On Some Aspects of Accuracy of the Geopotential Model EMG96 at the Territory of Canada

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Abstract

The geopotential model testing methodology has been theoretically developed by Burke et al. [1996]. The Working Group Global Geodesy Topics participated in the accuracy estimation of geopotential models for which the Geopotential Model Evaluation and Monitoring Network was established. This testing network covers about 70 % of the earth surface [Burke et al., 1996; Burša et al., 1997b] including the territory of Canada. The accuracy of the geopotential model EGM96 was tested on 1248 GPS/leveling points over the territory of Canada [Burša et al., 1999a; Burša et al., 1999b]. The Helmert orthometric heights of the GPS/leveling points are defined in the North American Vertical Datum 1988 (NAVD88), while the geodetic (ellipsoidal) heights of these points are defined in the International Terrestrial Reference Frame 1997 (ITRF97).
The accuracy of the geopotential model EGM96 on the GPS/leveling points over the territory of Canada is investigated in this paper. For the estimation of accuracy of the EGM96, Helmert's orthometric heights obtained from more recent adjustment of the vertical datum Adjustment January 1998 [Adj. Jan. 1998, see Véronneau et al., 2001] are used.

Resumen

La metodología para probar modelos geopotenciales ha sido desarrollada teóricamente por Burke et al [1996]. El Grupo de Trabajo sobre Tópicos de Geodesia Global ha participado en la estimación de la exactitud de modelos geopotenciales, para lo cual ha establecido la Red de Evaluación y Monitoreo de Modelos geopotenciales. Esta re de prueba cubre cerca del 70% de la superficie de la Tierra [Burke et al., 1996; Burša et al., 1997b] incluyendo el territorio del Canadá. La exactitud del modelo geopotencial EGM96 fue probada en 1248 puntos GPS sobre bancos de nivel en el territorio del Canadá [Burša et al., 1999a; Burša et al., 1999b]. Las alturas ortométricas de Helmert de los puntos GPS sobre bancos de nivel están definidas en el Datum Vertical de Norteamérica de 1988 (NAVD88), mientras que las alturas geodésicas (elipsoidales) de estos puntos están definidas en la Marco de Referencia Terrestre Internacional de 1997 (ITRF97).
Este artículo muestra los resultados de la exactitud del modelo geopotencial EGM96 en los puntos GPS sobre bancos de nivel en el territorio canadiense. Para la estimación de la exactitud del EGM96, las alturas ortométricas de Helmert empleadas fueron obtenidas del ajuste del datum vertical mas reciente realizado en enero de 1998 [Adj. Jan. 1998, ver Véronneau et al., 2001].

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Introduction

The practical application of the testing methodology requires the geocentric spherical coordinates ϕ and λ ; $\Omega \in (\phi, \lambda)$, $\Omega \in \Omega_o$ $\left(\Omega_o \in \langle -\pi/2 \le \phi \le \pi/2; 0 \le \lambda \le 2\pi \rangle \right)$, the geodetic heights $h(\Omega)$ above the geocentric reference ellipsoid, and Molodensky's normal heights $H^N(\Omega)$ of the testing points. Helmert's orthometric heights $H^O(\Omega)$ are not suitable for testing over continents because of their hypothetical character within the earth crust. On the other hand, Molodensky's theory of normal heights does not assume any hypothesis about the topographical density distribution. Therefore, the orthometric heights must be transformed into the normal heights.
Comparing the height anomalies evaluated from a geopotential model and determined from GPS and leveling, the accuracy of geopotential model can be estimated [Burke et al., 1996].

Relation between Helmert's orthometric and normal height

Helmert's [1890] defined the orthometric height

$$\forall\, \Omega \in \Omega_o : \qquad H^O(\Omega) = \frac{C[r_t(\Omega)]}{\bar{g}(\Omega)}, \tag{1}$$

so that the mean gravity along the plumb line $\bar{g}(\Omega)$ between the geoid surface $\left(\forall\, \Omega \in \Omega_o : r_g(\Omega)\right)$ and the physical surface of the earth $\left(\forall\, \Omega \in \Omega_o : r_t(\Omega) \cong r_g(\Omega) + H^O(\Omega)\right)$ is evaluated applying the Poincaré-Prey's gravity gradient

$$\begin{aligned} \forall\, \Omega \in \Omega_o : \qquad \bar{g}(\Omega) &\cong g[r_t(\Omega)] - \left.\frac{\partial g(r,\Omega)}{\partial t}\right|_{r=r_t(\Omega)} \frac{H^O(\Omega)}{2} \\ &\approx g[r_t(\Omega)] - \left(\left.\frac{\partial \gamma(r,\phi)}{\partial n}\right|_{r=r_t(\Omega)} + 4\pi G \rho_o \right) \frac{H^O(\Omega)}{2} \\ &= g[r_t(\Omega)] - \left.\frac{\partial \gamma(r,\phi)}{\partial n}\right|_{r=r_t(\Omega)} \frac{H^O(\Omega)}{2} - 2\pi G \rho_o H^O(\Omega). \end{aligned} \tag{2}$$

