Congrès annuel de la Société canadienne de génie civil

Annual Conference of the Canadian Society for Civil Engineering

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Moncton, Nouveau-Brunswick, Canada
4-7 juin 2003 / *June 4-7, 2003*

ON THE APPLICATION OF ROBUSTNESS ANALYSIS TO GEODETIC NETWORKS

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ABSTRACT: Geodetic control networks established for engineering construction (e.g., highways, railways, bridges, dams) typically have coordinates estimated by the method of least-squares and the 'goodness' of the network is measured by a precision analysis based upon the covariance matrix of the estimated parameters. When such a network is designed, traditionally this again is based upon measures derived from the covariance matrix of the estimated parameters. This traditional approach is based upon propagation of random errors.

In addition to this precision analysis, reliability (the detection of outliers/gross errors/blunders among the observations) has been measured using a technique pioneered by the geodesist Baarda. In Baarda's method a statistical test (data-snooping) is used to detect outliers. What happens if one or more observations are burdened with an outlier? It is clear that these outliers will affect the observations and produce incorrect estimates of the parameters. If the outliers are detected by the statistical test then those observations are removed, the network re-adjusted, and we obtain the final results.

In the approach described here, traditional reliability analysis (Baarda's approach) has been augmented with geometrical strength analysis using strain in a technique called robustness analysis. Robustness analysis is a natural merger of reliability and strain and is defined as the ability to resist deformations induced by the smallest detectable outliers as determined from internal reliability analysis.

This paper addresses the consequences of when outliers are not detected by Baarda's test. This may happen for two reasons (i) the observation is not sufficiently checked by other independent observations and (ii) the test does not recognize the gross error. By how much can these undetected errors influence the network? If the influence of the undetected errors is small the network is called robust, if it is not it is called a weak network.

1. INTRODUCTION

The earliest known published description of strain analysis in English seems to be Terada and Miyabe (1929). According to Pope (1966), in a series of papers in the Bulletin of the Institute for Earthquake Research of the University of Tokyo, Terada, Miyabe, Tsuboi and others extended these techniques and applied them to various areas in Japan and Taiwan. The next scientist interested in strain analysis was

Kasahara. In Kasahara (1957), (1958a), (1958b) and (1964), the work of Terada, Miyabe and Tsuboi were referenced and the analysis of the earlier workers were extended in some respects. Later Burford (1965) followed Terada and Miyabe. In Burford (1965) the components of strain for an arc of triangulation in Southern California was computed. Independently, Frank (1965) derived methods for computation of strain components and pointed out their advantages and disadvantages. All the above scientists are from seismology, geology or geophysics (Pope, 1966). Pope (the known first geodesist dealing with strain analysis) used this technique for application to repeated geodetic surveys to determine crustal movements.

The first use of strain to analyse the strength of a geodetic network was at the University of New Brunswick. This was performed by Thapa (1980). In the mentioned study, the impact of incompatible observations in horizontal geodetic networks was investigated using strain analysis. Vaníček et al. (1981) elaborated on this approach. In Dare and Vaníček (1982a) a new method for strain analysis of horizontal geodetic networks based on the measurement of network deformation was presented. Dare (1982b) developed a method for the strength analysis of geodetic networks using strain and the effect of scale change, twist or shear was studied. In Craymer et al. (1987) a program package called NETAN for the interactive covariance, strain and strength analysis of networks was introduced. Vaníček et al. (1991) combined into one technique, called “robustness analysis”, the reliability technique introduced by Baarda and the geometrical strength analysis method. Vaníček and Ong (1992) investigated the datum independence problem in robustness analysis. In Krakiwsky et al. (1993) further developments of robustness analysis such as singularities in robustness, precision of robustness measures and interpretation of robustness measures were given. Szabo et al. (1993) described robustness analysis of horizontal geodetic networks. Craymer et al. (1993a) and (1993b) presented findings about robustness analysis. Robustness analysis of horizontal geodetic networks was also studied by Ong (1993) and Amouzgar (1994). Vaníček et al. (1996) developed a more economical algorithm for searching for the most influential observations in large networks, investigated alternative methods of defining the local neighborhood for which strain measures are computed for each point, and purposed a method of network classification that takes into account both precision (random errors) and accuracy (systematic biases) of point positions. Vaníček et al. (2001) summarized the findings about robustness analysis and gave an explicit proof for the robustness datum independence.

In this study, further thoughts about robustness analysis are expressed. In Vaníček et al. (2001) a complete and detailed description of the potential network deformation in terms of three independent measures representing robustness in scale, orientation and configuration are given (these are also called ‘robustness primitives’). However, to evaluate networks some acceptable threshold values are needed. These threshold values are going to enable us to talk about robustness of the network. For instance if a geodetic network is being established for an engineering structure, it must be robust and its robustness can be evaluated using threshold values. If robustness primitives within the network go beyond the threshold values, we must redesign the network by changing the configuration until we obtain a robust network.

