Poisson DC and the Moritz analytical DC. The Poisson DC solves the Poisson integral equation which can be split into the near-zone and the far-zone contributions:

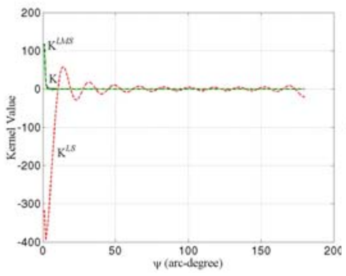
$$\Delta g(H,\Omega) = \frac{R}{4\pi r}\int_{\Omega_0} K(H,\Omega,\Omega')\Delta g(0,\Omega')d\Omega' + F_{\Delta g}(H,\Omega)$$

where the far-zone contribution can be evaluated as

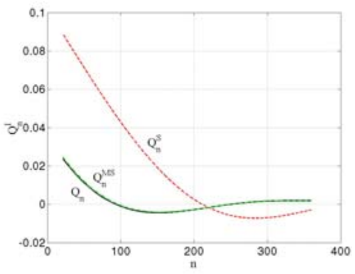
$$F_{\Delta g}(H,\Omega) = \frac{R\gamma}{2r}\sum_{n=2}^{\infty}(n-1)Q_n(H,\psi_0)\sum_{m=-n}^{n}c_{nm}Y_{nm}(\Omega)$$

The Poisson integral equation has no closed-form solution, thus it has to be solved numerically. The following issues have to be considered in the discrete Poisson DC: Poisson's kernel modification, evaluation of the far-zone contribution, selection of the radius for the near-zone cap size, discretization of Poisson's integral (point and mean models) and method for solving the discrete Poisson linear system of equations. The Moritz analytical DC approximately expands the gravity anomaly into a Taylor series. Whether it can provide a DC result as good as the Poisson one is an open question.

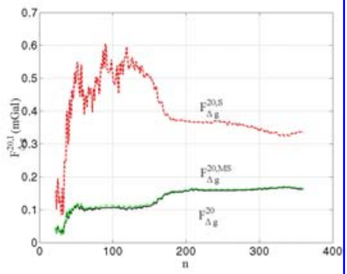
Poisson's kernel modification: The spheroidal Poisson kernel $K^{l,S}$ (Martinec, 1996) and the modified spheroidal Poisson kernel $K^{l,MS}$ (Vaníček et al., 1996) have been proposed in place of the standard Poisson kernel K. The former gives the 'real' far-zone contribution while the latter aims to reduce the real far-zone contribution. The three figures on the right side show that the real far-zone contribution given by $K^{l,S}$ has been significantly reduced by adopting $K^{l,MS}$, while K may be used as an alternative to $K^{l,MS}$. Furthermore, the far-zone contribution series displays a good convergence at lower degrees. After about degree 180, it shows negligible changes suggesting that the far-zone contribution can be evaluated from a global geopotential model taken to degree 180.



K, $K^{l,S}$ and $K^{l,MS}$ vs. ψ(1° - 180°). H = 2 km.



The truncation error coefficients Q_n^l. ψ_0=1°. H= 2 km.



The far-zone contribution $F_{\Delta g}$ vs. degree n. ψ_0=1°. H= 2 km.

A fast algorithm for evaluation of the far-zone contribution: The far-zone contribution can be evaluated by the following expression

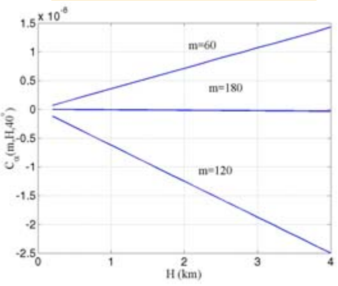
$$F_{\Delta g}(H,\Omega) = \frac{R\gamma}{2r}\sum_{m=0}^{k}[C_\alpha(m,H\theta)\cos mj\Delta\lambda + C_\beta(m,H\theta)\sin mj\Delta\lambda]$$

where

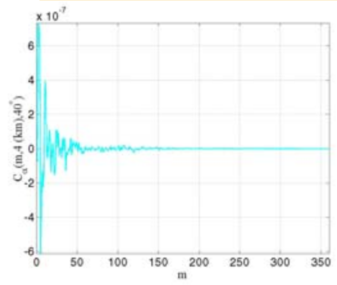
$$C_\alpha(m,H,\theta) = \sum_{n=m}^{k}[(n-1)Q_n(H,\psi_0)(c_{nm}P_{nm}\cos m\lambda_0 + d_{nm}P_{nm}\sin m\lambda_0)]$$

$$C_\beta(m,H,\theta) = \sum_{n=m}^{k}[(n-1)Q_n(H,\psi_0)(-c_{nm}P_{nm}\sin m\lambda_0 + d_{nm}P_{nm}\cos m\lambda_0)]$$

Instead of evaluating coefficients C_α and C_β at points of variable heights with the same latitude, we pre-tabulate them for a number of representative heights, then predict their values at the height of individual point by the linear interpolation with sufficient accuracy.



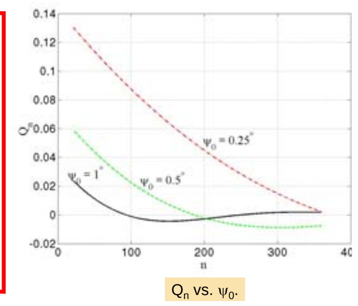
C_α(m, H, θ) vs. H.