In the above equations, $C[r_t(\Omega)]$ denotes the geopotential number, $\partial g(r,\Omega)/\partial t$ and $\partial \gamma(r,\phi)/\partial n$ stand for the actual and normal gravity gradient, G is Newton's gravitational constant, and the mean value of topographical density ρ_o is adopted instead of the actual topographical density $\rho(r,\Omega)$ distribution.
Molodensky's normal height $H^N(\Omega)$ is given by [Molodensky et al., 1960]

$$\forall\, \Omega \in \Omega_o : \qquad H^N(\Omega) = \frac{C[r_t(\Omega)]}{\bar{\gamma}(\Omega)}. \tag{3}$$

The mean value of normal gravity $\bar{\gamma}(\Omega)$ along the ellipsoidal normal between the surface of the reference ellipsoid and the telluroid in equation (3) is expressed by the following series expansion

$$\forall\, \Omega \in \Omega_o : \qquad \bar{\gamma}(\Omega) = \gamma\left[r_o(\phi) + H^N(\Omega)\right] - \sum_{k=1}^{\infty} \frac{1}{(k+1)!} \left.\frac{\partial \gamma(r,\phi)}{\partial n}\right|_{r=r_o(\phi)+H^N(\Omega)} \left[H^N(\Omega)\right]^k .$$

$$= \gamma\left[r_o(\phi) + H^N(\Omega)\right] - \left.\frac{\partial \gamma(r,\phi)}{\partial n}\right|_{r=r_o(\phi)+H^N(\Omega)} \frac{H^N(\Omega)}{2} - \ldots \ , \tag{4}$$

where $\gamma\left[r_o(\phi) + H^N(\Omega)\right]$ is the normal gravity at the telluroid $\left(\forall\, \Omega \in \Omega_o : r_o(\phi) + H^N(\Omega)\right)$.
Comparing equations (1) and (3), the difference between the normal and orthometric height, i.e., the geoid-quasigeoid correction $\delta H(\Omega)$, is equal to [Heiskanen and Moritz, 1967]

$$\forall\, \Omega \in \Omega_o : \qquad \delta H(\Omega) = H^N(\Omega) - H^O(\Omega) = H^O(\Omega)\frac{\bar{g}(\Omega) - \bar{\gamma}(\Omega)}{\bar{\gamma}(\Omega)} \, . \tag{5}$$

According to Martinec [1993], the term $\bar{g}(\Omega) - \bar{\gamma}(\Omega)$ on the right-hand side of equation (5) is approximately equal to the simple Bouguer gravity anomaly $\Delta g^{SB}\left[r_t(\Omega)\right]$, so that

$$\forall\, \Omega \in \Omega_o : \qquad \bar{g}(\Omega) - \bar{\gamma}(\Omega) \cong g\left[r_t(\Omega)\right] - \gamma\left[r_o(\phi) + H^N(\Omega)\right] - 2\pi G \rho_o H^O(\Omega) = \Delta g^{SB}\left[r_t(\Omega)\right]. \tag{6}$$

Assuming also $g\left[r_t(\Omega)\right] - \gamma\left[r_o(\phi) + H^N(\Omega)\right] \approx 0$, the geoid-quasigeoid correction $\delta H(\Omega)$ in equation (5) finally takes the following simple form

$$\forall\, \Omega \in \Omega_o : \qquad \delta H(\Omega) \approx -2\pi G \rho_o \frac{\left[H^O(\Omega)\right]^2}{\bar{\gamma}(\Omega)} \, . \tag{7}$$

Relation between height anomaly determined from geopotential model and GPS/leveling