2. RELIABILITY ANALYSIS

After geodetic networks for engineering construction (e.g., highways, railways, bridges, dams) control are physically established they are measured and point coordinates for the control points are estimated by the method of least-squares. What happens if one or more observations are burdened with an outlier (gross error/blunder)? It is clear that these outliers will affect the observations and produce incorrect estimates of the parameters. Therefore they must be detected and corrected. Generally in practice they are removed and the network is re-adjusted. To detect the outliers among the observations Baarda’s method of statistical testing (data-snooping) is used. What happens if outliers are not detected by Baarda’s test? This may happen for two reasons (i) the observation is not sufficiently checked by other independent observations and (ii) the test does not recognize the gross error. These situations were first investigated by Baarda (1968) (Vaníček et al. 2001).

Baarda's reliability theory is given in Baarda (1968). By using hypothesis testing, a statistical decision concerning postulated population parameters (mean μ and variance σ^2 etc.) is made. This is called the null hypothesis (H_0). For every null hypothesis there exists an infinite number of alternative hypothesis (H_1), each of which states that the population parameters have some other particular values. The probability α_0 of rejecting H_0 when in fact H_0 is true (Type I error) is called the significance level. The complementary probability (1-α_0) is called the confidence level. Likewise, a situation might arise in that H_0 is false but it is accepted. This is called (Type II error). The probability of making this decision is β_0. (1- β_0) is called the power of test (Vaníček et al. 1991).

By using Baarda's theory of reliability, Δl_i (the maximum value of an outlier in the ith observation which would not be detected by a statistical test with significance level α_0) can be estimated as fallows:

[1] $$\Delta l_i = \lambda_0(\alpha_0, \beta_0)\frac{\sigma_{l_i}}{\sqrt{r_i}}$$

where λ_0 is the value of the shift (non-centrality parameter) of the postulated distribution in the alternative hypothesis as a function of selected probabilities α_0 and β_0. σ_{l_i} is the a priori value of standard deviation of the ith observation. r_i is Baarda's redundancy number, which expresses the degree of influence on the estimated positions of the ith observation (Vaníček et al. 1991, Vaníček et al. 2001). Figure 1 illustrates the relation between α_0, β_0 and λ_0.

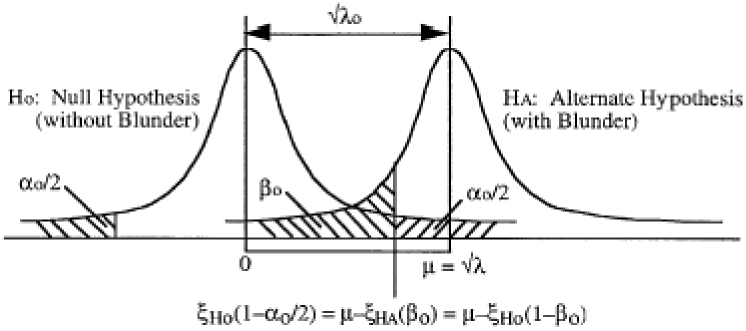

Figure 1. Relationship between α_0, β_0 and λ_0 (from Vaníček et al. 2001).

3. DESCRIPTION OF NETWORK DEFORMATION

To be able to measure the degree of robustness of a network, its degree of deformation has to be measured. Degree of deformation is described by means of displacements of individual points of the network. The estimates for displacements caused by outliers are given as follows (Vaníček and Krakiwsky, 1986).

[2] $$\Delta\hat{x} = (\mathbf{A}^T\mathbf{C}_l^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{C}_l^{-1}\Delta l$$

where **A** is the design matrix, C_l^{-1} is the covariance matrix of the observations, Δl is the maximum undetectable error vector and $\Delta\hat{x}$ is the displacement vector.

The problem with displacements is that their estimates are datum dependent. That is, these estimates depend not only on the geometry of the network, and accuracy of the observations but also on the selection of constraints for the adjustment (the points which are fixed during the Least Squares Estimation process). However, deformation description must reflect only network geometry, type and accuracy of the observations. Therefore the strain technique must be used (Vaníček et al. 2001).

