

# **THE EFFECT OF DATA DENSITY ON THE ACCURACY OF FOOT-LINE DETERMINATION THROUGH MAXIMUM CURVATURE SURFACE BY AUTOMATIC RIDGE-TRACING ALGORITHM**

Ziqiang Ou and Petr Vaníček

Department of Geodesy and Geomatics Engineering  
University of New Brunswick  
P. O. Box 4400 Fredericton, N. B.  
CANADA E3B 5A3

## **ABSTRACT**

The influence of data density on the accuracy of foot-line determination by the automatic ridge-tracing algorithm is investigated through a set of simulated bathymetric surfaces. The results show that no obvious differences exist for different sea bottom morphology, different depths and gradients of continental slope when data are dense enough, i.e., when the data interval is smaller than some 12.5km. When the data are very sparse, the accuracy of the foot-line determination becomes worse as the surfaces become more complicated or the gradients become steep. Approximately, the root mean square error of foot-line determination is equal to one third of the data interval and the areal error is  $23\text{km}^2/100\text{km}$  foot-line length per one kilometre of the data interval.

## **INTRODUCTION**

The United Nations Conference on the Law of the Sea III (UNCLOS III) [United Nations, 1983, pp.27-28] Article 76 defines the outer limits of the continental shelf over which the coastal state can claim jurisdiction. In order to apply fully the provisions of Article 76, a coastal state needs to know the accurate locations of five features [Macnab, 1994]: (1) the 200 nautical mile limit measured from the state's territorial sea baselines; (2) the 350 nautical mile limit measured from the same baselines; (3) the continental slope foot-line; (4) the 2500-metre isobath; and (5) the Gardiner line. Among these five features, the most important one is the continental slope foot-line which is defined in the UNCLOS III Article 76, Section 4(b).

Our initial steps for the continental slope foot-line determination were taken in the early nineties and are described in [Vaníček et al. 1994a, 1994b, 1994c]. Based on the foot-line definition in Article 76, Section 4(b), an algorithm for automatic determination of the foot-line through maximum curvature surface has been developed by Ou and Vaníček [1995]. The algorithm follows the legal definition of the foot in the UNCLOS III as closely as possible; it constructs the maximum curvature surface from bathymetric data and traces the ridges on this new

surface.

The maximum curvature can be evaluated either at the locations of the available soundings, or on some selected, regular or irregular grid of points. The former option is used by us because it best honours the collected data. The latter option implies the use of some approximation or interpolation and artifacts caused by the used approximation or interpolation technique may be expected to occur. Unfortunately, the preferred option may not give the optimal results either, if the bathymetric data density and configuration are not appropriate for the sea bottom morphology in the location [Vaní...ek et al., 1994c].

Among other things, we have discussed that it may not always be the best strategy for the purpose of foot-line determination to collect indiscriminately as much bathymetric data as possible. On the one hand, collecting bathymetric data is very time-consuming and costs a lot of money. On the other hand, when the density of collected soundings is too high for the actual sea bottom morphology, small bottom features cause the values of maximum curvature to become too large. These large values then dominate the maximum curvature surface, masking the gentler ridge associated with the continental slope foot-line. Conversely, the collected soundings may be too sparse for a meaningful determination of the foot-line. There is no mathematical technique to help us decide on the appropriate density, yet it is very important to match the data density with the sea bottom morphology.

To investigate the effect of data density on the accuracy of foot-line determination from real bathymetric data is very complicated because of the complexity of sea bottom morphology and the uncertainty of true foot-line location. Therefore, we opted for using simulated bathymetric surfaces in this study, to get at least some preliminary idea about the effects of data density. For all these simulated bathymetric surfaces, the theoretically correct foot-lines were computed analytically. A regular grid with a varying interval was used to sample the surfaces. The maximum curvature surfaces were constructed with these sampled data and the ridges automatically traced by the algorithm we have developed [Ou and Vanicek, 1995]. Finally, the accuracy of so determined foot-line was evaluated by comparing the recovered foot-line with the theoretically correct one. Two norms  $N_1$  and  $N_2$  are used to measure the accuracy. The  $N_1$  is given by the area enclosed by the two lines whose closeness we are trying to measure (a geometric criterion). The  $N_2$  is the discrete quadratic norm or the mean square deviation (a statistical criterion).

For each grid interval, we sample the bathymetric surface repeatedly 30 times. Each time we rotate and shift the grid randomly. Thus, correspondingly, we obtain 30 accuracy estimates of the foot-line determination from 30 samples. The mean value and standard deviation of the accuracy estimates are then calculated from these 30 accuracy estimates. The mean value reflects the average accuracy of the foot-line determination by our algorithm for the corresponding data density. The standard deviation reflects the influence of the varying location of sampling points on the accuracy of foot-line determination, i.e., the uncertainty in this average accuracy.