The disturbing gravity potential $T(r,\Omega)$ reads [Pick et al., 1973]

$$\forall\, \Omega \in \Omega_o,\, r \in \Re^+ \left(\Re^+ \in \langle 0, +\infty)\right): \qquad T(r,\Omega) = W(r,\Omega) - U(r,\phi) + W_o - U_o \, , \tag{8}$$

where $W(r,\Omega)$ is the actual gravity potential, $U(r,\phi)$ is the normal gravity potential, W_o is the gravity potential at the geoid surface, and U_o defines the normal gravity potential at the surface of the geocentric reference ellipsoid. The disturbing gravity potential $T(r,\Omega)$ is harmonic above the earth surface, i.e., satisfies the Laplace equation $\forall\, \Omega \in \Omega_o,\, r > r_t(\Omega) : \Delta T(r,\Omega) = 0$.
The normal gravity potential $U(u,\beta)$ can be expressed in a form of the ellipsoidal coordinates u and β ,

$$\tan\beta = \frac{\sqrt{u^2(r,\phi) + a^2 e^2}}{u(r,\phi)} \tan\phi \, , \qquad u^2(r,\phi) = \frac{r^2 - a^2 e^2}{2}\left(1 + \sqrt{1 + \frac{4 a^2 e^2 r^2 \sin^2\phi}{\left(u^2(r,\phi) - a^2 e^2\right)^2}}\right), \tag{9}$$

as follows [Somigliana, 1929; see also Heiskanen and Moritz, 1967, eqn. 2-62]

$$\forall \beta \in \langle -\pi/2, \pi/2\rangle,\, u \in \Re^+ :$$
$$U(u,\beta) = \frac{GM}{ae} \arctan\frac{ae}{u(r,\phi)} + \frac{1}{3}\omega^2 a^2 \frac{q(u,\beta)}{q_0} P_{2,0}(\sin\beta) + \frac{1}{2}\omega^2\left(u^2(r,\phi) + a^2 e^2\right)\cos^2\beta \, , \tag{10}$$

where

$$q(u,\beta)=\frac{1}{2}\left[\left(1+3\frac{u^2(r,\phi)}{a^2e^2}\right)\arctan\frac{a\,e}{u(r,\phi)}-3\frac{u(r,\phi)}{a\,e}\right],\quad q_o=\frac{1}{2}\left[\left(1+3\frac{b^2}{a^2e^2}\right)\arctan\frac{a\,e}{b}-3\frac{b}{a\,e}\right]. \qquad (11)$$

The normal gravity potential U_o at the level rotation ellipsoid is given by (ibid, eqn. 2-61)

$$U_o=\frac{GM}{a\,e}\arctan\frac{a\,e}{b}+\frac{1}{3}\omega^2a^2. \qquad (12)$$

The parameter GM denotes the geocentric gravitational constant, ω is the mean angular velocity of the earth spin, and $e=\sqrt{a^2-b^2}/a$ stands for the first numerical eccentricity [Bomford, 1971] while a and b are the semi-axes of the ellipsoid.
The actual gravity potential $W(r,\Omega)$ in equation (8) is given by [Burša and Kostelecky, 1999]

$$\forall\,\Omega\in\Omega_O,\ r>r_t(\Omega):$$

$$W(r,\Omega)=\frac{GM}{r}\left[1+\sum_{n=2}^{\bar{n}}\left(\frac{a_o}{r}\right)^n\sum_{m=0}^{n}\left(C_{n,m}\cos m\lambda+S_{n,m}\sin m\lambda\right)P_{n,m}(\sin\phi)\right]+\frac{1}{2}\omega^2r^2\cos^2\phi, \qquad (13)$$

where $P_{n,m}(\sin\phi)$ are the Legendre associate functions (of degree n and order m) for the argument of sine of the geocentric spherical latitude ϕ; $C_{n,m}$ and $S_{n,m}$ are the geodynamic coefficients of the harmonic expansion of the earth gravity field, a_o is an arbitrary parameter of length (usually the semi-major axis of the geocentric reference ellipsoid), and $\bar{n}$ stands for the maximum degree of the retained harmonics.
The height anomaly $\varsigma(\Omega)$ is either defined as a difference between the geodetic height $h(\Omega)$ and the normal height $H^N(\Omega)$, i.e.,

$$\forall\,\Omega\in\Omega_O: \qquad \varsigma(\Omega)=h(\Omega)-H^N(\Omega)=h(\Omega)-H^O(\Omega)-\delta H(\Omega), \qquad (14)$$

or can be evaluated from the geopotential model by applying the Bruns formula [1878] to the disturbing gravity potential $T[r_t(\Omega)]$ which is stipulated at the earth surface, so that [Molodensky et al., 1960]

$$\forall\,\Omega\in\Omega_O: \qquad \varsigma(\Omega)=\frac{T[r_t(\Omega)]}{\gamma[r_o(\phi)+H^N(\Omega)]}. \qquad (15)$$

