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On the indirect effect in the Stokes–Helmert method of geoid determination

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Abstract. The classical integral formula for determining the indirect effect in connection with the Stokes–Helmert method is related to a planar approximation of the sea level. A strict integral formula, as well as some approximations to it, are derived. It is concluded that the cap-size truncated integral formulas will suffer from the omission of some long-wavelength contributions, of the order of 50 cm in high mountains for the classical formula. This long-wavelength information can be represented by a set of spherical harmonic coefficients of the topography to, say, degree and order 360. Hence, for practical use, a combination of the classical formula and a set of spherical harmonics is recommended.

Key words. Geoid · Helmert condensation · Indirect effect · Remove–restore

1 Introduction

Geoid determination by Stokes' well-known formula requires that all topographic masses are removed or reduced to a layer on or below the geoid. The effect of restoration of these masses is the indirect effect. The most common reduction method is Helmert's second condensation method, where the topographic masses are reduced to a surface layer at sea level in a spherical approximation of the geoid. This classical method, using a planar approximation, has been extensively discussed, by, for example, Heiskanen and Moritz (1967), Wichiencharoen (1982), Vaníček and Kleusberg (1987), Wang and Rapp (1990) and Sjöberg (1994). Sjöberg (1994, 1995, 1997) and Nahavandchi and Sjöberg (1998) used a spherical harmonic approach to derive the indirect effect. Common to all the above methods is that they are limited to the second or third power of elevation.

A recent description of the Stokes–Helmert method for geoid determination was given by Vaníček and Martinec (1994). The specific problem of determining the indirect effect was treated by Martinec and Vaníček (1994), who pointed out that the classical formula may be severely biased.

In the present paper, we will start to compare the formulas based on planar approximation with those based on spherical harmonics. Further on, a rigorous surface integral formula for the indirect effect will be derived (assuming a spherical approximation of the geoid and constant topographic density). The strict formula is finally expanded in a Taylor series for comparison with the previous approximate solutions.

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2 The indirect effect to power H^3

The classical formula (see e.g. Wichiencharoen 1982) for determining the indirect effect on the geoid for Helmert's second condensation method is

$$N_{\mathrm{I}}(P)^{\text{classic}} = \frac{-\pi\mu H_P^2}{\gamma} - \frac{\mu R^2}{6\gamma} \iint_\sigma \frac{H^3 - H_P^3}{\ell_0^3}\, \mathrm{d}\sigma \qquad (1)$$

where

$\mu = G\rho_0$, G being the universal gravitational constant, and ρ_0 the density of topography, assumed to be constant;
H, H_P = orthometric heights of the running and computation points, respectively;
$\ell_0 = R\sqrt{2(1-\cos\psi)} = 2R\sin\frac{\psi}{2}$;
R = mean Earth surface radius;
σ = the unit sphere;
ψ = geocentric angle between computation point P and running point on the sphere;
γ = mean normal gravity.

Sjöberg (1994, 1995, 1997) developed the indirect effect in terms of (surface) spherical harmonics to the power H^2, and Nahavandchi and Sjöberg (1998) extended this approach to the power H^3. The results can be summarized as

$$N_{\mathrm{I}}(P) = -\frac{2\pi\mu}{\gamma}\sum_{n=0}^{\infty}\frac{n-1}{2n+1}H_n^2(P) + \frac{2\pi\mu}{3R\gamma}\sum_{n=0}^{\infty}\frac{n(n-1)}{2n+1}H_n^3(P) \tag{2}$$

where

$$H_n^{\nu}(P) = \frac{2n+1}{4\pi}\iint_{\sigma} H^{\nu}P_n(\cos\psi)\,\mathrm{d}\sigma;\ \ \nu = 2,3 \tag{3}$$

and $P_n(\cos\psi)$ is Legendre's polynomial of order n.

