Impact of Digital Elevation Models on Geoid Modelling

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Abstract

For the provinces of New Brunswick, Alberta and British Columbia, two Digital Elevation Models (DEM) are available. The first DEM, which is based on the Digital Terrain Elevation Data (DTED) standard, is derived from federal maps 1:250000, has a resolution of 3 arcsec and a vertical accuracy ranging from 25 m to 150 m. The second DEM is derived from provincial maps 1:10000, and has a resolution of 25 m to 100 m depending on the terrain roughness and a vertical accuracy of 5 m. From these two models, Geodetic Survey Division has derived two sets of mean elevations on 30 arcsec grid. For geoid modelling, it is important to use an accurate high-resolution DEM. In this contribution, we discuss the differences in terms of the so-called "Direct Topographical Effect " (DTE), which is one of the two most significant topographical effect that have to be evaluated when a precise geoid is being compiled. As the DTE is an effect on gravity, we compute first the differences of the affected gravity anomalies and then estimate the difference in the compiled geoid. The difference between the two DEM's is also depicted.

1. Introduction and formulation of the problem

The knowledge of the geoid is important in geodesy as the geoid is the most natural representation of the Earth shape. Moreover, geoid has become very important for the determination of orthometric heights from geodetic (ellipsoidal) heights obtained by GPS.

There are presently four research groups in Canada, whose goal is to produce as accurate as possible geoid for Canada. These are: the group within the Geodetic Survey Division in Ottawa, the group in the Department of Geodesy and Geomatics Engineering at the University of New Brunswick, the group in the Department of Geomatics at the University of Calgary and the group in the Department of Oceanography at Dalhousie University. Each group follows more or less a different scheme for the geoid computation as there are many different approaches how to solve this problem. Moreover there are two different sets of the elevation data in three Canadian provinces: New Brunswick, Alberta and British Columbia. In these three provinces, provincial governments have put out their own Digital Elevation Model (DEM), which had been compiled independently from the Canadian Digital Elevation Data (CDED) used by the federal government. There are more kinds of the input data necessary for the precise geoid computation. Digital elevation model is just one part of the input data, but important part, especially for the correct determination of the topographical influence. In this contribution we want to show and discuss the differences in the geoid model when different elevation data are used. The differences above 1 cm are significant.

2. Information about two available Digital Elevation Models

In Tab. 1, there are basic information about the CDED and about the provincial DEM in New Brunswick, Alberta and British Columbia. According this information, the vertical accuracy of the provincial DEM is much better. The variation of the vertical accuracy of the federal CDED is shown in Appendix 1. Of course, the worst accuracy is in the Rocky Mountains.

Tab. 1: Basic information about the CDED and the provincial DEM

	CDED	DEM_AB	DEM_BC	DEM_NB
derived from	1:250 000	1:20 000	1:20 000	1:10 000
resolution	3" (g)	25 – 100 m (p)	25 m (g)	25 – 70 m (p)
vertical accuracy	25 – 150 m	3 – 5 m (90%)	10 m (90%)	2.5 m (90%)

In Tab. 1, CDED means the Canadian Digital Elevation Data (Federal DEM), DEM_AB, BC, NB means the provincial Digital Elevation Model in Alberta, British Columbia and New Brunswick respectively. The symbol (g) means that the model was derived in the regular grid format, (p) means the model was derived in the point list format. The symbol (90%) indicates that the actual vertical accuracy should be valid for the 90 % of the points.
From the CDED as well as from the provincial DEM the mean heights in the regular grid 30 by 30 arc seconds and 5 by 5 arc minutes were derived. These files serve as the input files for the geoid computation. If we compare these files, we obtain maximum and minimum differences shown in Tab.2. The comparison was performed separately in New Brunswick and in the western provinces Alberta and British Columbia. This table indicates that the one of these models, maybe both, contains few blunders in the Rocky Mountains.

