Computational Methods for the Discrete Downward Continuation of the Earth Gravity

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Outline

- Poisson’s kernel modification
- The discrete Poisson downward continuation (DDC) and Moritz’s analytical downward continuation (ADC)
- Downward continuation of the refined Bouguer anomaly
Poisson’s Kernel Modification

Poisson’s integral:
\[ \Delta g^I(r, \Omega) = \frac{R}{4\pi r} \int_{\Omega_0} K_I(r, \psi, R) \Delta g^I(R, \Omega') d\Omega' + F^I_{\Delta g, I}(r, \Omega) \]

The spheroidal Poisson kernel:
\[ K_{WG}(r, \psi, R) = K(r, \psi, R) - \sum_{n=0}^{I} (2n + 1) \left( \frac{R}{r} \right)^{n+1} P_n(\cos \psi) \]

The modified spheroidal Poisson kernel:
\[ K_{VK}(r, \psi, R) = K(r, \psi, R) - \sum_{n=0}^{I} \frac{2n + 1}{2} t_n(H, \psi_0) P_n(\cos \psi) \]
The Far Zone Contribution

The far zone contribution:

\[ F_{\Delta g}^l (r, \Omega) = \frac{R \gamma}{2r} \sum_{n=l+1}^{\infty} (n-1)Q_n^l (H, \psi_0) \sum_{m=-n}^{n} C_{nm} Y_{nm} (\Omega) \]

The truncation error coefficients:

\[ Q_n^l (H, \psi_0) = \int_{\psi_0}^{\pi} K_l (r, \psi, R) P_n (\cos \psi) \sin \psi \, d\psi \]

The standard deviation:

\[ m_F^l (r, \Omega) = \frac{R \gamma}{2r} \left( \sum_{n=l+1}^{\infty} (n-1)^2 \left( Q_n^l \right)^2 \sum_{m=0}^{n} dC_{nm}^2 Y_{nm}^2 \right) \frac{1}{2} \]
Moritz’s Analytical Downward Continuation
(Moritz, 1980, Section 45)

\[ \Delta g(r_g, \Omega) = \sum_{n=0}^{\infty} g_n, \]

\[ g_0 = \Delta g(r, \Omega), \]
\[ g_1 = -H \cdot L_1(g_0), \]
\[ g_2 = -H \cdot L_1(g_1) - H^2 \cdot L_2(g_0), \]
\[ \vdots \]
\[ g_n = -\sum_{r=1}^{n} H^r \cdot L_r \cdot g_{n-r}. \]

\[ L(g) = \frac{R^2}{2\pi} \int_{\Omega'} \frac{g - g_p}{d^3} d\Omega' - \frac{1}{R} g_p, \]

\[ L_n = \frac{1}{n!} L^n = \frac{1}{n} LL_{n-1}. \]
The DDC for the GPM98a Synthetic Data (degrees 21-1800)
The ADC for the GPM98a Synthetic Data (degrees 20-1800)
Errors of the DDC and the ADC for the GPM98a Data (21-1800)
The DDC of the Residual Helmert Gravity Anomaly (above degree 20)
The ADC of the Residual Helmert Gravity Anomaly (above degree 20)
The Difference Between the DDC and the ADC
The Downward Continuation of the Refined Bouguer Anomaly

\[- \frac{\partial T^h(r, \Omega)}{\partial r} - \frac{2}{r} T^h(r, \Omega) = \Delta g^h(r, \Omega) + \varepsilon\]

\[T^h(r, \Omega) = T(r, \Omega) - V^t(r, \Omega) + V^{ct}(r, \Omega)\]

\[- \frac{\partial T^{rb}(r, \Omega)}{\partial r} - \frac{2}{r} T^h(r, \Omega) = \Delta g^h(r, \Omega) + \frac{\partial V^{ct}(r, \Omega)}{\partial r} + \varepsilon\]
$\psi_0 = 1 \text{ arc-degree}$
$\psi_0 = 2 \text{ arc-degrees}$
The Difference Between the 1-deg. Cap and the 2-deg. Cap
Conclusions

- The standard Poisson kernel functions as well as the modified spheroidal Poisson kernel in reducing the far zone contribution.
- The analytical downward continuation agrees with the discrete Poisson's within 10% of the total downward continuation effect.
- The DC of the refined Bouguer gravity anomaly faces a challenge of evaluation.