Equation (2) can be reformulated as an integral similar to Eq. (1). To achieve this, we first rewrite Eq. (2) as follows:

$$N_{\mathrm{I}}(P) = \frac{\pi\mu}{\gamma}\left[-H_P^2 - \frac{H_P^3}{2R} + \sum_{n=0}^{\infty}\frac{3}{2n+1}H_n^2(P) + \sum_{n=0}^{\infty}\left(\frac{n}{3}+\frac{1}{2}\frac{1}{2n+1}\right)\frac{H_n^3(P)}{R}\right] \tag{4}$$

where we have used the notation

$$H_P^{\nu} = \sum_{n=0}^{\infty}H_n^{\nu}(P);\ \ \nu = 2,3$$

Inserting Eq. (3), and considering that

$$\sum_{n=0}^{\infty}P_n(\cos\psi) = \frac{R}{\ell_0} \tag{5}$$

and (Heiskanen and Moritz 1967, p. 39)

$$-\frac{1}{R}\sum_{n=0}^{\infty}nH_n^3(P) = \frac{R^2}{2\pi}\iint_{\sigma}\frac{H^3 - H_P^3}{\ell_0^3}\,\mathrm{d}\sigma \tag{6}$$

we arrive at

$$N_{\mathrm{I}}(P) = \frac{\pi\mu}{\gamma}\left[-H_P^2 + \frac{3R}{4\pi}\iint_{\sigma}\frac{H^2}{\ell_0}\,\mathrm{d}\sigma - \frac{H_P^3}{2R} + \frac{R^2}{6\pi}\iint_{\sigma}\left(\frac{H_P^3 - H^3}{\ell_0^3} + \frac{3}{4}\frac{H^3}{R^2\ell_0}\right)\mathrm{d}\sigma\right] \tag{7}$$

In view of the fact that

$$\frac{R}{4\pi}\iint_{\sigma}\frac{\mathrm{d}\sigma}{\ell_0} = 1 \tag{8}$$

we finally obtain

$$N_{\mathrm{I}}(P) = \frac{2\pi\mu}{\gamma}H_P^2 + \frac{3R\mu}{4\gamma}\iint_{\sigma}\frac{H^2 - H_P^2}{\ell_0}\,\mathrm{d}\sigma - \frac{R^2\mu}{6\gamma}\iint_{\sigma}\frac{H^3 - H_P^3}{\ell_0^3}\left(1 - \frac{3}{4}\frac{\ell_0^2}{R^2}\right)\mathrm{d}\sigma \tag{9}$$

Comparing the classical formula [Eq. (1)] with the new one [Eq. (9)], we obtain the difference

$$\delta N_{\mathrm{I}}(P) = N_{\mathrm{I}}^{\mathrm{classic}} - N_{\mathrm{I}}^{\mathrm{new}} = -\frac{3\pi\mu}{\gamma}H_P^2 - \frac{3R\mu}{4\gamma}\iint_{\sigma}\frac{H^2 - H_P^2}{\ell_0}\,\mathrm{d}\sigma - \frac{\mu}{8\gamma}\iint_{\sigma}\frac{H^3 - H_P^3}{\ell_0}\,\mathrm{d}\sigma \tag{10}$$

or, in view of Eq. (8)

$$\delta N_{\mathrm{I}}(P) = -\frac{3R\mu}{4\gamma}\iint_{\sigma}\frac{H^2}{\ell_0}\,\mathrm{d}\sigma - \frac{\mu}{8\gamma}\iint_{\sigma}\frac{H^3 - H_P^3}{\ell_0}\,\mathrm{d}\sigma \tag{11}$$

or, in spectral form

$$\delta N_{\mathrm{I}}(P) = -\frac{3\pi\mu}{\gamma}\bar{H}_P^2 + \frac{\pi\mu}{2R\gamma}\left(H_P^3 - \bar{H}_P^3\right) \tag{12}$$

where

$$\bar{H}_P^{\nu} = \sum_{n=0}^{\infty}\frac{1}{2n+1}H_n^{\nu}(P) \tag{13}$$

Formula (12) shows that there are some long-wavelength differences of power H^2 between the classical and new formulas. (The terms of power H^3 are less than a few centimetres, and they can therefore be neglected in most cases.) The most likely explanation for this difference is that the classical method suffers from the planar approximation. This can be seen from the following example. For a smooth topography, the first term on the right-hand side of Eq. (11) can be approximated by

$$-\frac{3R\mu}{4\gamma}\iint_{\sigma}\frac{H^2}{\ell_0}\,\mathrm{d}\sigma \doteq -\frac{3\pi\mu H_P^2 s_0}{R\gamma} \doteq -0.027 s_0 H_P^2\ \ (\mathrm{mm})$$

where s_0 is the maximum polar radius (in km) and H_P is also given in km. For example, with $s_0 = 555$ km (corresponding to a geocentric radius of about 5°) and $H_P = 1$ km, this term becomes −1.5 cm, but for $H_P = 6$ km it ranges to −50 cm! (Cf. Martinec and Vaníček 1994). Hence $-\delta N_{\mathrm{I}}$ in Eq. (12) can be regarded as a correction to the classical method, which leads to the formula

$$N_{\mathrm{I}}^{\mathrm{new}} = N_{\mathrm{I}}^{\mathrm{classic}} - \delta N_{\mathrm{I}} \tag{14}$$