Tab. 2: Differences between the CDED and the provincial DEM

difference	min (m)	max (m)
CDED – DEM (30" × 30") in NB	-92	58
CDED – DEM (5' × 5') in NB	-21	27
CDED – DEM (30" × 30") in AB and BC	-671	1509
CDED – DEM (5' × 5') in AB and BC	-169	406

The differences (5' × 5') plotted as a map are shown in Appendix 2.

3. Solution to the problem and presentation of the results

The scheme in Fig.1 represents the simplified scheme of the geoid determination process according the Stokes-Helmert approach improved at UNB (Vaníček et al., 1999). According this scheme it can be seen how we solved the problem, how we transferred the height differences into the geoid differences.

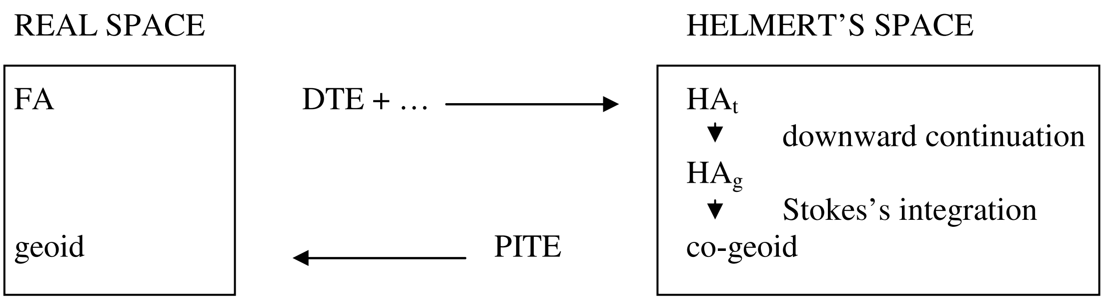

Fig. 1: The simplified Stokes-Helmert scheme for the geoid determination

We start with the free-air gravity anomaly (FA) in the real space. These values are transformed into the Helmert space by adding the Direct Topographical Effect (DTE) and some other little correction terms. What we obtain is the Helmert gravity anomaly on the earth topography (HA_t). These values are harmonic because in the Helmert space there are not masses above the geoid. All topographical masses are mathematically removed and substituted by the infinitely thin condensation layer located directly on the geoid (Vaníček and Martinec, 1994). So if the Helmert gravity anomaly is harmonic above the geoid, we can perform a downward continuation from the earth surface to the geoid using the Poisson's integral. The Helmert gravity anomaly on the geoid (HA_g) now serves as a boundary condition for the geodetic boundary value problem and of course as an input to the Stokes's integration. After Stokes's integration we obtain geoid in the Helmert space or co-geoid. In order to transform the co-geoid back into the real space, the Primary Indirect Topographical Effect (PITE) has to be added. These two terms, DTE and PITE, are the most significant values affected by the digital elevation model. DTE and PITE are defined by the following formulas (Martinec and Vaníček, 1994a), (Martinec and Vaníček, 1994b)

$$DTE = \frac{\partial \delta V}{\partial r}, \qquad (1)$$

$$PITE = \frac{\delta V}{\gamma}, \qquad (2)$$

$$\delta V = V^t - V^{ct}. \qquad (3)$$

In these equations, $\partial/\partial r$ is the radial derivative, γ is the normal gravity, δV is the residual potential defined by equation (3), V^t is the gravitational potential of the topographical masses and V^{ct} is the gravitational potential of the condensation layer.

We computed both the DTE and the PITE using the CDED as well as using the provincial DEM. Then the DTE had to be integrated by the Stokes's integration in order to obtain its influence on the geoid. Finally the total influence of the topography on the geoid was obtained as the DTE after Stokes's integration plus the PITE. The results are shown in Tab. 3 and in Appendix 3.
In New Brunswick area there are not significant differences between geoid models when different elevation data are used. In the Rocky Mountains there is a different situation as it can be seen in Tab.3 and in Appendix 3. The differences here are significant, systematically negative and have mostly the long-wavelength character. So the geoid computed using the CDED is lower than the geoid computed using the provincial DEM. In the vicinity of Mt. Fairweather (4660 m) there is probably something wrong in one of the elevation data set.

