On the downward continuation of Helmert's gravity anomalies

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This is a progress report on our investigation of the downward continuation of Helmert's gravity anomalies from the surface of the earth onto the geoid, considered in the "Helmert space". The first question to which we seek the answer is: "Is this a well-posed problem in the Hadamard sense?" In other words:

1) Does the solution exist ?
2) If it exists, is it unique?
3) If it exists and if it is unique, is the solution continuous?

For Helmert's anomalies, the answer to all three Hadamard's conditions is affirmative, and we conclude that we are dealing with a well posed problem. We note that the fact that small changes in the input data (anomalies on the earth surface) may cause large changes in the solution (anomalies on the geoid), does not strictly violate the third Hadamard condition as the solution is still continuous (in the narrow sense). Consequently, the Picard condition is also satisfied.

The discretised problem is then automatically also well posed, as can be seen from the discrete Picard condition which is always satisfied. The question of interest, however, is whether the numerical solution to the discretised problem above is unstable in the sense that errors in the input data are unreasonably magnified by the downward continuation. It is well known that the numerical representation (discretisation) of the Poisson integral for downward continuation leads to a system of linear equations, the matrix of which depends only on topographical heights and the discretisation step. This matrix is non-symmetrical and increasingly ill-conditioned for decreasing discretisation steps and increasing topographical heights. The answer to the question of numerical instability can be sought in the frequency domain. In this contribution we make an attempt to formulate the answer.