INTERNATIONAL HYDROGRAPHIC ORGANIZATION

GEODETIC COMMENTARY TO TALOS MANUAL

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Geodetic Commentary to TALOS Manual[1]

Geodesy is the oldest earth science. Its domain of interest encompasses the size and shape of the earth, the earth's gravity field, and the time variations of all these. As a by-product of these interests, geodesists have developed an expertise in the accurate positioning of points on the surface of the earth; so much so, that in some circles, geodesy has became a synonym for the science and craft of position determination. Needless to say, this is a very limited view of geodesy, but in this context it should prove useful to show that geodesists are the specialists who have something meaningful to say about positioning. The requirement for precise positioning in boundary delimitation, brings geodesists into the circle of professionals vitally interested in the Law of the Sea.

In its centuries old history of describing the earth, geodesy has developed special theories, methods, and techniques, based on an advanced knowledge of mathematics and physics, which one has to study to become a geodesist. A full-fledged geodesist would normally possess a postgraduate university degree in geodesy.

This Commentary begins with the above statements as a justification for the existence of a special geodetic commentary. The purpose of the Commentary is to make the user of the TALOS Manual aware of the subtleties in the geodetic content of the Manual and some of the pitfalls of the geodetic techniques used to carry out the technical operations implied by UNCLOS. It is, of course, out of the question to explain properly all the necessary geodetic concepts in this Commentary; this would require a textbook. A serious user of the TALOS Manual should acquire one of the existing textbooks on geodesy, such as Bomford, *Geodesy,* Groten, *Geodesy and the Earth's Gravity Field,* Vaníček and Krakiwsky, *Geodesy. the Concepts,* Torge, *Geodesy.* For proper citations see the References.

This Commentary is arranged as a series of explanatory notes and comments on the geodetic topics encountered in the Manual. As individual notes are keyed to the Manual, the Commentary should be read together with the Manual. Furthermore the Commentary contains references to geodetic literature that the user can consult on his own.

1.2 - **Chart Projections** - The surface of the earth, which includes the sea surface, is not projected directly onto a map. A point is first projected from the surface of the earth onto the reference ellipsoid (horizontal datum) used in the region, along the normal to the reference ellipsoid; the relation between a reference ellipsoid and a horizontal datum will be explained in section 2.4.3. Mathematically, this projection is represented by formulae which give the sought geodetic coordinates on the horizontal datum (latitude *(b* and longitude 1) as functions of given coordinates on the surface of the earth [Vaníček and Krakiwsky, 1986;

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[1] Prepared by P. Vaníček with contributions by the following members of the IAG GALOS Committee: G. Carrera, W.A. van Gein, B.G. Harsson, A.J. Kerr, M. Kumar, F. Madsen, L.E. Sjoberg, J. Weightman and J. Zund.

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The senior author often had to make a decision as to which contribution from the other authors to include and which one to leave out. Thus the responsibility for the material and reference selection rests with him. The same goes for any factual or conceptual errors.

Comments added by the International Hydrographic Bureau to assist readers in relating the text to the hydrographic discipline are denoted by ... (text)... *.

§15.4]. Depending on what coordinates the position is available in, these transformation equations are more or less complicated. Similarly, known geometrical quantities such as distances, horizontal angles, or azimuths, which are available at the earth surface, can be projected onto the horizontal datum by means of well known and accurate equations. These equations are normally reduced to equations for "correcting" or "reducing" these quantities from the earth surface onto the horizontal datum [ibid.; §16.2].

In the second step, the geodetic latitude and longitude on the horizontal datum are projected onto the mapping plane (i.e., the plane in which the chart is drawn) by the "mapping equations". These equations relate the x, y coordinates on the chart to the geodetic coordinates φ, λ on the horizontal datum [Clark, 1968; Pearson, 1990; Mailing, 1992]. Similarly, the above listed geometrical quantities are projected from the horizontal datum onto the mapping plane by means of equations for "corrections" or "reductions" from the horizontal datum to the mapping plane [Bomford, 1980, § 8]**.** Thus the positions (as well as the above introduced geometrical quantities) on the chart are related precisely to those on the earth surface and vice versa. We note that the equations relating the geodetic coordinates (φ, λ with the map coordinates x, y are known as the "inverse mapping equations". Similarly, in the transformation from the mapping plane to the earth surface, the corrections to the above geometric quantities are applied with reversed signs.

