Automatic Tracing of the Foot of the Continental Slope

ZIQIANG OU
PETR VANÍČEK

Department of Geodesy and Geomatics Engineering
University of New Brunswick
Fredericton, New Brunswick, Canada

The UNCLOS III (Article 76, Section 4(b)) defines the foot of the continental slope as the point of maximum change in the gradient at its base. It is impossible to locate so defined a foot and thus to trace the foot-line objectively by eye. In this study we show a method designed automatically to detect and trace the foot-line of the continental slope from an irregular array of bathymetrical data. Our algorithm first transforms the bathymetric surface to a maximum curvature surface. On this new surface, the foot-line corresponds to one of the ridges; instead of tracing the foot-line on the bathymetric surface, we now can trace the ridges on the maximum curvature surface. The tracing of the ridges can be done automatically and objectively and the foot-line is identified as being one of these ridges. We devote particular attention to the case when the ridge-line is not defined, i.e., to the case when the point of maximum gradient change becomes a region of maximum gradient change.

Keywords continental slope, foot line, ridge line, bathymetric surface, maximum curvature surface, geometric uncertainty

The United Nations Conference on the Law of the Sea III (UNCLOS III) (United Nations, 1983, pp. 27–28), Article 76, Section 4(b), defines the foot of the continental slope as "the point of maximum change in the gradient at its base." In mathematics, the change of the gradient is called the curvature. The UNCLOS III document thus speaks of the foot of the continental slope as the point at which the curvature is at the maximum. The foot line, defined as the line that connects all the foot points, can therefore be understood as the line consisting of all the points where the curvature of the slope, at its base, is at a maximum (Vaníček et al., 1994a, 1994b, 1994c).

How can the foot line be traced objectively? Even though in some cases the human eye can trace the foot line rather well, such tracing is always subjective and no two people will come up with the same result. Using an objective mathematical formulation of the problem, the location of the foot line depends on the method used for the computations. Basically, there are two choices to locate the foot line: to use two-dimensional profiles or

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Address correspondence to Ziqiang Ou, Department of Geodesy and Geomatics Engineering, University of New Brunswick, P.O. Box 4400, Fredericton, N.B., Canada E3B 5A3. E-mail: zqou@jupiter.sun.csd.unb.ca.

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a three-dimensional approach. In the former case, the foot line is obtained by connecting the maximum curvature points on successive adjacent profiles. In the latter case, the location of foot-line points are determined directly on the surface of maximum curvature. It is worth pointing out that these two techniques will result theoretically in two different solutions. We will illustrate this in the following sections using a simulated bathymetric surface.

In this study, we follow an approach proposed by Vaníček et al. (1994a), based on a three-dimensional surface of maximum curvature constructed directly from bathymetry, on which the foot line corresponds to one of the ridges. The tracing of the ridges can be done automatically, i.e., objectively; this is why we became interested in this technique. In the earlier articles (Vaníček et al., 1994a, 1994b), the automatic recognition and tracking of the ridges had not been implemented. This constitutes the main contribution that this article makes toward the solution of the problem.

The ridges of maximum curvature extend along the directions of minimum curvature, yet the directional information on the normal principal curvature has been so far neglected in the construction of the maximum curvature surface, even though this information is very useful for tracing the ridges. Another focus of this study is on using this directional information in the development of an automatic recognition and tracing algorithm for the foot line.

Our approach is illustrated and demonstrated on an approximately regular data set, i.e., all data form a topological quadrilateral network and are stored as a two-dimensional array. The technique can be easily extended to deal with totally irregular data. Numerical experiments have shown that if the data set is too irregular, however, the accuracy of foot-line determination may be severely affected. Therefore, some data regularization, such as the one used by Vaníček et al. (1994a), would be necessary in a real situation.

Construction of the Surface of Maximum Curvature

The sea floor is characterized by irregularly spaced depth data (i.e., bathymetric data), from which the shape of the sea floor has somehow to be reconstructed. To follow the most objective approach that honors the data well, we decided to construct the surface of maximum curvature only from observed bathymetric data. This means that all the partial derivatives that need to be used in calculating normal curvatures to construct the maximum curvature surface are evaluated numerically from discrete bathymetric data.