3 A strict formula for the indirect effect

At the point P on the sphere of radius R, the topographic potential at point P can be written (for constant density)

$$V^{\mathrm{t}}(P) = \mu\iint_{\sigma}\int_{r=R}^{R+H}\frac{r^2\,\mathrm{d}r}{\ell}\,\mathrm{d}\sigma \tag{15}$$

where $\ell = \sqrt{r^2 + R^2 - 2rRt}$, r is the geocentric radius of topographic point and $t = \cos\psi$. The corresponding potential of a condensed layer according to Helmert's second condensation method becomes

$$V^{c}(P) = \mu R^2 \iint_\sigma \frac{H}{\ell_0}\, d\sigma \tag{16}$$

corresponding to a surface layer of density $\rho_0 H$ on the sphere of radius R. By differencing and dividing by γ (Bruns' formula), we obtain the indirect geoid effect

$$\begin{aligned} N_{\mathrm{I}}(P) &= \frac{1}{\gamma}\left(V^{t}(P) - V^{c}(P)\right) \\ &= \frac{\mu}{\gamma}\iint_\sigma \left(f(H,t) - \frac{R^2 H}{\ell_0}\right) d\sigma \end{aligned} \tag{17}$$

where we have introduced the kernel function

$$f(H,t) = \int_R^{R+H} \frac{r^2 dr}{\ell} \tag{18}$$

This formula can be directly integrated to (cf. Martinec and Vaníček 1994)

$$\begin{aligned} f(H,t) =& \frac{R+H+3Rt}{2}\ell - \frac{R+3Rt}{2}\ell_0 \\ &+ R^2 P_2(t) \ln\left|\frac{R+H-Rt+\ell}{R-Rt+\ell_0}\right| \end{aligned} \tag{19}$$

or

$$f(H,t) = \begin{cases} 0 & \text{if } H = 0 \\ \frac{R+3Rt}{2(\ell+\ell_0)} H\left(H + \frac{\ell_0^2}{R}\right) + \frac{H\ell}{2} & \\ +R^2 P_2(t) \ln\left|\frac{R+H-Rt+\ell}{R-Rt+\ell_0}\right| & \text{if } H > 0 \end{cases} \tag{19a}$$

Therefore, Eq. (17), with the function $f(H,t)$ provided by Eq. (19) or Eq. (19a), is a rigorous integral formula for the indirect effect.

4 Taylor expansion of the strict formula

For completeness, we now derive the power series of N_{I} from the strict formula of Eq. (17) above. We start from a Taylor expansion of $f(H,t)$ around $f(0,t)$:

$$f(H,t) = f(0,t) + \sum_{k=1}^{\infty} f^{(k)}(0,t)\frac{H^k}{k!} \tag{20}$$

where $f^{(k)}$ is the k-th radial derivative (i.e. with respect to H) of $f(H,t)$. It follows directly from Eq. (18) that

$f(0,t) = 0$ and $f^{(1)}(0,t) = R^2/\ell_0$

Thus we can rewrite Eq. (17) as

$$N_{\mathrm{I}}(P) = \frac{\mu}{\gamma}\sum_{k=2}^{\infty}\iint_\sigma f^{(k)}(0,t)\frac{H^k}{k!}\, d\sigma \tag{21}$$

From (with the notation $r = R + H$)

$$f^{(1)}(H,t) = \frac{r^2}{\ell} = r\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^n P_n(t)$$

we obtain

$$\begin{aligned} f^{(2)}(H,t) &= \frac{3r}{2\ell} - \frac{r(r^2-R^2)}{2\ell^3} \\ &= -\sum_{n=0}^{\infty}(n-1)\left(\frac{R}{r}\right)^n P_n(t) \end{aligned} \tag{22}$$

and

$$\begin{aligned} f^{(3)}(H,t) &= \frac{3}{4\ell} - \frac{5}{4}\frac{r^2-R^2}{\ell^3} - \frac{\ell}{2}\frac{\partial}{\partial r}\left(\frac{r^2-R^2}{\ell^3}\right) \\ &= \frac{1}{R}\sum_{n=0}^{\infty} n(n-1)\left(\frac{R}{r}\right)^{n+1} P_n(t) \end{aligned} \tag{23}$$