Tab. 3: Minimum and maximum values in computed files

file	area	min.	max.	units
DTE	NB	-5.595	2.394	mGal
DTE_DIF	NB	-0.304	0.176	mGal
DTE_S	NB	-0.015	0.017	m
DTE_S_DIF	NB	-0.002	0.001	m
PITE	NB	-0.098	0.046	m
PITE_DIF	NB	-0.002	0.002	m
TOTAL	NB	-0.113	0.058	m
TOTAL_DIF	NB	-0.004	0.002	m
DTE	AB, BC	-40.591	44.949	mGal
DTE_DIF	AB, BC	-19.725	7.825	mGal
DTE_S	AB, BC	-0.014	1.433	m
DTE_S_DIF	AB, BC	-0.118	0.015	m
PITE	AB, BC	-0.889	0.434	m
PITE_DIF	AB, BC	-0.148	0.107	m
TOTAL	AB, BC	-0.274	1.487	m
TOTAL_DIF	AB, BC	-0.162	0.088	m

In Tab. 3, DTE means the Direct Topographical Effect on gravity (computed using CDED), DTE_DIF is a difference between the two DTE files (using CDED – using provincial DEM), DTE_S is the DTE after Stokes's integration, DTE_S_DIF is a difference between the two DTE_S files, PITE is the Primary Indirect Topographical Effect on geoid, PITE_DIF is a difference between the two PITE files, TOTAL is the total topographical influence on geoid, TOTAL_DIF is a difference between the two TOTAL files.

4. Conclusions

- The differences between the geoid models computed using the CDED and provincial DEM are significant in the Canadian Rocky Mountains. The differences are on the decimeter level so they are significant from the "one-centimeter geoid" point of view;
- They are systematically negative, and the geoid model computed using the CDED data is lower than one computed from provincial DEM data;
- The differences between the two geoid models have mostly long wavelength character;
- There appears to be something seriously wrong with either the provincial DEM or the CDED data in the vicinity of Mt. Fairweather in British Columbia.

Acknowledgement

The authors would like to thank Pavel Novák, whose software was used for the computation.

Reference

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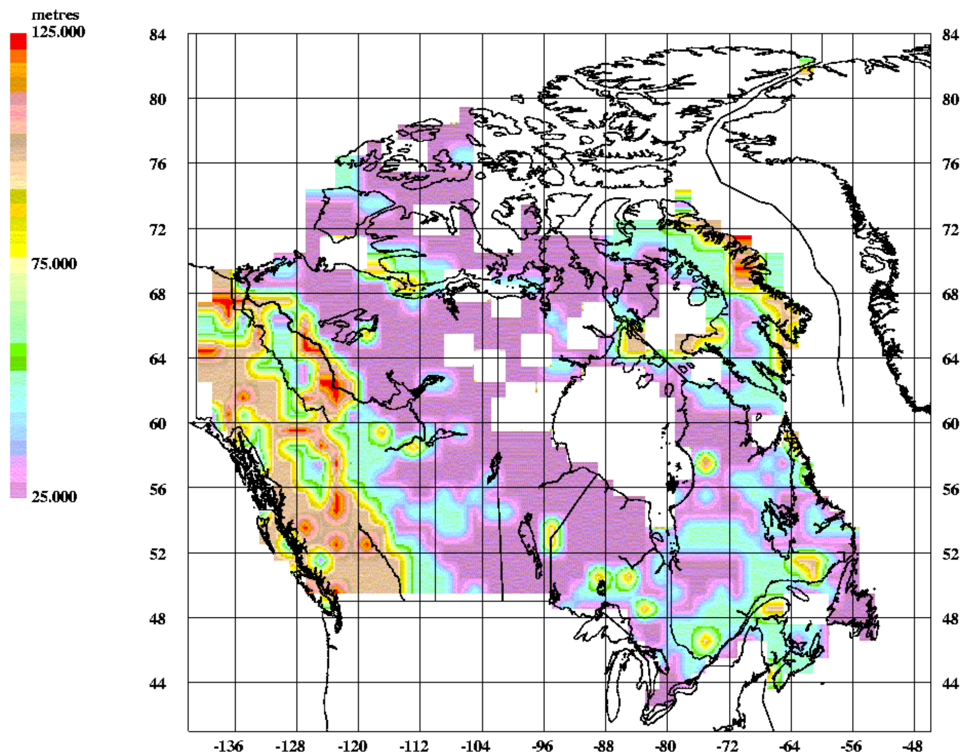
APPENDIX 1