2.2 **The Geoid-** Ignoring for the moment that soundings on charts are referred to a low water chart datum, a topic that will be addressed later*, the geoid is the reference surface for heights (orthometric or dynamic) used in mapping. As such it is often called a "vertical datum" and the heights referred to it are commonly known as "heights above mean sea level". The practical realisation of the vertical datum is normally achieved by accepting a mean sea level at the location of tide gauges along the sea shore. This realisation (definition) carries with it some inherent errors that may reach well over one metre. The local mean sea level is determined indirectly, by studying the tide-gauge record for a certain time period and is thus tacitly valid for that time period. For a more extensive discussion of these concepts, see for instance [Vančeк, 1991].

Terminological note: the term "vertical datum" is in some surveying circles used to mean one, or the whole network of vertical control points (benchmarks) and/or the permanent tide gauges. This usage, is somewhat confusing and should be avoided. The geoid is, of course, not the only vertical datum used. Chart datums, treated in section 3, are examples of vertical datums used for compiling nautical charts. Other possible choices of a vertical datum are now being vigorously debated in hydrographic circles due to the need for a precise international reference for digital data*.

Before starting to discuss the geoid itself, the two basic height definitions as they are used in geodesy shall be reviewed, leaving alone the concept of "normal" heights. "Orthometric" heights are the everyday heights used in surveying practice and in mapping. The orthometric height of a point is defined as the length of the section of plumb line between the geoid and the point. Thus, clearly, the orthometric height of any point located on the geoid equals to zero. "Dynamic" heights are used whenever it is necessary to deal with phenomena where the laws of physics play a dominant role. This situation is encountered, for instance, in hydrological investigations. The dynamic height of a point is defined in such a way that all the points on the same level surface (an equipotential surface of the earth gravity field) have the same dynamic height and if one point has a larger dynamic height than another point, it is guaranteed that a fluid would flow from the higher point to the lower point. This behaviour is not guaranteed for orthometric heights. The dynamic height of any point located on the geoid is equal to zero. (Section 2.5.3 deals with yet another height, "geodetic" height, which is not used in practice because it refers to the horizontal datum.) Readers interested in learning how levelled height differences obtained from field measurements are transformed into one of the two proper heights, orthometric or dynamic, should consult [Vaníček and Krakiwsky, 1986; §19.21.

The geoid is probably the most important surface in geodesy. Generations of geodesists have been cormputing the geoid from different kinds of measurernents ranging from the astronomically determined deflections of the vertical, to gravitv, analvsis of satellite orbital analvsis, to satellite altimetry, striving for an ever increasing accuracy. Two broad families of techniques are used for the geoid computations and, correspondingly, two broad families of results are available: global solutions and regional solutions. Global solutions are available in terms of equations (involving a number of functions), regional solutions are given by numerical values on a prescribed geographical grid, see for instance [Vaníček and Christou, 1994; chapter 4]. In both cases, the geoid is described at each point by its departure, the "geoidal height" or "geoid undulation", from a horizontal daturn usually a global horizontal datum, which will be explained in section 2.4.3.

When one wishes to use the geoidal height, it is thus absolutely essential to know to which horizontal datum it is referred. This may sound paradoxical to readers who have not been much exposed to geodesy. It shows, in its simplest form, that the geoid supplies the vital link between horizontal and vertical positions. Geoidal heights are an indispensable component in converting horizontal positions and heights (above the sea level) into true three-dimensional positions.

The short wavelength features (up to several hundred kilometres) of the geoid are now becoming quite well known, with errors in decimetres, or even centimetres in some parts of the world. Long wavelength features, obtainable only from satellite orbit analysis, are still known with a somewhat lower accuracy. Regional solutions, capable of depicting much shorter wavelength features, are usually much more accurate than global solutions. Errors in global solutions can reach several metres, particularly in the mountains. This is due to the finiteness of the functional series that describe the global solution [Vaníček and Christou, 1994; chapter 21 which smoothes out the shorter wavelength features. If the geoidal height is to be used in any calculations, its uncertainty, both systematic and random, must also be taken into account.