In the limit $H \to 0$, or equivalently $r \to R$, we obtain

$$f^{(2)}(0,t) = \frac{3R}{2\ell_0} - \frac{\delta(1-t)}{2} = -\sum_{n=0}^{\infty}(n-1)P_n(t) \tag{24}$$

and from the last sum of Eq. (23)

$$\begin{aligned} f^{(3)}(0,t) &= \frac{1}{R}\sum_{n=0}^{\infty} n(n-1)P_n(t) \\ &= \frac{1}{2R}\sum_{n=0}^{\infty}\left[(2n+1)n - \frac{3}{2}(2n+1) + \frac{3}{2}\right]P_n(t) \\ &= \frac{1}{2R}\left(D(t) - \frac{3}{2}\delta(1-t) + \frac{3R}{2\ell_0}\right) \end{aligned} \tag{25}$$

where we have introduced the notations

$$\delta(1-t) = \sum_{n=0}^{\infty}(2n+1)P_n(t)$$

and

$$D(t) = \sum_{n=0}^{\infty}(2n+1)nP_n(t)$$

From Eq. (6), we obtain

$$\begin{aligned} \frac{R^3}{2\pi}\iint_\sigma \frac{H_P^3 - H^3}{\ell_0^3}\, d\sigma &= \sum_{n=0}^{\infty} nH_n^3(P) \\ &= \frac{1}{4\pi}\iint_\sigma \sum_{n=0}^{\infty}(2n+1)nP_n(t)H^3\, d\sigma \\ &= \frac{1}{4\pi}\iint_\sigma D(t)H^3\, d\sigma \end{aligned} \tag{26}$$

and we also have

$$\frac{1}{4\pi}\iint_\sigma \delta(1-t)F(Q)\, d\sigma_Q = F(P)$$

for any continuous function F and $t = \cos\psi_{PQ}$, which defines the properties of $D(t)$ and $\delta(1-t)$. By inserting Eqs. (24) and (25) into Eq. (21), we finally arrive at

$$N_{\mathrm{I}}(P) = \frac{2\pi\mu}{\gamma}H_P^2 + \frac{3R\mu}{4\gamma}\iint_\sigma \frac{H^2 - H_P^2}{\ell_0}\,\mathrm{d}\sigma - \frac{R^2\mu}{6\gamma}\iint_\sigma \frac{H^3 - H_P^3}{\ell_0^3}\left(1 - \frac{3}{4}\frac{\ell_0^2}{R^2}\right)\mathrm{d}\sigma + O(H^4)$$

i.e. the third-order Taylor expansion of the strict formula of Eq. (17) equals our previous formula Eq. (9).

5 Numerical investigations

In order to numerically investigate the indirect effect, we have applied the classical and new integrals, Eqs. (1) and (7) respectively. We have also applied the classical formula corrected with the harmonic expansion of Eq. (14) as well as the strict integral of Eq. (17) with $f(H,t)$ given by Eq. (19a). The results of the spherical harmonic approach are limited to the third power of elevation H. The harmonic coefficients of heights H_n^2 and H_n^3 are determined from Eq. (3). For this, a $30 \times 30'$ Digital Terrain Model (DTM) is generated using the Geophysical Exploration Technology (GETECH) $5 \times 5'$ DTM (GETECH 1995a). This $30 \times 30'$ DTM was averaged using area weighting. Since the interest is in continental elevation coefficients, the heights below sea level are all set to zero. The spherical harmonic coefficients are computed to degree and order 360. The parameter $\mu = G\rho$ is computed using $G = 6.673 \times 10^{-11}$ m^3 kg^{-1} s^{-2} and $\rho = 2670$ kgm^{-3}. $R = 6371$ km and $\gamma = 981$ Gal are also used in the computations. Strictly, Bruns' formula requires γ to be computed for geodetic latitude ϕ. However, since N_{I} reaches at most 2 m, γ is set to a constant. This approximation is of the order of the flattening of the ellipsoid (−0.003), which is at most 6 mm.

Two test areas of size $2 \times 2°$ are chosen. Both of these test areas are in Sweden. The area A is limited by latitudes 57 and 59°N and longitudes 13 and 15°E, located in the south of Sweden. The topography in this area varies from 40 to 340 m. The area B is limited by latitudes 63 and 65°N and longitudes 13 and 15°E, located in north-west Sweden. The topography in this area is more rugged than in test region A and varies from 276 to 1051 m.