By analogy with equation (10), the normal gravity $\gamma(r,\phi)$ can be defined by [Heiskanen and Moritz, 1967, eqn. 2-64]

$$\forall\beta\in\langle-\pi/2,\pi/2\rangle,\ u\in\Re^+:$$

$$\gamma(u,\beta)=\left|\,\mathrm{grad}\,(u,\beta)\right|=\sqrt{\frac{u^2(r,\phi)+a^2e^2}{u^2(r,\phi)+a^2e^2\sin^2\beta}}\sqrt{\left(\frac{\partial U(u,\beta)}{\partial u}\right)^2+\frac{1}{u^2(r,\phi)+a^2e^2}\left(\frac{\partial U(u,\beta)}{\partial\beta}\right)^2}. \qquad (16)$$

The partial derivatives $\partial U(u,\beta)/\partial u$ and $\partial U(u,\beta)/\partial\beta$ in equation (16) read [ibid, eqn. 2-66]

$$\frac{\partial U(u,\beta)}{\partial u}=-\frac{GM}{u^2(r,\phi)+a^2e^2}-\frac{1}{3}\frac{\omega^2a^2a\,e}{u^2(r,\phi)+a^2e^2}\frac{q'(u,\beta)}{q_o}P_{2,0}(\sin\beta)+\omega^2u(r,\phi)\cos^2\beta, \qquad (17)$$

$$\frac{\partial U(u,\beta)}{\partial \beta} = a^2\omega^2\left(\frac{q(u,\beta)}{q_o} - \frac{u^2(r,\phi)}{a^2} - e^2\right)\sin\beta\cos\beta\,, \tag{18}$$

and the abbreviation $q'(u,\beta)$ is given by [ibid, eqn. 2-67]

$$q'(u,\beta) = -\frac{u^2(r,\phi)+a^2e^2}{ae}\,\frac{\partial q(u,\beta)}{\partial u} = 3\left(1+\frac{u^2(r,\phi)}{a^2e^2}\right)\left(1-\frac{u^2(r,\phi)}{ae}\arctan\frac{ae}{u(r,\phi)}\right)-1. \tag{19}$$

Numerical investigation

The accuracy of the geopotential model EGM96 has been tested on 1443 GPS/leveling points over the territory of Canada. Helmert's orthometric heights were used for the definition of the vertical datum NAVD88 as well as Adj. Jan. 1998. Therefore, Helmert's orthometric heights $H^O(\Omega)$ are first transformed to Molodensky's normal heights $H^N(\Omega)$ according to the approximate formula in equation (7) if the simple Bouguer gravity anomalies $\Delta g^{SB}[r_t(\Omega)]$ are not available (see equation 6). The approximate relation between the geoid-quasigeoid correction $\delta H(\Omega)$ and the height is shown in figure 1.

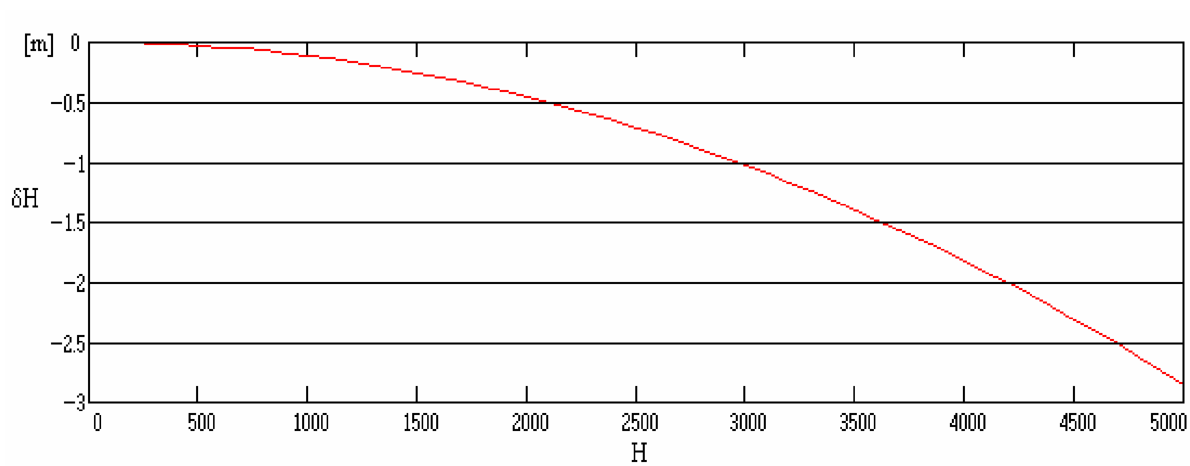

Figure 1. The geoid-quasigeoid correction [cm].

Depending on whether the direct, direct and indirect, or no zero-frequency tidal corrections are removed from the observations, the analysis of geopotential model is defined for the zero-tide, tide-free, or the mean tidal reference system [IAG SC3 Rep., 1995]. Since the complete tidal correction has been applied into the definition of the NAVD88 [Balazs and Young, 1982] and also Adj. Jan. 1998, the orthometric heights are assumed to be in the tide-free reference system [Burša et al., 1999a].

