Project ACQ#SID

Effect of terrain on orthometric height

Marcelo Santos, Robert Tenzer, Petr Vaníček

University of New Brunswick, Department of Geodesy and Geomatics Engineering,
Fredericton, N.B., E3B 5A3, Canada

Objective:

Investigate the effect of terrain on orthometric height, within the context of the "rigorous" definition of orthometric height.The rigorous definition of orthometric height states that the mean value of the gravity along the (curved) plumbline between the Earth's surface and the geoid is defined in an integral sense (*Heiskanen and Moritz*, 1967). *Helmert* (1890) applied the Poincaré-Prey's vertical gradient of gravity to the observed gravity at Earth's surface in order to obtain the approximate mean value of gravity along the plumbline.. *Niethammer* (1932) and *Mader* (1954) took the effect of topography into account (see, e.g., *Heiskanen and Moritz*, 1967). *Vaníček et al.* (1995) introduced the correction to Helmert's orthometric height due to the lateral variation of topographical density. Similar numerical results were also shown by *Allister and Featherstone* (2001) and *Tenzer and Vaníček* (2002). *Véronneau* (2002) and *Hwang and Hsiao* (2003) studied the effect of topography on orthometric heights.

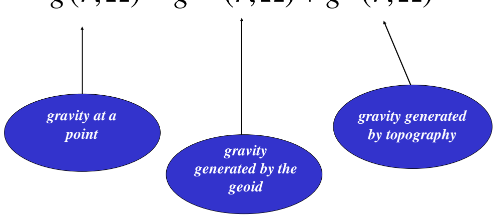
In this paper the analytical downward continuation of the observed gravity at the Earth's surface is used to evaluate the approximate value of the mean gravity along the plumbline. The relation between Poincaré-Prey's gravity gradient and the analytical downward continuation of gravity is formulated, and the corrections to Helmert's orthometric height arising from the relation are discussed. It is shown that the effect of terrain on orthometric heights can be as high as 17 cm in mountainous regions.

Definition

Length of plumbline between the geoid and the Earth's surface. For a numerical evaluation, knowledge of mean value of gravity along the plumbline required. Mean value of gravity along the plumbline is a function of mass density distribution of Earth and on shape of Earth's surface.

Decomposition of actual gravity

$$g(r,\Omega) = g^{NT}(r,\Omega) + g^{T}(r,\Omega)$$



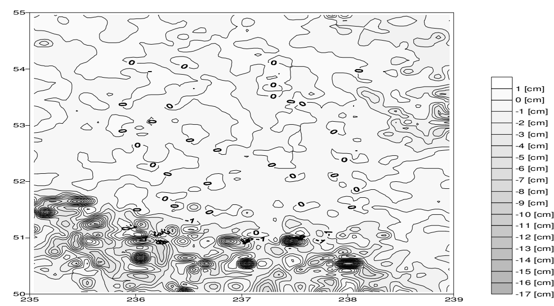
Mean gravity generated by topography : our approach

$$\bar{g}^{TC}(\Omega) = \bar{g}^{TC}(\rho_0;\Omega) + \bar{g}^{TC}(\delta\rho;\Omega)$$

The inclusion of terrain term in our approach

1. Mean gravity generated by topography expressed in terms of potential. Solution more accurate.
2. It is composed of a contribution coming from the mean mass density plus a correction due to density variations.
3. Dominant term represents the change in the roughness part of the Secondary Indirect Topographical Effects keeping a direct relationship with the topography of constant density of ρ_0, from the geoid to the surface of the Earth, divided by the height of the point of interest.
4. Numerical evaluation is similar to the one applied in the geoid computation, and is rather simple.

Correction due to the terrain effect



Mean value of gravity along plumbline: comparison with other authors

Helmert (1890)

$$\bar{g}^{H}(\Omega) = g(r_t(\Omega)) + \frac{\partial\gamma(H,\phi)}{\partial H}\frac{H^{O}(\Omega)}{2} - 2\pi G\rho_0 H^{O}(\Omega)$$

Niethammer (1932)

$$\bar{g}^{N}(\Omega) = \bar{g}^{H}(\Omega) - g^{TC}(r_t(\Omega)) + \bar{g}^{TC}(\Omega)$$

Mader (1950)

$$\bar{g}^{M}(\Omega) = \bar{g}^{H}(\Omega) - \frac{1}{2}(g^{TC}(r_t(\Omega)) - g^{TC}(r_g,\Omega))$$

Our approach

$$\bar{g}(\Omega) \cong \bar{g}^{H}(\Omega) - \delta g^{NT}(r_t(\Omega)) + \overline{\delta g}^{NT}(\Omega) - g^{TC}(r_t(\Omega)) + \bar{g}^{TC}(\Omega)$$

Concluding remarks

1. Comparison shows that terrain effect missing in Helmert's and that effect of gravity disturbance (and also term due to irregularities in density) also missing in Niethammer's and Mader's orthometric heights.
2. Numerical comparison to be carried out using synthetic gravity field.

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