First, the strict formula is computed with a global integration. The integration area is extended outside the test areas and limited to cap size (ψ) of 20°. A $2.5 \times 2.5'$ DTM (GETECH 1995b) is used inside the cap size of 20°. Further out, we have applied a $30 \times 30'$ DTM. This result is then used as the 'true' solution to compare with all other methods. Figure 1 shows the indirect effect computed by the strict formula [Eq. (17)] for area B. It ranges from −1.53 to 7.43 cm.

For the classical and new integral approaches, a $2.5 \times 2.5'$ DTM (GETECH 1995b) is also used. The integration area is again extended outside the test areas, but limited to cap sizes of 3 and 12°. Figures 2–4 show the indirect effect computed by the classical [Eq. (1)] and new integral [Eq. (7)], as well as the classical formula corrected with the harmonic expansion [Eq. (14)] methods with 12° integration area in test region B.

Figures 2 and 3 are similar in shape with minor differences. The plots range from −4.30 to 1.03 cm (Fig. 2) and −2.99 to 1.79 cm (Fig. 3). Both of them include the local contributions with pure integral formulas, but they

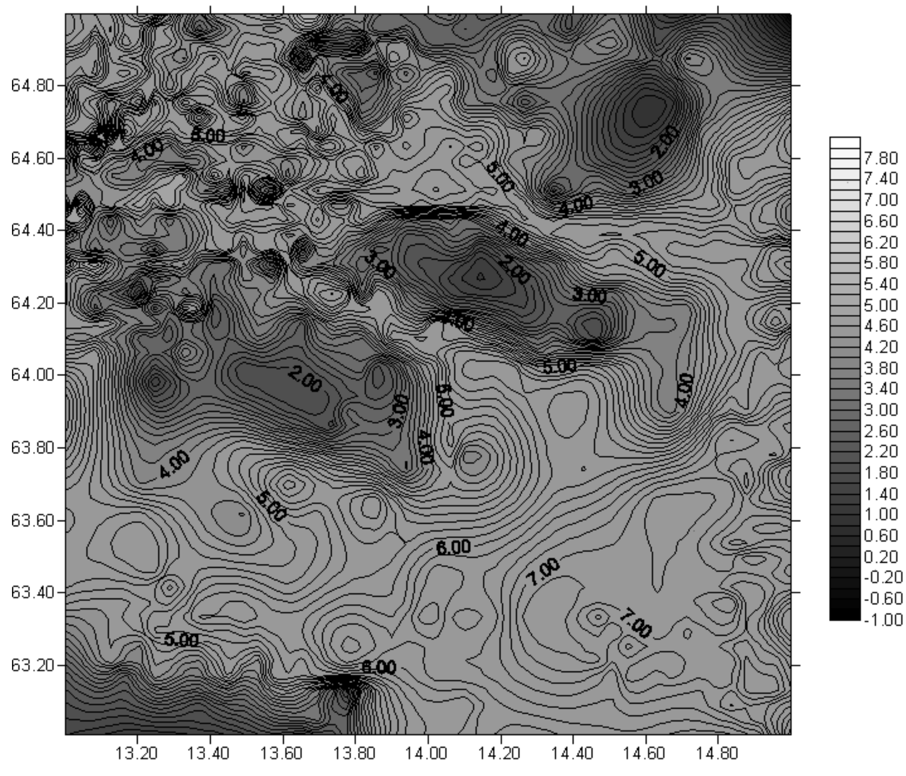

Fig. 1. The indirect effect computed by strict method [Eq. (17)] with a global integration in test region B. Contour interval is 0.2 cm