The sea floor can be expressed as a continuous function of horizontal position (x, y):

$$z = z(x, y) \tag{1}$$

where z is the depth. A direction in the tangent plane at P is conveniently described by the components of the unit vector in this direction. These components are called direction coefficients and are denoted by (l, m). The normal curvature k at point P in a direction specified by directional coefficients (l, m) is given by (Willmore, 1959, p. 97)

$$k = Ll^2 + 2Mlm + Nm^2 \tag{2}$$

with an auxiliary condition

$$El^2 + 2Flm + Gm^2 = 1 \tag{3}$$

Here (l, m) are reckoned in the tangent plane to the bathymetric surface at point P, and E, F, and G are the elements of the first Gaussian fundamental form of the surface (Smirnov, 1964, p. 371), i.e.,

$$E = 1 + \left(\frac{\partial z}{\partial x}\right)^2 \qquad F = \frac{\partial z}{\partial x}\frac{\partial z}{\partial y} \qquad G = 1 + \left(\frac{\partial z}{\partial y}\right)^2 \tag{4}$$

and L, M, and N are the elements of the second Gaussian fundamental form of the surface,

$$L = \frac{1}{H}\frac{\partial^2 z}{\partial x^2} \qquad M = \frac{1}{H}\frac{\partial^2 z}{\partial x\,\partial y} \qquad N = \frac{1}{H}\frac{\partial^2 z}{\partial y^2} \tag{5}$$

with

$$H = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} \tag{6}$$

As l, m vary, subject to constraint (3), the normal curvature will vary as well. Its extreme values may be found by making use of Lagrange's multiplers:

$$k = Ll^2 + 2Mlm + Nm^2 - \lambda(El^2 + 2Flm + Gm^2 - 1) \tag{7}$$

where λ is Lagrange's multipler. Letting the first-order partial derivatives equal zero, we obtain

$$\frac{1}{2}\frac{\partial k}{\partial l} = Ll + Mm - \lambda El - \lambda Fm = 0 \tag{8}$$

$$\frac{1}{2}\frac{\partial k}{\partial m} = Ml + Nm - \lambda Fl - \lambda Gm = 0 \tag{9}$$

Multiplying Eq. (8) by l, Eq. (9) by m, adding them, and considering Eqs. (2) and (3), we obtain $\lambda = k$. Thus, by eliminating l and m from Eqs. (8) and (9), the maximum and minimum normal curvatures (principal curvatures) can be obtained as the roots of the following quadratic equation:

$$(EG - F^2)k^2 - (EN + GL - 2FM)k + (LN - M^2) = 0 \tag{10}$$

At the same time, the principal directions corresponding to the principal curvatures are obtained by eliminating λ from Eqs. (8) and (9):

$$(EM - FL)l^2 + (EN - GL)lm + (FN - GM)m^2 = 0. \tag{11}$$

To solve Eq. (11), we divide it by l^2 and realize that $m/l = \tan\alpha$, where α is the angle that the principal direction of curvature makes with the x axis. We obtain two roots (i.e., two angles α_1 and α_2), which characterize the two mutually orthogonal principal directions.

In our case, the bathymetric surface is not known analytically. Partial derivatives can be evaluated in many ways, for example, through finite differences (Ames, 1969) or by fitting a local algebraic surface (Vaníček et el., 1994a). Since our data are distributed irregularly, the simple finite differences do not work very well. In the following, we use a method similar to that used by Vaníček et al. (1994a), which can work with irregular data. Instead of fitting a local algebraic surface, however, the function $z(x,y)$ is developed into a Taylor series up to second order, valid for $P_i = (x_i, y_i)$ in the neighborhood of the computation point $P_0 = (x_0, y_0)$:

$$\begin{aligned} z(x_i, y_i) = z(x_0, y_0) + \frac{\partial z}{\partial x}\Delta x_i + \frac{\partial z}{\partial y}\Delta y_i + \frac{1}{2}\frac{\partial^2 z}{\partial x^2}\Delta x_i^2 \\ + \frac{\partial^2 z}{\partial x\,\partial y}\Delta x_i\,\Delta y_i + \frac{1}{2}\frac{\partial^2 z}{\partial y^2}\Delta y_i^2 \end{aligned} \tag{12}$$

where

$$\Delta x_i = x_i - x_0 \qquad \Delta y_i = y_i - y_0 \tag{13}$$

Thus, if there are, say, five data points around (x_0, y_0), we can obtain five linear Eqs. (12) for the five partial derivatives we are looking for. If there are more than five data points around (x_0, y_0), the least-squares technique may be used. We observe that when the data points are distributed regularly and only five adjacent points are used to determine the partial derivatives, Eqs. (12) are equivalent to the well-known finite-difference formulas. For example, for the case shown in Figure 1, we can obtain five Eqs. (12), which involve the five partial derivatives to be estimated, where h and v are the coordinate increments in the x and y directions, respectively. The following finite-difference approximations are obtained by solving these equations:

$$\frac{\partial z}{\partial x} = \frac{1}{2h}\left[z(x+h,y) - z(x-h,y)\right] \tag{14}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2v}\left[z(x,y+v) - z(x,y-v)\right] \tag{15}$$

$$\frac{\partial^2 z}{\partial x\,\partial y} = \frac{1}{hv}\left[z(x-h,y-v) - z(x-h,y) - z(x,y-v) + z(x,y)\right] \tag{16}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{h^2}\left[z(x+h,y) - 2z(x,y) + z(x-h,y)\right] \tag{17}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{1}{v^2}\left[z(x,y+v) - 2z(x,y) + z(x,y-v)\right] \tag{18}$$

Now, the principal curvatures K_1 and K_2 can be calculated from Eq. (10). We are interested only in the foot line of the continental slope, which is a concave feature and, correspond-

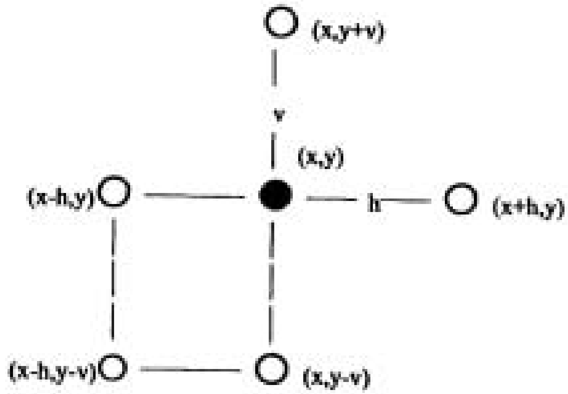

Figure 1. The neighborhood for the computation of partial derivatives.

ingly, the curvature we are interested in should be positive. Therefore, the surface of maximum curvature is constructed by the following algorithm (Vaníček et al., 1994a):

$$K_{max} = \begin{cases} \max(K_1, K_2) & \max(K_1, K_2) \geq 0 \\ 0 & \max(K_1, K_2) < 0 \end{cases} \tag{19}$$

Clearly, like the bathymetric surface, the surface of maximum curvature is also a numerical surface. It is defined only on the bathymetrical data points, where there are enough adjacent points to determine the partial derivatives numerically. At the same time, the corresponding principal directions can be determined from Eq. (11).

As an illustrative example, we simulate a "bathymetric" surface by the following analytical formula:

$$z = 2\left[\frac{\exp(-a) - \exp(a)}{\exp(-a) + \exp(a)} + 2\right] \tag{20}$$

with

$$a = 2\left[x - 2 + \sin\left(\frac{\pi y}{2}\right)\right] \tag{21}$$

on a regular rectangular grid (see Figure 2). Since the surface is generated by an analytical formula, we can compute the foot line analytically. The location of the foot line thus becomes known and is shown by asterisks in Figure 2.