In order to test the sensitivity of our algorithm to the sea bottom morphology, three different cases are considered in our simulation:

Case 1: different shapes of sea bottom are represented by 3 different surfaces, 8 different data densities are used for each surface and 30 different samples are taken for each data density.

Case 2: different depths of continental slope are represented by 3 different values, 8 different data densities are used for each depth and 30 different samples are taken for each data density.

Case 3: different gradients of continental slope are represented by 3 different values, 8 different data densities are used for each gradient and 30 different samples are taken for each data density.

Each case thus consists of 720 samples. They are discussed in detail in the following sections.

### DIFFERENT SHAPES OF SEA BOTTOM

To learn the effects of different sea bottom morphology on the foot-line determination, three different surfaces are generated. They are plotted in Figs. 1 to 3.

Figure 1: Simulated surface #1 (scales in kilometres)

Figure 2: Simulated surface #2 (scales in kilometres)

The scales of these simulated surfaces are analogous to the scales of the continental shelf features off the east coast of Canada. For each surface, the theoretical foot-line is computed analytically. Eight square grids with different grid intervals are used to simulate eight different data densities and for each data density 30 different samples are taken by randomly rotating and shifting the sampling grid over the surface. Two criteria are used to evaluate the accuracy: the root mean square error and the area enclosed between the recovered foot-line and the

theoretical foot-line.

Figure 3: Simulated surface #3 (scales  
in kilometres)

Figure 4: Averages of root mean  
square errors for different  
surfaces, calculated from 30  
samples for each data density.

The average of 30 root mean square errors of the foot-line determination for each data density are plotted in Fig. 4. It shows that the accuracy becomes worse as the surfaces become more complicated and the data become sparse. But no obvious differences exist among them when the data are dense enough, when for example, the data interval is smaller than 25km.

Figure 5: Averages of areal errors (unit:  $\text{km}^2/100\text{km}$  of foot-line length) for different surfaces calculated from 30 samples of each data density.

Figure 6: The average root mean square errors and their 95% confidence intervals for surface #2, calculated from 30 samples of each data density.

The average of 30 areal errors of the foot-line determination for each data density are plotted in Fig. 5. It shows that no significant difference exists among them either.

Figure 7: The average areal errors (unit:  $\text{km}^2/100\text{km}$  of foot-line length) and their 95% confidence intervals for surface #2 calculated from 30 samples of each data density.

Figs. 6 and 7 show the dispersions of results for the surface #2. The dispersions for surfaces #1 and #3 give similar results and we do not show them here. In Figs. 6 and 7, the solid line is the mean value and the dashed lines due to its 95% confidence interval (i.e., 1.96 times of standard deviation). Both are calculated from the 30 samples for each data density. While the mean value shows the average accuracy of the foot-line determination by our algorithm, the standard deviation reflects the uncertainty with which the accuracy has been determined.

From Figs. 6 and 7, we can see that the confidence interval becomes wider as the sampling data become sparser. This means that the sparser the sampling data, the more dispersive are the results and the bigger the influence of the location of sampling data on the accuracy of foot-line determination.

#### DIFFERENT DEPTHS OF CONTINENTAL SLOPE FOOT-LINE

In order to learn the possible influences of the different continental slope foot depths on the foot-line determination, three different depths are considered: 4, 6 and 8km. The profiles of the continental slope are plotted in Fig. 8. For this investigation, only the surface #1 (as shown in Fig. 1) is used. Please note that in Fig. 8 the slope gradients for different profiles are still suffered small changes.

The average of 30 root mean square errors and the average of 30 areal errors of the recovered foot-lines for the three depths and each data density are shown in Figs. 9 and 10 respectively. From these results, it can be concluded that our algorithm is insensitive to the depth of continental slope foot. We note that, since the confidence intervals for different depths are similar to those shown in Figures 6 and 7, the dispersions are not shown here to avoid repetition.

Figure 8: The profiles of different continental slope foot depths; scales in kilometres.

Figure 9: The average root mean square errors for different slope foot depths, calculated from 30 samples for each data density.

Figure 10: The average areal errors (unit:  $\text{km}^2/100\text{km}$  of foot-line length) for different continental slope foot depths, calculated from 30 samples for each data density.

## DIFFERENT GRADIENTS OF CONTINENTAL SLOPE

Again, only surface #1 (as shown in Fig. 1) is used in this study. Three different values of continental slope gradient are considered. The profiles of the continental slope are plotted in Fig. 11.