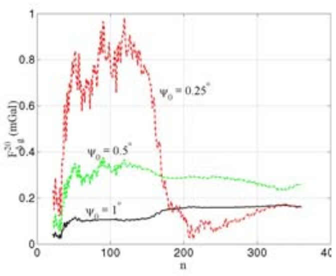
C_α(m, H, θ) vs. m.

Selection of the radius for the near-zone cap: The criterion for the selection of the radius of the near-zone cap is that the far-zone contribution can be accurately evaluated or be small enough to be considered negligible. The far-zone contribution is routinely computed from a global geopotential model. Therefore, the radius must be determined in terms of the accuracy of the model, which is, in turn, represented by the accuracy of the model coefficients and the maximum degree and order of the model.

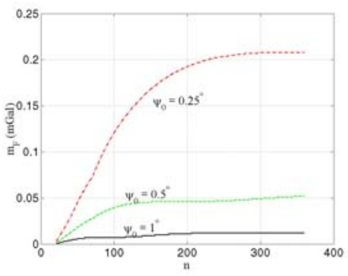
The results on the right side demonstrate that one-half arc-degree can be selected as the radius of the near-zone cap even in mountainous areas where the maximum elevation does not exceed 2 km when the EGM96 is used.



Q_n vs. ψ_0.



$F_{\Delta g}$ vs. ψ_0.

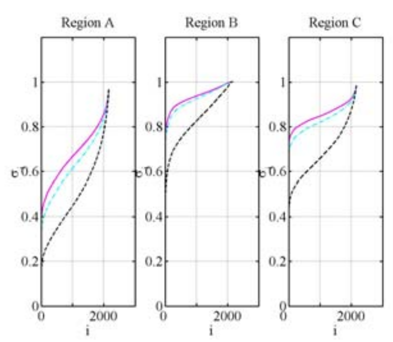


m_F vs. ψ_0.

The discrete models of the Poisson integral equation: We evaluate the singular value spectra (see the right figure) of the coefficient matrices for the point-point, point-mean and mean-mean discrete models for three different regions. The results indicate that the mean-mean model possesses the biggest condition number while the point-point model displays the smallest condition number. The differences among spectra show that the elevation rather than the relative relief is the dominant factor in determining the condition of the coefficient matrix.

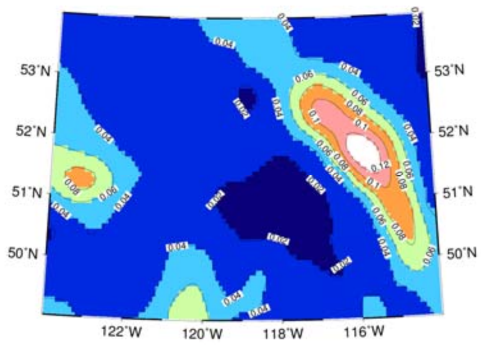
Statistics of the mean 5' by 5' DTM for the test regions A, B and C. Unit: m.

Region	Min.	Max.	Mean	StdDev	R.M.S.
A	459	2682	1599	431	1656
B	0	942	316	202	376
C	488	1122	684	122	694

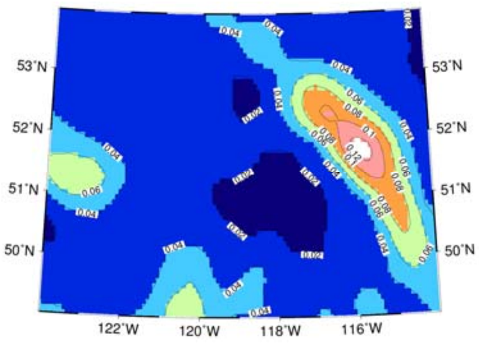


The singular value spectra of the coefficient matrices for the point-point model (solid lines), point-mean (dotdash lines) and mean-mean model (dash lines) on the 5' by 5' grid.

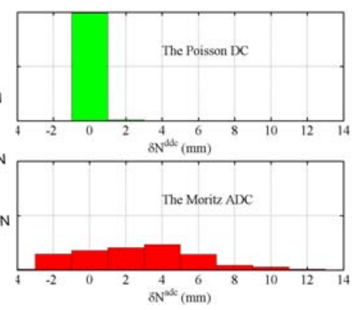
The Moritz analytical downward continuation: The Moritz ADC is compared to the Poisson DC using the synthetic data generated from degrees 21-1800 of GPM98a (Wenzel, 1998). The results show that the Moritz ADC introduces an error of about 10% of the total DC effect while the error of the Poisson DC is smaller than 1 cm (see figures below). When both methods are used to evaluate the DC effects on the Helmert gravity anomaly in the Canadian Rocky Mountains, the difference accounts for 10% of the total DC effect, which translates to 10 mgal in gravity and 10 cm in the geoid height at maximum (not shown here).



The point-point Poisson DC effect on the geoid from the GPM98a synthetic data. C.I.: 0.01 m.



The point-point Moritz DC effect on the geoid from the GPM98a synthetic data. C.I.: 0.01 m.



The error distributions of the Poisson DC and the Moritz DC against the GPM98a synthetic data.

Conclusions: The modified spheroidal Poisson kernel (MSPK) significantly reduces the real far-zone contribution given by the unmodified spheroidal Poisson kernel, while the standard Poisson kernel works as efficiently as the MSPK. A fast algorithm is proposed to evaluate the far-zone contribution. The radius of the near-zone cap must be determined in terms of the accuracy of the global geopotential model. When the EGM96 is used, one-half arc-degree can be selected as the radius of the near-zone cap in mountainous areas where the maximum elevation does not exceed 2 km. The Moritz DC causes a relative error of 10% of the total DC effect in the Canadian Rocky Mountains.

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