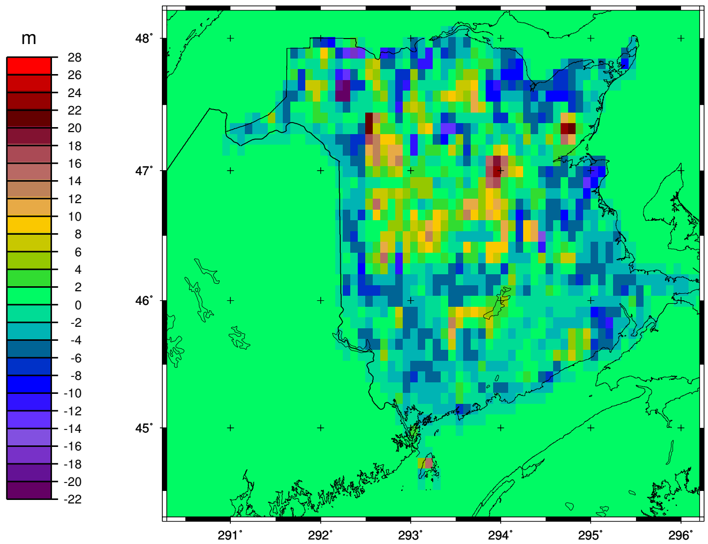


Accuracy of CDED

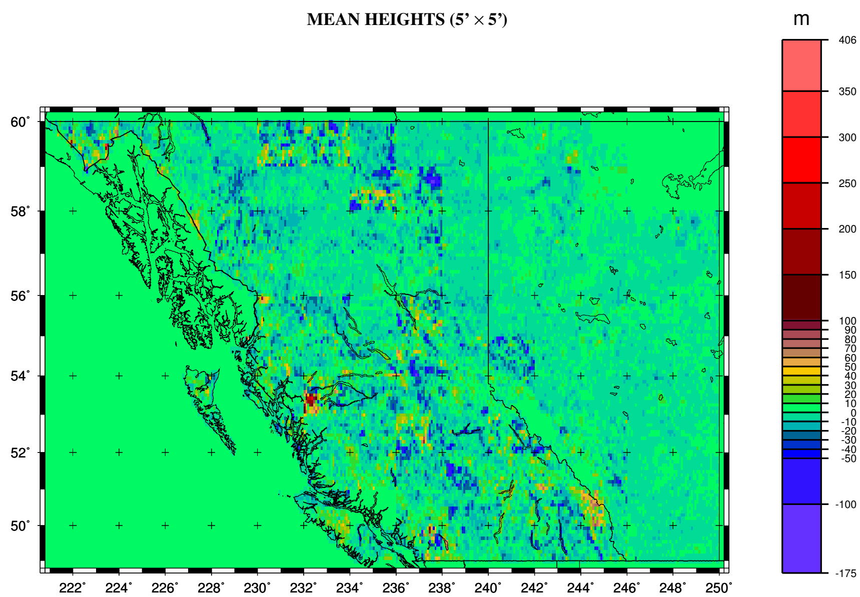
APPENDIX 2

DIFFERENCES BETWEEN DEM's (FEDERAL - PROVINCIAL) IN NEW BRUNSWICK

MEAN HEIGHTS (5' × 5')