The residuals of height anomalies $\Delta\varsigma(\Omega)$ on the GPS/leveling points, where Helmert's orthometric heights are available in the vertical datum NAVD88 [Mainville, 1997], are computed according to the following equation

$$\Delta\varsigma(\Omega) = h(\Omega) - H^O(\Omega) - \delta H(\Omega) - \varsigma_{EGM96}(\Omega)\,. \tag{20}$$

The result is shown in figure 2. Subsequently, the residuals of height anomalies $\Delta\varsigma(\Omega)$ on the GPS/leveling points for Helmert's orthometric heights defined in the vertical datum Adj. Jan. 1998 are shown in figure 3.

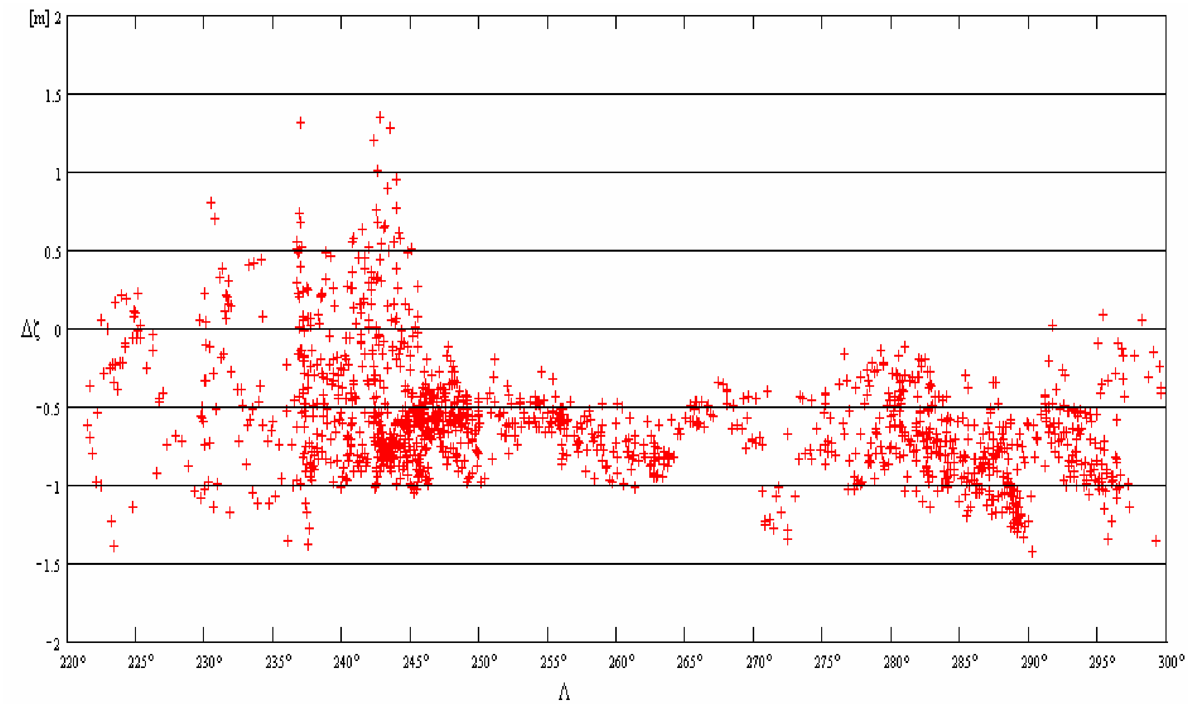

Figure 2. Differences between the height anomalies computed from EGM96 and GPS/leveling (NAVD88) for the parameters of the best fitting tide-free reference ellipsoid.