At sea the geoid can now be determined directly by satellite altimetry. There have been several satellite altimetry missions flown in the recent past. The most recent mission, yielding the most accurate sea surface, is the TOPEX/POSEIDON described in [AGU, 1994]. Because satellite altimetry measures the height of instantaneous sea level above a geocentric datum - see the next section - the geoid obtained from this system is only approximate, accurate to perhaps a metre.

2.4.3 **Geocentric Datum (Worldwide Datums)** - First the relation between a "reference ellipsoid" and a "horizontal datum", geocentric or local (geodetic), has to be explained. A horizontal datum is a reference ellipsoid. But for the reference ellipsoid of a selected size and shape to be of any use as a coordinate reference surface, its position and orientation within the earth have to be uniquely defined. There are several techniques of positioning and orientating the reference ellipsoid within the earth and the reader will find a detailed description of these techniques in [Vaníček and Krakiwsky, 1986; § 18. 1]. Since a horizontal datum is nothing else but a (properly positioned and orientated) reference ellipsoid, we use these two terms interchangeably when there is no danger of confusion. In some surveying circles, the term horizontal datum (geocentric or local) is understood a little differently. There, it is used for the totality of the reference ellipsoid and the geodetic control network points. This usage is somewhat confusing and should be avoided.

The Geodetic Reference System GRS80 [IAG,1980] is the internationally recommended geocentric geodetic system and should normally be used by international organisations. A geodetic reference system is defined by specifying:

1) the Cartesian geocentric coordinate system's orientation,
2) the reference ellipsoid's size and shape,
3) the system's rotation rate, and
4) the "normal gravity field".

It may be noted that the word "geocentric" implies the position of the coordinate system's origin at the centre of the earth. In the applications of geodetic positioning described in the Manual only the geometrical aspects of the reference system are actually needed.

Most techniques for determining positions (coordinates) use already known positions to relate the new positions to. To determine a position in a specific coordinate system therefore usually requires a knowledge of some positions in that coordinate system. Thus positioning in the GRS80 would require a knowledge of some positions in that system. But there are no positions specified as being a part of the definition of GRS80 and this is where the World Geodetic System, WGS84 becomes useful. The WGS 84, which includes coordinates (positions) of some of the satellite tracking stations, is one of the practical realizations of GRS80 [Defense Mapping Agency, 1987] and as such should be used here. Thus, by using the WGS 84, countries really conform with the IAG's recommendation that GRS80 be used for geodetic work. Note that the North American Datum, NAD 83 horizontal datum is another realization of GRS80.

2.4.4 **Transformation Between Geodetic Datums -** Since the parameters of transformation between two geodetic datums cannot be determined directly, coordinates (positions) of a set of identical points on both datums must be used to determine them. These positions are always distorted due to the presence of both systematic and random errors. This fact makes the determination of transformation parameters very tricky and a great deal of care must be taken to do it properly. For an in-depth study of the problem the reader is referred to [Vaníček, 1992]. IHO Special Publication No. 60 (S-60) lists transformation parameters world-wide for transformations between numerous local (regional) datums and WGS84.*

Note that the parameter called "scale correction" or "scale difference" is not a transformation parameter for two datums. The "scale correction" reflects the (average) difference in scales of the coordinates used for the parameter determination. It is thus the simplest mathematical model for the difference of systematic distortions in the two sets of coordinates.

2.5.2.2 **Geodetic Measurements** - The "terrestrial horizontal geodetic positioning" described in this section is sometime referred to as being of the relative mode: it determines the horizontal positions of the (new) points on the coastline relative to the horizontal positions of the existing geodetic control points. As a consequence, the new positions are subject to all the position errors contained in the existing geodetic control network plus all the errors contained in the new measurements. This is known as the "propagation of errors", or propagation of positional uncertainties.

These uncertainties are of two different kinds (assuming that gross errors had been already eliminated): random and systematic, the latter being often referred to as "systematic distortions". Each of these uncertainties propagates differently; while the random uncertainties propagate with the square root of the distance (or even slower in a well braced network with lots of redundancies), see for instance [Vaníček and Krakiwsky, 1986, §18.3], systematic distortions usually propagate much faster. In networks composed of terrestrial observations Le. ' older networks, adjusted with respect to the "point of origin", random uncertainties propagate radially from the origin, while systematic distortions have a regional character. Either of the two components May reach many metres, even several tens of metres.