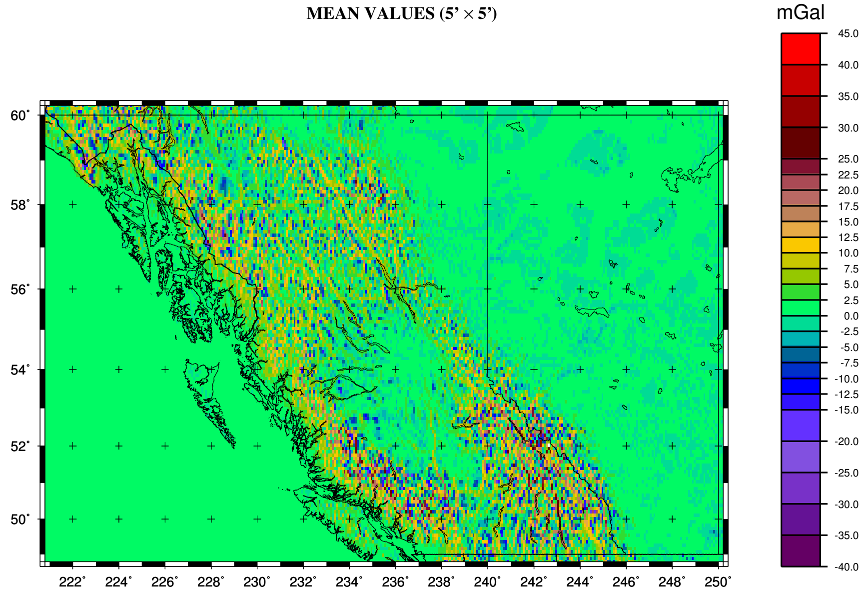


DIFFERENCES BETWEEN DEM's (FEDERAL - PROVINCIAL)
IN ALBERTA AND BRITISH COLUMBIA
MEAN HEIGHTS (5' × 5')