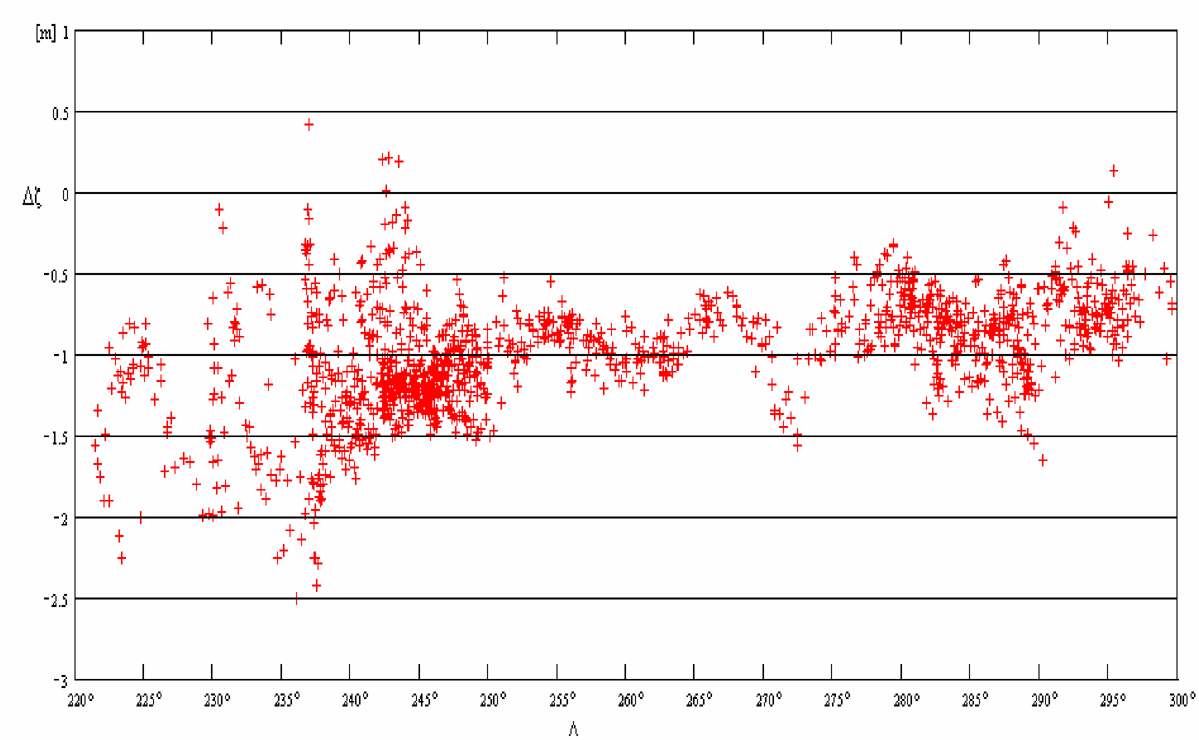

Figure 3. Differences between the height anomalies computed from EGM96 and GPS/leveling (Adj. Jan. 1998) for the parameters of the best fitting tide-free reference ellipsoid.

In both cases the parameters $a = 6378136.52$ m and $f^{-1} = 298.260310$ of the best fitting mean earth ellipsoid in the tide-free reference system [Burša et al., 1999a] are adopted for the evaluation of the normal gravity field. For the fundamental physical parameters, i.e., the geocentric gravitational constant GM, the mean angular velocity of the earth spin ω, the second zonal Stokes parameter $C_{2,0}$ in the tide-free system, and the gravity potential on the geoid W_o, the following values are adopted: $GM = (398600441.8 \pm 0.8) \times 10^{6}\ \mathrm{m^3\ s^{-2}}$ [Ries et al., 1992], $\omega = 7292115 \times 10^{-11}\ \mathrm{rad\ s^{-1}}$ [IAG SC3 Rep., 1995], $C_{2,0} = -(1082626.7 \pm 0.1) \times 10^{-9}$ [IAG SC3 Rep., 1995], and $W_o = 62636855.8 \pm 0.5\ \mathrm{m^2\ s^{-2}}$ [Burša et al., 1997a].

Furthermore, in this numerical investigation the residuals of height anomalies $\Delta\varsigma(\Omega)$, where the parameters of the reference ellipsoid GRS80 are used for the definition of the normal gravity field, are shown in figures 4 and 5. Since the normal gravity potential U_o on the level rotation ellipsoid is not equal to the actual gravity potential W_o on the geoid, i.e., $W_o \neq U_o$, the equation (8) is used for the computation of the disturbing gravity potential referred to the earth surface.