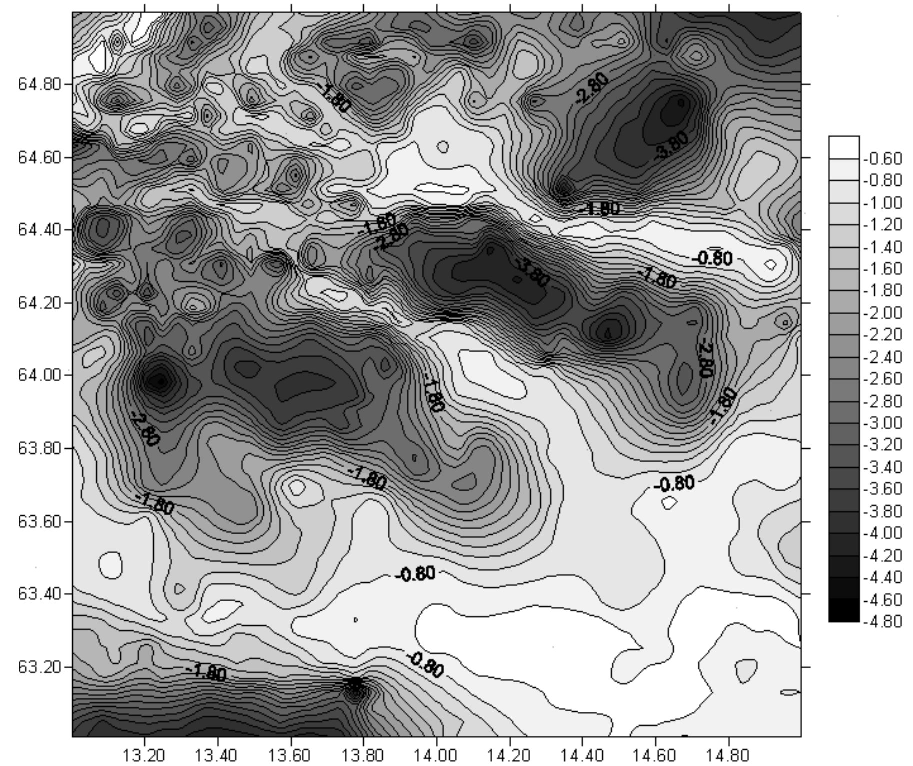

Fig. 2. The indirect effect computed by classical method [Eq. (1)] with 12° integration area in test region B. Contour interval is 0.2 cm

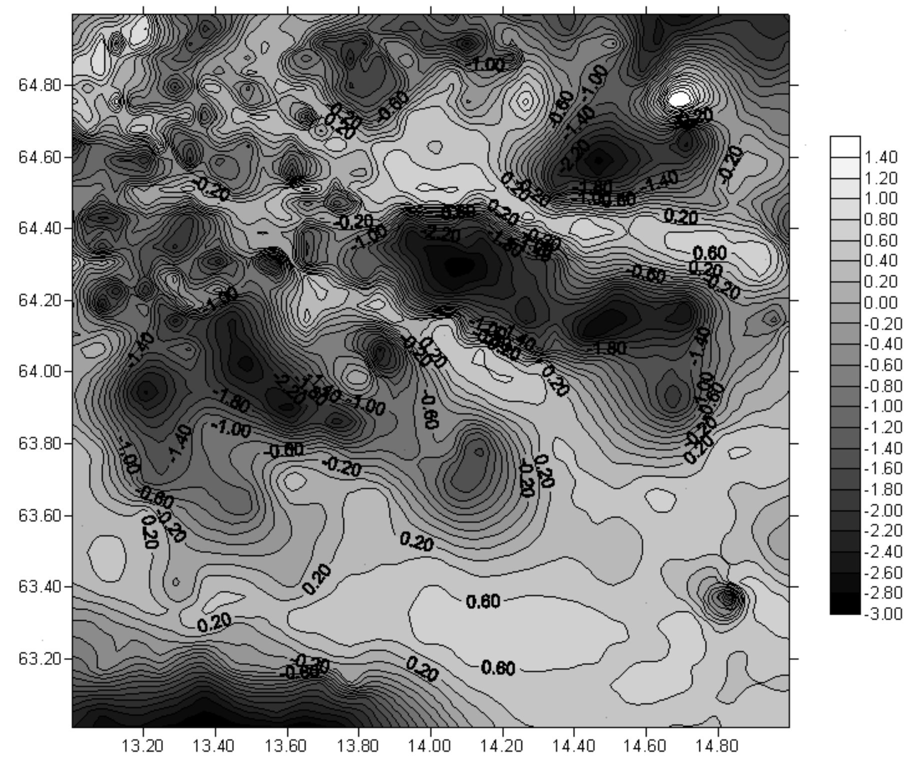

Fig. 3. The indirect effect computed by new integral method [Eq. (7)] with 12° integration area in test region B. Contour interval is 0.2 cm

are lacking the far-zone contribution outside the cap of radius 12°.

Figures 1 ('true' solution) and 4 (corrected classical solution) are similar in shape and only slightly different in magnitude. In both these formulas, the local information and long-wavelength contributions are present. Finally, Fig. 5 depicts the smooth long-wavelength spherical harmonic representation of the indirect effect [Eq. (2)], ranging from 4.23 to 6.96 cm. This suggests that the classical formula corrected with the harmonic expansion [Eq. (14)] is the best alternative to the strict formula in this study.