Let us denote a displacement of a point as follows

$$[3]\ \mathbf{\Delta x}_i = \begin{bmatrix} \Delta x_i \\ \Delta y_i \end{bmatrix} = \begin{bmatrix} u_i \\ v_i \end{bmatrix}$$

Then the deformation or gradient matrix for the points is given as

$$[4]\ \mathbf{E}_i = \begin{bmatrix} \frac{\partial u_i}{\partial x} & \frac{\partial u_i}{\partial y} \\ \frac{\partial v_i}{\partial x} & \frac{\partial v_i}{\partial y} \end{bmatrix}$$

$$[5]\ \mathbf{E} = \frac{1}{2}(\mathbf{E} + \mathbf{E}^T) + \frac{1}{2}(\mathbf{E} - \mathbf{E}^T)$$

$$[6]\ \mathbf{E} = \mathbf{S} + \mathbf{A}$$

The matrix **S** describes symmetrical differential deformation and the matrix **A** (it should not to be confused with design matrix already introduced) describes anti-symmetrical differential deformation at a point. These can be decomposed further as

$$[7]\ \mathbf{S} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \\ \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) & \frac{\partial v}{\partial y} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix}$$

$$[8]\ \mathbf{A} = \begin{bmatrix} 0 & \frac{1}{2}(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}) \\ \frac{1}{2}(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}) & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}$$

$\varepsilon_X, \varepsilon_{Y,} \varepsilon_{XY}$ are the strain components. ω describes a differential rotation at the point of interest.

As mentioned above, network deformation should not be depend on the choice of a datum. In Vaníček et al. (2001) it is shown that scale change has only a second order and thus negligible effect on the deformation matrix, while translations of the datum origin and rotations of the coordinate system have no effect at all.

4. COMPUTATION OF DEFORMATION MATRIX AND ROBUSTNESS PRIMITIVES

The computation of deformation matrix is given in detail in Vaníček et al. (2001). Therefore only the result formulae are given here.

$$[9]\quad E_i = \mathbf{T}_i (\mathbf{A}^T \mathbf{C}_l^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{C}_l^{-1} \Delta l$$

If Δl_i from Eq. 1 and $\mathbf{E}_i$ from Eq. 4 are substituted in Eq. 2, we obtain

$$[10]\quad \begin{bmatrix} \dfrac{\partial u_i}{\partial x} \\ \dfrac{\partial u_i}{\partial y} \\ \dfrac{\partial v_i}{\partial x} \\ \dfrac{\partial v_i}{\partial y} \end{bmatrix} = \mathbf{T}_i (\mathbf{A}^T \mathbf{C}_l^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{C}_l^{-1} \left(\lambda_0 (\alpha_0, \beta_0) \frac{\sigma_{l_i}}{\sqrt{r_i}} \right)$$

where $\mathbf{T}_i$ is a matrix based upon coordinates of points and connections: -its computation is given in Vaníček et al. (2001).

Thus the robustness primitives are obtained as follows (Vaníček et al. 1991; Vaníček et al. 2001).

$$[11]\quad \sigma = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right),\ \tau = \frac{1}{2}\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right),\ \nu = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),\ \omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

where σ is mean strain or dilation, τ is pure shear, ν is simple shear and ω describes a differential rotation at the point of interest.

5. COMPUTATIONS OF THRESHOLD VALUES FOR ROBUSTNESS PRIMITIVES

After calculating robustness primitives and initial conditions for the network (derivations are not given here), the displacements for each point can be computed as follows:

$$[12]\quad \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \sigma + \tau & \upsilon - \omega \\ \upsilon + \omega & \sigma - \tau \end{bmatrix} \begin{bmatrix} x_i - x_0 \\ y_i - y_0 \end{bmatrix}$$

If we examine the right side of the formula all components of the matrices are known. In this case if we assign reasonable values for displacements for the points, we can calculate a threshold value for each primitive by setting the others to zero. For example, if we let $\sigma=0$, $\omega=0$ and $\upsilon=0$ and assume the displacement in x (the direction u) is 10 cm, we can calculate the value for σ as follows:

[13] $\sigma = \frac{10}{x_i - x_0}$

If we apply the same approach for each primitive, computing a threshold value for each primitive can be computed.

6. CONCLUDING REMARKS

To be able to construct and monitor engineering structures (e.g., highways, railways, bridges, dam) geodetic networks must be established, measured and evaluated. To obtain reliable results the networks has to be evaluated for their ability to resist errors. For this purpose Baarda's statistical testing method (data-snooping) is used. To see the effect of when outliers are not detected by Baarda's test, robustness analysis is applied. The robustness of a network is given in terms of three independent measures representing robustness in scale, orientation and configuration (are also called robustness primitives); however, to evaluate networks some acceptable threshold values are needed. For this purpose the gradient matrix is defined using robustness primitives and initial conditions are formulated. By using these means, computing threshold values for robustness primitives seems realistic. Calculating threshold values would enable us to talk about robustness of networks. Moreover they should help to design the network. If robustness primitives within the design of network go beyond threshold values, we need to consider redesigning the network by changing the configuration until we obtain a robust network.

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