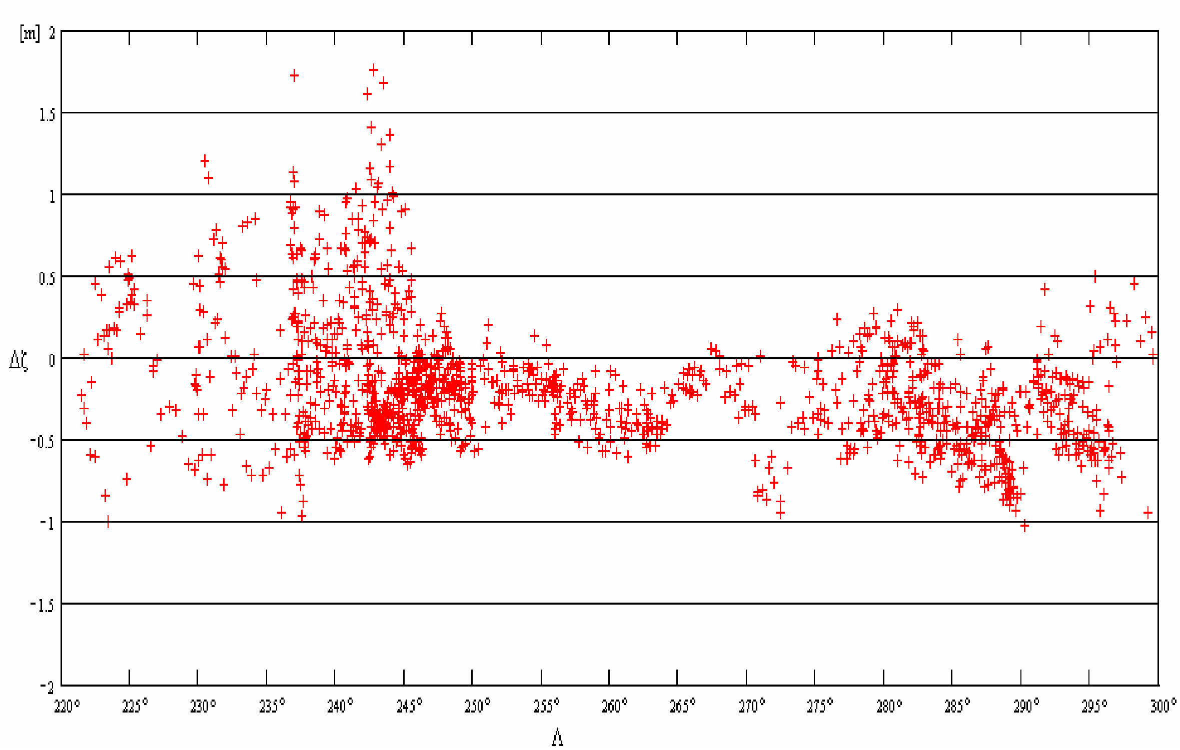

Figure 4. Differences between the height anomalies computed from EGM96 and GPS/leveling (NAVD88) for the parameters of the geocentric reference ellipsoid GRS80.

The mean anomalous height offsets $\overline{\Delta\varsigma}(\Omega)$ and the residuals $\sigma_{\Delta\varsigma}(\Omega)$ evaluated according to the following equations, where N is the number of point

$$\overline{\Delta\varsigma}(\Omega) = N^{-1} \sum_{n=1}^{N} \Delta\varsigma_n(\Omega), \qquad \sigma_{\Delta\varsigma}(\Omega) = \sqrt{(N-1)^{-1} \sum_{n=1}^{N} \left[\Delta\varsigma_n(\Omega) - \overline{\Delta\varsigma}(\Omega)\right]^2}\,, \tag{21}$$

are summarized in Table 1.