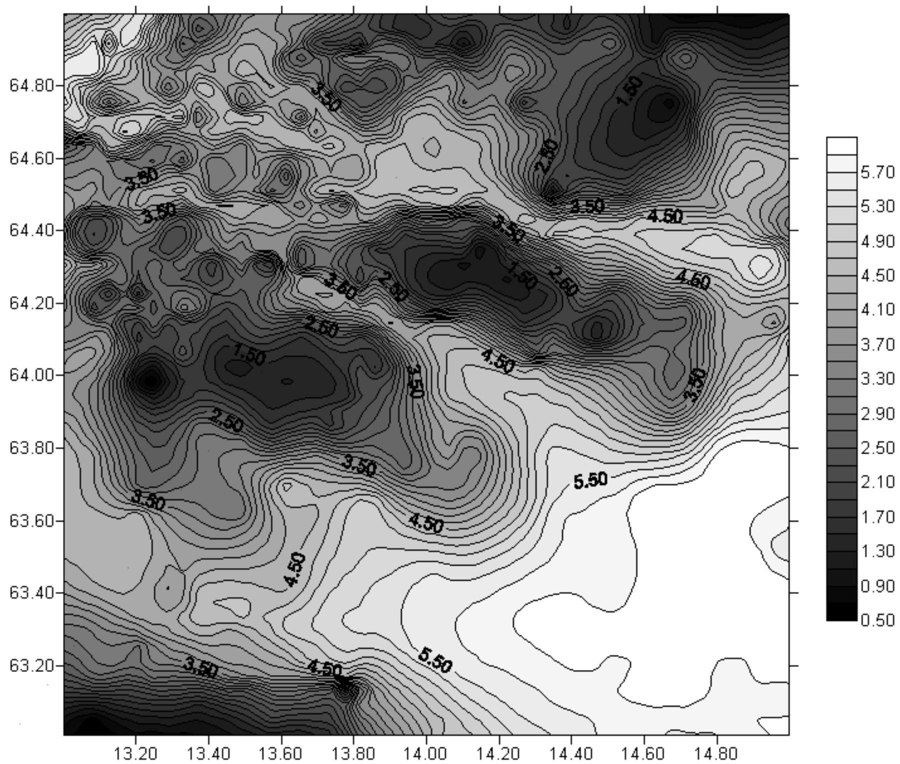

Fig. 4. The indirect effect computed by classical formula corrected with the harmonic expansion [Eq. (14)] with 12° cap size in test region B. Contour interval is 0.2 cm