Figure 11: The profiles of continental slope for different gradients (scales in kilometres)

For each data density and slope gradient, we repeatedly sample the surface up to 30 times. Each time we randomly rotate and shift the sampling grid and totally get 30 samples for each case. The average of 30 root mean square errors and the average of 30 areal errors of the recovered foot-lines for each data density and each slope gradient are shown in Figs. 12 and 13 respectively. Dispersions are not plotted here.



Figure 12: The average root mean square errors for different slope gradients, calculated from 30 samples for each data density.

Figure 13: The average areal errors (unit:  $\text{km}^2/100\text{km}$  of foot-line length) for different slope gradients, calculated from 30 samples for each data density.

The Figs. 12 and 13 both show that the accuracy of the foot-line determination becomes worse as the gradient gets steeper and the data get sparser. There is no discernable difference between the results when data are dense enough, i.e., when the grid is smaller than 12.5km by 12.5km.

## SUMMARY AND CONCLUSIONS

Determining the optimal sampling interval for sounding data collected for the continental slope foot-line determination is an important problem. In general, a small sampling interval is sensitive to morphological noise and redundant details may distort the overall picture. On the other hand, a large sampling interval is robust to morphological noise, but it may be insensitive to some necessary details. How to balance them in the real situation is still a problem which needs to be investigated further. This study is only the first step approaching the solution and it can give us only a preliminary view of how the data density affects the accuracy of foot-line determination.

Summarizing the above described cases, we simulated different shapes of the sea bottom, different depths of the continental slope foot and different gradients of the continental slope. In total, we should have 270 samples for each data density. But since the surface #1 (as shown in Fig. 1) is used in all three cases, the data sampled from this surface are repeatedly used in three different cases. Thus, we have really generated only 210 independent samples for each data density. We can now calculate the mean values and standard deviations from all the independent samples and obtain the average accuracy of foot-line determination by our algorithm and its 95% confidence interval. These take into account the influence of different sea bottom morphology, different depths and gradients of continental slope. The results are plotted in Figs. 14 and 15 respectively.

The confidence interval in Figs. 14 and 15 becomes very wide when the data are very sparse. In other words, the accuracy of the foot-line determination vary widely with the shape of sea bottom, the gradient of slope and the location of data points when the data are very sparse. Even so, their mean values show a very stable trend. As a *rule of thumb*: the root mean square error of foot-line determination is approximately equal to one third of the data interval and the areal error is  $23\text{km}^2/100\text{km}$  of foot-line length per one kilometre data interval.

Figure 14: Summary of the root mean square error and its 95% confidence interval calculated from 210 samples, i.e., the samples for different surfaces, different slope depths and different slope gradients.

Figure 15: Area error (unit:  $\text{km}^2/100\text{km}$  of foot-line length) and its 95% confidence interval calculated from 210 samples, i.e., the samples for different surfaces, different slope depths and different slope gradients.

We should mention that the accuracy looks rather poor. The reason is in the fact that we honour the collected data and use only the given data locations to work with, i.e., no approximation and no interpolation are used in our algorithm. The accuracy would be easy to improve by introducing some approximation or interpolation techniques at the cost of introducing a certain degree of arbitrariness and some artifacts.

## ACKNOWLEDGMENTS

The first author would like to acknowledge that his stay at the University of New Brunswick has been supported by the Natural Sciences and Engineering Research Council of Canada International Fellowship Programme.

## REFERENCES

- Macnab R., 1994. Canada and article 76 of the law of the sea: defining the limits of Canadian resource jurisdiction beyond 200 nautical miles in the Atlantic and Arctic oceans. Geological Survey of Canada, Open File 3209.
- Ou, Z. and Vaní...ek, P., 1996. Automatic tracing of the foot of the continental slope. **Marine Geodesy**, 19(2): pp. 181-195.

United Nations, 1983. **UNCLOS III**, New York.

Vaní...ek, P., Wells, D.E. and Hou, T., 1994a. Determination of the foot of the continental slope. Contract Report for Geological Survey of Canada, Atlantic Geoscience Centre, Dartmouth, N.S.

Vaní...ek, P., Wells, D.E. and Hou, T., 1994b. Continental slope foot-line determination: Geometrical aspects. Proceedings of the LOS Article 76 Workshop, Ed. W. Wells, Fredericton, N.B., 14-15 April, Department of Geodesy and Geomatics Engineering, University of New Brunswick, pp.57-67.

Vaní...ek, P., Wells, D.E., Hou, T. and Ou, Z., 1994c. First experiences with continental slope foot-line determination from real bathymetric data. Proceedings of the International Symposium on Marine Positioning INSMAP 94, Hannover, Germany, pp. 385-397.