In geodetic practice the random error component at each point is described by an elliptical "confidence region" also called an "absolute error ellipse". The larger the error ellipse, the larger the random positional uncertainty. The absolute error ellipses grow in size with distance from the origin of the network. In more modern networks, where the terrestrial observations are adjusted together with some selected satellite positions (possibly also with position differences determined by the very accurate radioastronomical technique known as "Very Long Baseline Interferometry", (VLBI)), absolute error ellipses have more homogeneous sizes [Vaníček and Krakiwsky, 1986; § 18.3].

Unlike random uncertainties, systematic distortions of a control network can be modelled by mathematical formulae of one kind or another. This, of course, requires a specialised knowledge of the network and can be done only by the responsible national agencies in charge; for an example see [Junkins, 1991].

The user of a control network should be well aware of the positional uncertainties of the points he wishes to use, including the systematic distortions, if these are known. The best source of information on these are the pertinent national agencies. The user, having disposed of the known systematic distortions, should obtain the part of the "covariance matrix", cf. for instance [Mikhail, 1976: §4.4.2], pertaining to the points he wishes to use from the agency who carried out the adjustment of the control network. This covariance matrix contains all the necessary information about the random uncertainties in the adjusted control network. The appropriate part of the complete network's covariance matrix should then be used in the adjustment of the coastline survey in the proper manner, i.e., to weight properly the positions of the used control points in the new adjustment, see for instance [Vaníček and Lugoe, 1986].

When the new survey of coastline points is adjusted, the covariance matrix of the adjusted coastline positions is obtained as a by-product in the adjustment. This covariance matrix again contains all the information about the random uncertainties in the adjusted positions and can be easily converted into absolute confidence regions [Mikhail, 1976; §11.5.2]. These ellipses portray the random uncertainties in the new positions in the context of the control network. To obtain (adjusted) positions, without the accompaniment of at least the random uncertainty, is unacceptable in geodesy. The rationale for this requirement is easily understood: the uncertainty represents a rudimentary quality control. It is not difficult to realize that a position with an uncertainty of, say, 10 km may not be acceptable, while an uncertainty of a few centimetres would indicate a very accurately determined position. Naturally, the ultimate use of the position dictates the required accuracy.

In many applications it is more important to know the relative uncertainties of one position with respect to other positions, than to know the absolute uncertainties. Relative uncertainties of random origin are also readily obtained from the covariance matrix of the adjusted positions. These uncertainties closely reflect the random errors in the observations. Relative systematic distortions can be generated from the mathematical description of these distortions.

A few words about vertical measurements and the necessary vertical control are also in order. "Geodetic vertical control" consists of a network of levelling benchmarks the (orthometric) heights of which, as already indicated in section 2.2, are referred to the geoid via the local mean sea levels. As with the horizontal control, there are errors, both random and systematic, associated with vertical control points. As with horizontal control, these errors should be propagated into the newly determined heights of coastal points.

2.5.3 **Positioning by Satellite** - As stated in the Manual, positions can now be determined also by measurements to geodetic/navigation satellites. This can be done in one of the following two modes: using point positioning or using relative (differential) positioning with respect to one or more nearby points whose positions are already known. The latter mode is inherently more precise by at least one order of magnitude, see for instance [Seeber, 1993; §4]. The actual accuracy depends on the instrumentation, duration and care devoted to the observations, the kind of and the way in which the GPS software is used. On these matters the interested reader would be well advised to consult one of the existing texts such as [Wells *et al.,* 1987; Rizos, 1990; Hofmann-Wellenhof *et al_* 1992; Seeber, 1993; Leick, 1995].

Both modes of positioning by satellite naturally yield three-dimensional positions. These positions, or position differences, are usually expressed in three-dimensional geocentric Cartesian coordinates, or coordinate differences, which can subsequently be transformed into geodetic latitude (b, longitude X and height h, or their differences [Vaníček and Krakiwsky, 1986; §15.4]. Because one starts with geocentric Cartesian coordinates, these "curvilinear coordinates" are also referred to a geocentric horizontal datum as discussed in section 2.4.3. For example, GPS determined curvilinear coordinates, or coordinate differences, are referred to WGS 84. It is important to realize that heights (or height differences) determined in this way are of the geodetic variety and cannot be used in lieu of the usual heights above the sea level, e.g., in lieu of the orthometric heights. As explained in the Manual, the geoidal height, or geoidal height difference, with respect to the appropriate geocentric horizontal datum, must be known to carry out the transformation.