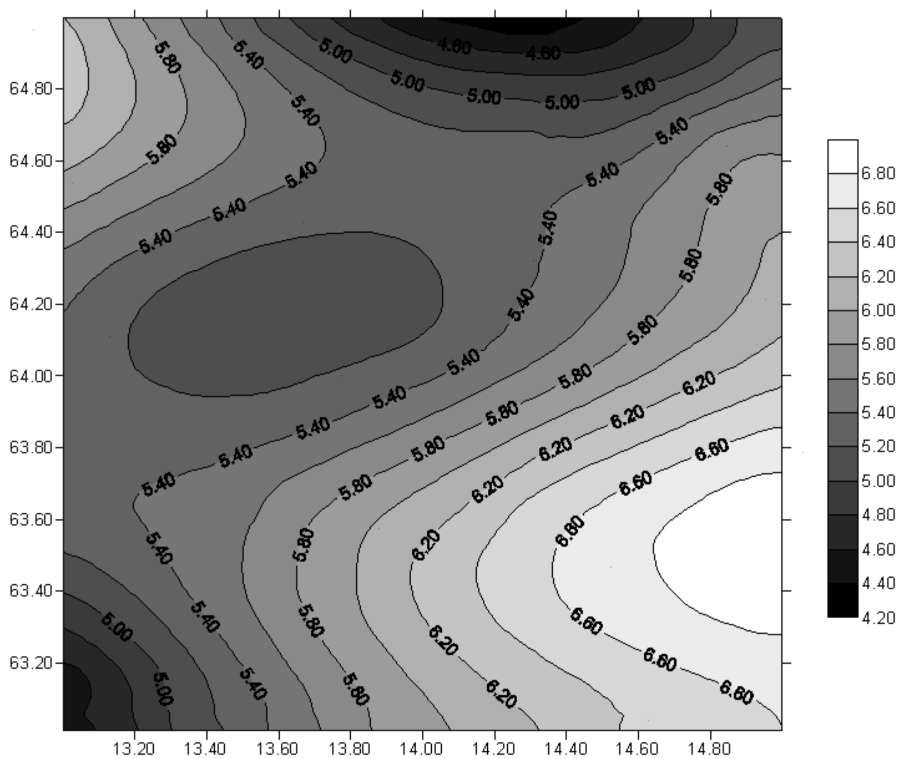

Fig. 5. The indirect effect computed by the spherical harmonic method [Eq. (2)] in test region B. Contour interval is 0.2 cm

In order to obtain further insight into how the methods differ, the classical, new integral and harmonic expansion approaches and the classical formula, corrected with the harmonic expansion, are subtracted from the strict method. These differences are presented for the two test areas using integration areas of 3 and 12° in Tables 1 and 2, respectively. These tables show errors of the classical formula, Eq. (1), as well as the new integral formula Eq. (7). We note also that the errors even increase with cap size (see Tables 1 and 2), revealing that an integration area of 12° is too small to account for the long-wavelength signal. Also, the pure harmonic ex-

Table 1. Error of approximate formulas for indirect effect in test regions A and B with 3° cap size

	Test region A				Test region B			
	Eq. (1)	Eq. (7)	Eq. (14)	Eq. (2)	Eq. (1)	Eq. (7)	Eq. (14)	Eq. (2)
Max	−1.71	1.63	−0.25	2.31	−2.69	2.25	−0.95	4.30
Min	−2.52	−2.38	−1.01	−2.12	−4.61	−3.85	−1.81	−4.03
Ave	−2.25	−1.23	−0.55	0.11	−3.60	−2.14	−1.01	0.99
SD	0.48	0.63	0.20	0.85	0.54	0.78	0.30	1.02

Units in cm
Eq. (1) = classical formula; Eq. (7) = new formula; Eq. (14) = corrected classical formula; Eq. (2) = harmonic expansion

Table 2. Error of approximate formulas for indirect effect in test regions A and B with 12° cap size

	Test region A				Test region B			
	Eq. (1)	Eq. (7)	Eq. (14)	Eq. (2)	Eq. (1)	Eq. (7)	Eq. (14)	Eq. (2)
Max	−1.91	1.85	−0.41	2.31	−5.05	1.54	−0.31	4.30
Min	−3.12	−2.94	−0.71	−2.12	−8.24	−9.75	−1.55	−4.03
Ave	−2.74	−1.35	−0.65	0.11	−6.38	−5.04	−0.84	0.99
SD	0.43	0.65	0.22	0.85	0.57	0.80	0.29	1.02

Units in cm
Eq. (1) = classical formula; Eq. (7) = new formula; Eq. (14) = corrected classical formula; Eq. (2) = harmonic expansion

pansion of the indirect effect [Eq. (2)] is not a good alternative, because it lacks local details.

In both test regions (A and B), the classical formula corrected with the harmonic expansion [Eq. (14)] agrees better with the strict formula than the other two methods, especially when the cap size is increased from 3 to 12°. Tables 1 and 2 show that the mean of differences between two methods decreases from 1.01 to 0.84 cm. This justifies our belief that the best alternative for the computation of the indirect effect is the use of Eqs. (14) and (12).

6 Conclusions

The indirect geoid effect is composed of both local effects and long-wavelength contributions. This implies that most formulas studied in this paper may have some numerical problems in representing these different signals. Our strict formula, Eq. (17), as well as its approximations, Eqs. (7) and (9), require that the integration area covers most of the globe to include the long wavelengths. However, a pure set of spherical harmonics, Eq. (2), truncated to, say, degree 360 will not contain the local details. For the test areas A and B in Sweden, the long-wavelength contribution from the harmonic expansion is of about the same significance as the short-wavelength signal of the local integrals. In the classical formula, Eq. (1), such long-wavelength information, ranging to −50 cm, is missing even for a global integration area. We conclude that our Eqs. (14) and (12) may be a suitable compromise between the local contribution [represented by the classical formula of Eq. (1)] and a set of spherical harmonics, Eq. (12), that is subtracted to account for the long-wavelength signal.

Finally, all the surface integrals above could be slightly generalized from a constant to a laterally variable topographic density (μ), simply by putting μ inside the integrals.

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