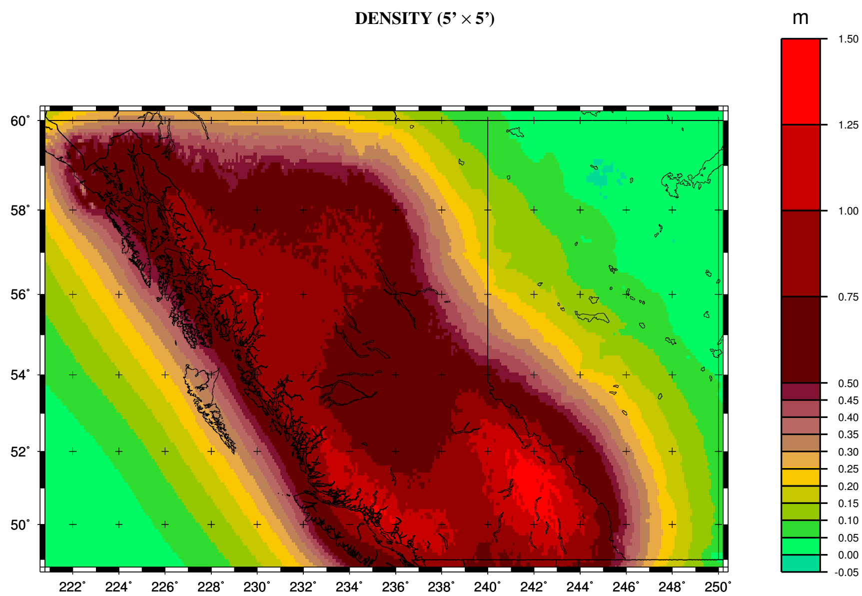


APPENDIX 3

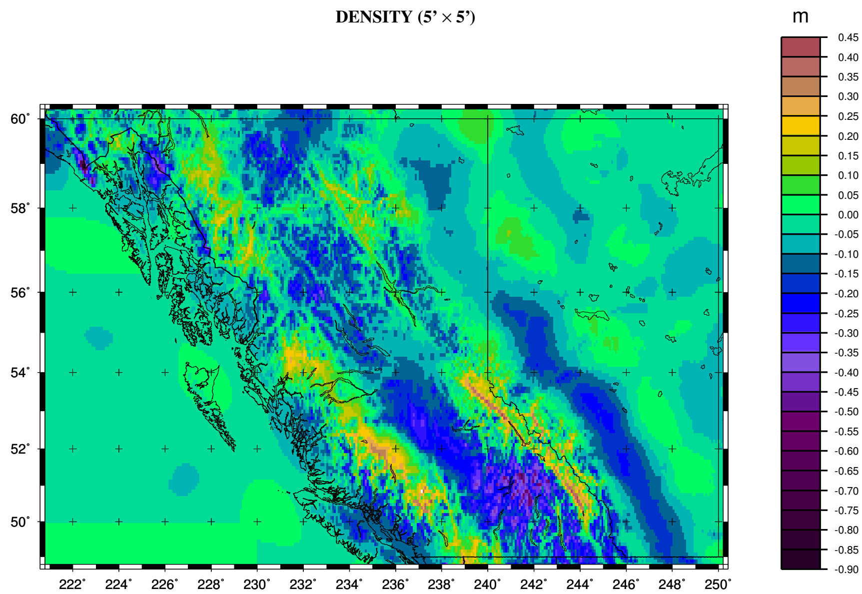
DIRECT TOPOGRAPHICAL EFFECT
IN ALBERTA AND BRITISH COLUMBIA
MEAN VALUES (5' × 5')



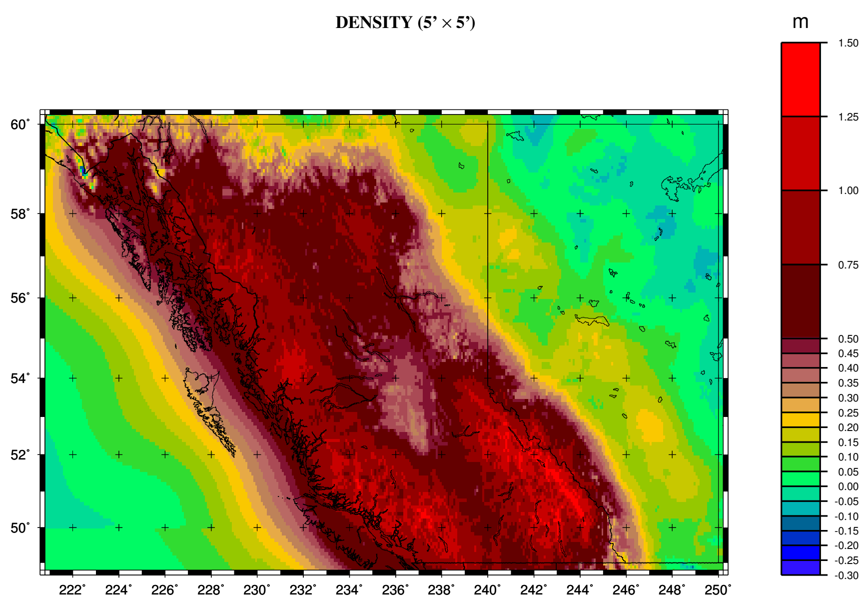
INFLUENCE OF THE DIRECT TOPOGRAPHICAL EFFECT ON THE GEOID
IN ALBERTA AND BRITISH COLUMBIA
DENSITY (5' × 5')



PRIMARY INDIRECT TOPOGRAPHICAL EFFECT
IN ALBERTA AND BRITISH COLUMBIA
DENSITY (5' × 5')



TOTAL TOPOGRAPHICAL EFFECT ON THE GEOID
IN ALBERTA AND BRITISH COLUMBIA
DENSITY (5' × 5')



DIFFERENCES BETWEEN TOTAL TOPOGRAPHICAL EFFECTS ON THE GEOID IN ALBERTA AND BRITISH COLUMBIA

DENSITY (5' × 5')