To transform the horizontal coordinates, or coordinate differences, from the geocentric datum to the local datum, one must use the same procedure as hinted at above, in section 2.4.4. When determining the transformation parameters, one must take into account the fact that satellite determined positions contain errors, resulting in positional uncertainties much like the uncertainties resulting from terrestrial measurements, cf. section 2.5.2.2.

3. **Chart Datums (Vertical) -** To provide the mariner with a margin of safety in terms of depth measurement all charted depths are referred to chart datum which is equated to the datum of tidal predictions and defined by the IHO as a plane so low that the tide will not frequently fall below it. Thus, unlike heights on land maps which are normally referred to Mean Sea Level, depths on charts are referred to a low water level. For the determination of chart datum it is necessary to observe heights of points above the low water.* Thus the height of low water below the mean sea level must be determined. This is done by analyzing the records of tide gauges from the vicinity of the points of interest, which requires a specific expertise. Note that the local sea level variations are caused not only by sea-tides (also called astronomical tides in hydrography)*, but also by other phenomena such as storm surges, currents, wind action, barometric pressure variations, thermohaline changes, etc. Even though these non-tidal variations may be occasionally as large as the tidal variations, they are not normally considered in the analysis. For more detailed discussion the reader is referred to monographs such as [Hill, 1962-, Warren and Wunsch, 19811, or the overview paper by [Stommel, 1963].

Both the sea level and land heights are also undergoing secular changes of a varying speed (as well as periodic changes which do not present a real problem because they can be averaged out, one way or another) see [Lambeck, 1988]. While the sea level rises globally by 1 or 2 mm per year, the land rise or subsidence may reach several centimetres per year and more, in particularly active regions. This may seem trivial along most coasts but when the coast has a low gradient, the effect of such changes may be fairly significant. For example, through the post-glacial rise of the earth crust in the Hudson Bay area, Canada is steadily gaining many hundreds of square kilometres of territory each year [Walcott, 1972]. A similar situation exists in the Bay of Bothnia in Fennoscandia.

Transformations between different kinds of vertical datums is a fairly involved problem. Various approximate methods exist. For a reader interested in a more technical exposure of the problem we recommend the paper by [Vaníček, 1994]. A practical experience with a transformation in Fennoscandia has been described, for example, by [Pan and Sjoberg, 1993].

4.3 **Baselines Formed by Straight Lines** - While the determination of baseline points, or more precisely positions of baseline points in terms of their horizontal coordinates φ, λ on a horizontal datum or x, y on a mapping plane with their requisite uncertainties (cf., section 2.5.2.2), should by now be fairly clear, we should stop and think about the "straight line" that connects two adjacent baseline points, the baseline itself. A line which is straight in the given coordinate system (in the case of maps this coordinate system is given by the map projection) on a given surface is known as the "geodesic line", i.e., it the shortest possible line connecting the two end points.

Depending on what surface we take the straight lines to be in, we get different baselines. A baseline that is straight on a Mercator chart, say, will generally not be straight on a chart which uses the Lambert projection. Neither of the two will be the same as the "straight line" on the horizontal. datum, except for some special cases. The differences can be quite significant, particularly for longer baselines as illustrated in the Manual. It thus becomes absolutely imperative to specify the surface (the horizontal datum and, if applicable, the chart projection) on which the baselines are reckoned. It should also be noted that, except for straight lines on a Mercator chart (rhumb lines or loxodromes), the azimuth of the straight line generally changes from point to point. As pointed out in the Manual, "straight lines" on the horizontal datum should be the preferred choice here.

Nowadays, when the positions of baseline points are expressed in coordinates, rather than dots on a chart, also the generation of the baselines as strings of positions given in coordinates is desirable. These coordinates have to be generated mathematically. To generate a string of positions on a straight line on a mapping plane is, of course, quite simple once the interval between the adjacent points has been decided upon. On the horizontal datum, the proposition is somewhat more complicated; these coordinates must be generated using the (fairly complex) equation of a geodesic line or from an approximation of this equation [Vincenty, 1975; Pearson, 1990]. This requires a specialized geodetic knowledge.