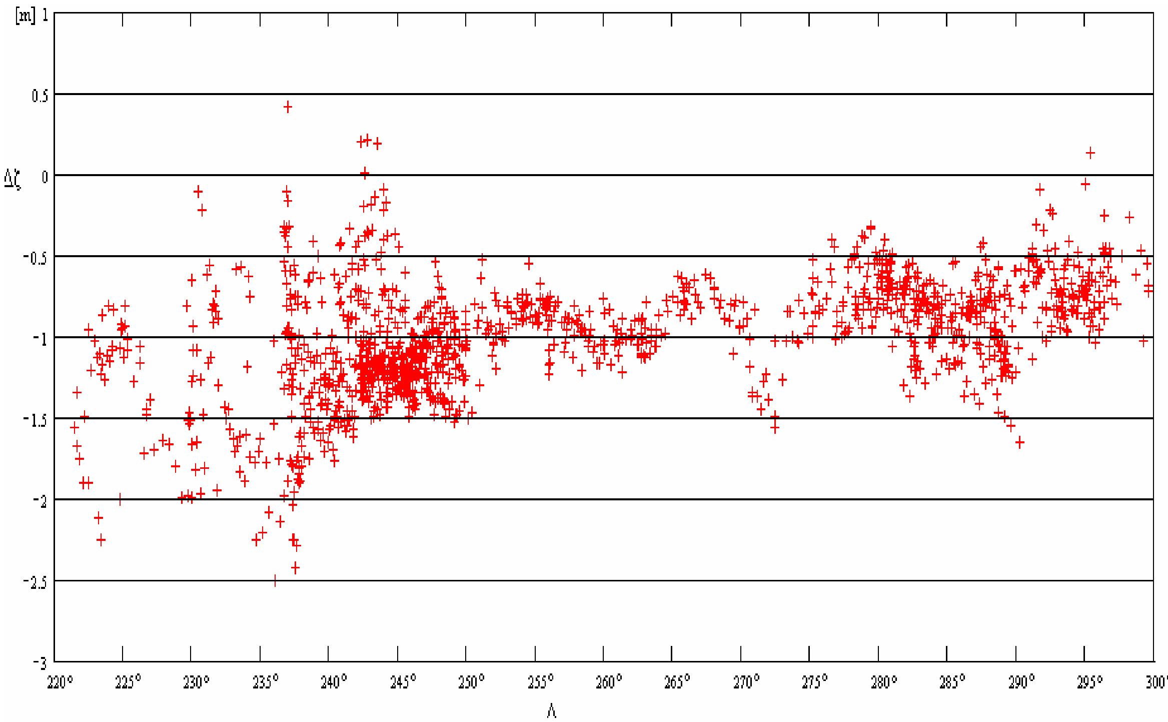

Figure 5. Differences between the height anomalies computed from EGM96 and GPS /leveling (Adj. Jan. 1998) for the parameters of the geocentric reference ellipsoid GRS80.

Vertical datum	Reference ellipsoid	$\overline{\Delta\varsigma}(\Omega)$	$\sigma_{\Delta\varsigma}(\Omega)$
NAVD88	Best fitting	–0.57 m	±0.38 m
	GRS80	–0.17 m	±0.38 m
Adj. Jan. 1998	Best fitting	–1.00 m	±0.35 m
	GRS80	–0.60 m	±0.35 m

Table 1. The mean anomalous height offsets $\overline{\Delta\varsigma}(\Omega)$ and the residuals $\sigma_{\Delta\varsigma}(\Omega)$ for the vertical datum NAVD88 and vertical datum Adj. Jan. 1998.

The dependence of the mean offsets $\overline{\Delta\varsigma}(\Omega)$ and residuals $\sigma_{\Delta\varsigma}(\Omega)$ due to the retained harmonics of the EGM96 is shown in Table 2.

Conclusions

As follows from the results of the numerical investigation summarized in Tables 1 and 2, the telluroid computed from the orthometric heights in the vertical datum NAVD88 approximates the EGM96 telluroid better than the telluroid computed based on the orthometric heights in the vertical datum Adj. Jan. 1998. This corresponds to the fact, that the gravity and height data of NAVD88 were used for the determination of the geodynamic coefficients $C_{n,m}$ and $S_{n,m}$ of the geopotential model EGM96. Since better accuracy of Helmert's orthometric heights adjusted in the vertical datum Adj. Jan. 1998 is assumed, the systematic errors of the geopotential model EGM96 over the territory of Canada may be expected. These errors have the long-frequency character from the east to west coast of Canada with amplitude about 1 meter (see Figures. 3 to 5). As also follows from Table 2, this systematic trend can be indicated for all the frequency spectrum of the EGM96, i.e., also for the low-frequency part which can be precisely determined using only the satellite data.

Degree of the EGM96	NAVD88		Adj. Jan. 1998.	
	$\overline{\Delta\varsigma}(\Omega)$	$\sigma_{\Delta\varsigma}(\Omega)$	$\overline{\Delta\varsigma}(\Omega)$	$\sigma_{\Delta\varsigma}(\Omega)$
n = m = 20	-0.89 m	±1.65 m	-1.32 m	±1.65 m
n = m = 30	-0.87 m	±1.35 m	-1.30 m	±1.23 m
n = m = 40	-0.80 m	±1.09 m	-1.24 m	±1.07 m
n = m = 50	-0.82 m	±0.94 m	-1.25 m	±0.93 m
n = m = 100	-0.72 m	±0.68 m	-1.16 m	±0.67 m
n = m = 150	-0.67 m	±0.53 m	-1.10 m	±0.51 m
n = m = 180	-0.66 m	±0.47 m	-1.09 m	±0.45 m
n = m = 200	-0.65 m	±0.45 m	-1.05 m	±0.42 m
n = m = 250	-0.60 m	±0.41 m	-1.03 m	±0.38 m
n = m = 300	-0.58 m	±0.40 m	-1.01 m	±0.36 m
n = m = 350	-0.57 m	±0.39 m	-1.01 m	±0.35 m
n = m = 360	-0.57 m	±0.38 m	-1.00 m	±0.35 m

Table 2. The dependence of the mean offsets $\overline{\Delta\varsigma}(\Omega)$ and the residuals $\sigma_{\Delta\varsigma}(\Omega)$ due to the retained harmonics of the EGM96.

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