It must be pointed out that each position on the straight line has its own positional uncertainty, that can be shown in much the same way as the positional uncertainties of the baseline points. These uncertainties should be evaluated at the same time as the positions of detailed points on the straight line through the propagation of errors discussed in section 2.5.2.2. Once again, it should be emphasized that to produce a position, even of a detailed point on a straight baseline, without evaluating its uncertainty makes little sense.

4.4 **Archipelagic Straight Baselines** - The additional complication encountered here is the necessity to evaluate areas enclosed by straight baselines. When the baselines are straight on a map, the projected area can be evaluated fairly simply by using analytical geometry. Unless the work is carried out in an equal-area map projection, the projected area has then to be corrected for the map distortion to get the correct value, i.e., the value of the area on the horizontal datum. This is not a simple task and is better left to a specialist in mathematical cartography. Even if the baselines are defined as "straight lines" on a horizontal datum then the evaluation of the area is not a simple task either. (The same remark applies to the determination of an area of a bay, a task encountered when bay closing lines are constructed as part of the normal straight baseline.) The integrals needed in the evaluation are quite complicated and many different approaches have been devised [Danielsen, 1989, 1994; v. Geinand Gillissen, 1993, Gillissen, 1994]. Here again, the task is better left to someone with a specific knowledge of geodesy. Needless to say the value of the area of interest will vary depending on what surface is used for the definition of the baselines encompassing the area.

It should by now be obvious that areas, being computed from positions subject to uncertainties, also have uncertainties associated with them. The value of an area of interest should thus be given by some limiting probable values which can be derived from the positional uncertainties of the circumferential baselines.

5.2 **Limits Based on Distance -** As stated in the Manual, an accurate determination of limits of maritime zones (called limit(s) in the following text) defined by a specific distance from baselines is not simple. The idea, of course, goes back to the time when graphical methods on charts were still the order of the day and when drawing a circle with a pair of compasses around a point on the chart was still an acceptable practice. Nowadays, the limits, as any other boundary, should be determined mathematically, by a string of horizontal coordinates φ, λ on the selected horizontal datum, or x, y, on the selected chart. Perhaps, it is useful to remind the reader that one kind of horizontal coordinate pairs can be transformed to the other exactly, using the concepts described above in section 1.2.

Before proceeding any further, it should be discussed why the traditionally used graphical approach to any maritime boundary delimitation should be avoided. In marine navigation, numerical navigation algorithms that give the ship's track in terms of horizontal coordinates, have become the norm. Graphical display of the track based on the computed coordinates, is provided to the captain as an additional convenience, rather than as the primary information. These navigation algorithms rely mostly on satellite derived positions in a selected coordinate system, but other positioning systems are also used. Clearly, in the case of marine trespassing, it will be increasingly the mathematical evidence that will carry the weight in any dispute, because using mathematical evidence, the fact of trespassing can be established unequivocally, up to the errors in the boundary and ship positions.

Limits or boundaries are best computed on a horizontal datum, where length distortions inherent in any chart projection do not pose problems. For every detail point on a baseline a string of points out at sea can be computed, that have the prescribed distance from. the baseline point. Repeating this procedure for each detail baseline point, one ends up with a mesh of points. Those points that are the furthest away from the shore make out the required limit; a special algorithm for discarding the points that do not define the limit has to be employed here. Carrera [1989] designed an alternative technique, based on an iterative approach, that directly traces the outer limit.

It is of interest to note that a limit that is equidistant throughout its length and therefore parallel to a straight baseline (created by a geodesic line of one kind or another, cf. section 4.3), is itself not a geodesic line. This fact is, of course, irrelevant once the coordinates of detail points along the length of the line are computed. An obvious but interesting consequence of this is that a line parallel to a loxodrome - a geodesic on a Mercator chart - is itself not a loxodrome. It is no longer a line of constant azimuth, thus invalidating the argument often offered in support of defining baselines as straight lines on Mercator charts. It may be worth reiterating that the outer limits will be different for different definitions of straight lines making the straight baselines.

For the reason discussed in section 2.5.2.2, a geodesist is vitally interested in knowing the accuracy of any position dealt with. Yet, the UNCLOS III is silent about the treatment of positional uncertainties in establishing the marine boundary. This dilemma is discussed in more detail here. When it comes to a potential dispute of trespassing, it becomes important to establish the accuracy of the navigational algorithm used by the alleged trespasser. But it is equally important also to know the accuracy of the claimed limit in dispute. Disregarding the former here, a few words about the latter seem necessary. To determine the positional uncertainty of the outer limit is a relatively simple technical task, once it is known how to propagate errors (both random and systematic, taking into account that the seaward extension of the baselines is done on the basis of exactly defined, i.e., errorless distances); the -concept of error propagation was shown already in sections 2.5.2.2 and 4.3. The courts will have to establish a precedent as to when the trespassing is considered to have taken place on the basis of a specific probability associated with the positional uncertainties in question.

5.7.8 **The Foot of the Slope** - The idea of using the maximum change of gradient at the base of the continental slope dates back to the age when graphical techniques were still used routinely. The idea pre-supposes that the direction of the continental slope foot-line[2] is known, the direction of a profile perpendicular to this foot-line can therefore be detected and bathymetric points along this profile collected. Yet, generally, the directions of individual segments of the foot-line are not known. Neither do bathymetric measurements along the stipulated profiles exist: in most places bathymetric data exist either along some other, arbitrary profiles, or on an irregular mesh as collected by one of the multibeam systems [Vaníček et al., 1994]. Hence the graphic approach described in the Manual will probably need supplementing with an analytical approach.*

To get the horizontal position of the foot-line from available bathymetric data requires a more sophisticated technique. The right methodology has not yet been formulated, but some attempts are already underway; for an example see [ibid.]. A large part of the problem is the non-uniform density of collected bathymetric data. Too high or too low a density of bathymetric data can have formidable effects on the location of the detected foot-line. Investigations undertaken to-date indicate that too low a density (undersampling) might pose less problems.*

An additional problem. is presented by the very concept of the maximum change of the gradient. There are real life situations, where the maximum change does not occur at a point but over a whole stretch of the pertinent profile. This happens when the transition from the slope to the abyssal plane has a shape of a circular cylinder. In these regions the foot-line taken according to the UNCLOS III definition, becomes a foot-strip of a finite width.

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[2] Foot-line: The line that connects the points defining the foot of the slope.

To evaluate the positional uncertainty of the foot-line is a complex task. The uncertainty is affected by the uncertainties (errors) in both the horizontal positions of bathymetric soundings as well as the soundings themselves. The latter, i.e., the depth uncertainty, will be fairly significant since the foot is usually at a considerable depth.

6.2 The Equidistance Method - For an automated computation of points defining the equidistance line, the connections between the baseline points in the opposite states are best chosen using the Delaunay triangulation [Brassel and Reif, 1979]. This technique insures that always the closest baseline points are connected for the purpose of finding the appropriate mid-point. For connections of a reasonable length (perhaps up to 100 km), normal sections on the reference ellipsoids can be used instead of geodesic curves. Under these circumstances, the differences for the two curves in both the length and azimuth at the mid-point, are insignificant and would have a trivial effect on the equidistant line while saving a lot of computer time.

It is repeated once more that the positional uncertainty of the equidistance line should be determined, for reasons spelled in section 5.2. This is done by propagating the uncertainties of the pertinent baseline points on the opposite coasts into the point defining the equidistance line and further to the detail points on that line. It should be noted that if the coordinates of baseline points at the opposite coasts had been originally determined on two different horizontal datums (before one set of points was transformed into the horizontal datum of the second set) then the positional uncertainties of the equidistance line rnay be very large.

6.4.3 Prolongation of Land Boundaries - When a straight boundary is "prolonged" (extended) seaward along a parallel of latitude, a meridian or an azimuth,* it is important to elucidate the definition of this prolongation. If the extended boundary is deemed desirable to be a geodesic on the reference ellipsoid and to follow a specific azimuth at the coastline, it should be borne in mind that a geodesic curve on the reference ellipsoid does not have generally a constant azimuth. The positional uncertainty of such extended boundary will be constant along its length.

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