The maximum curvature surface, computed from Eqs. (10) and (19), is plotted in Figure 3. The partial derivatives are obtained from Eq. (12) by the least-squares technique from eight adjacent points. The contour plot of the maximum curvature surface with an overlay of the theoretical foot line is shown in Figure 4. From this figure we can see that the theoretically correct foot line is located accurately on the ridge of the maximum curvature surface. This suggests that the construction of the maximum curvature surface

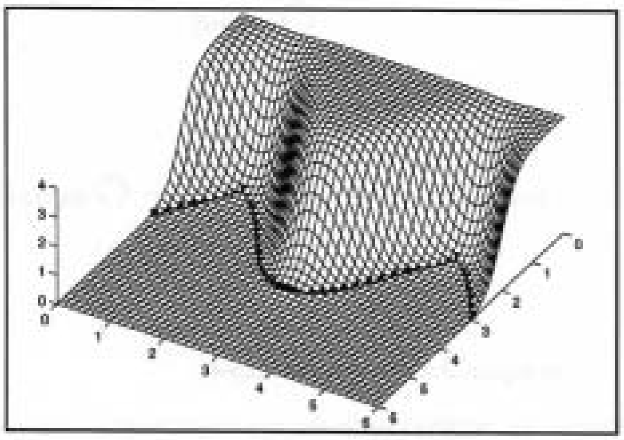

Figure 2. Simulated bathymetric surface and theoretical location of the foot line.

is correct and the numerical evaluation of partial derivatives by the Taylor series [Eq. (12)] is accurate enough for our purpose.

Figure 5 shows the principal directions that correspond to the minimum curvature as determined by Eq. (11). The other principal direction is normal to it. The length of the arrows denotes the magnitude of the maximum curvature. It is clear that the ridges extend along the directions of the minimum curvature. In our automatic tracing algorithm described later, this directional information is used to tell the computer in which direction the next ridge point should be expected.

For curiosity's sake, we also determinated the foot line from two-dimensional profiles running horizontally (i.e., parallel with the horizontal axis of Figure 5). Clearly, the two approaches give slightly different results. This is because the direction of the maximum curvature in three dimensions is different from the running direction of the profiles. If the

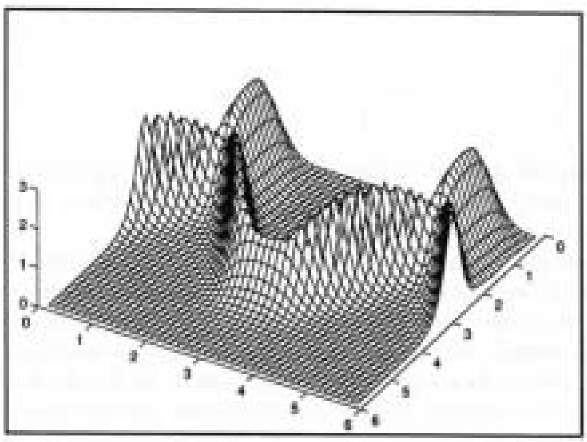

Figure 3. Maximum curvature surface corresponding to the bathymetric surface shown in Figure 2.

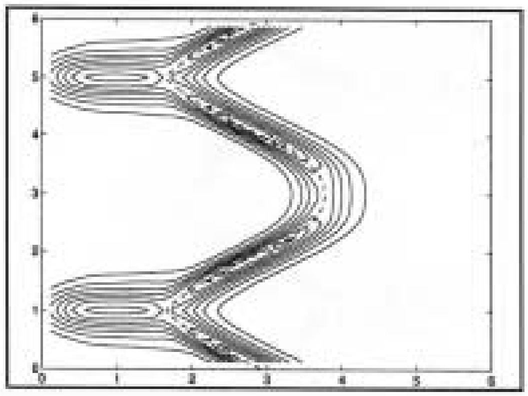

Figure 4. Contour plot of the maximum curvature surface with an overlay of the correct foot line (dashed).

profiles were taken along the direction of maximum gradient of the slope, both approaches would give the same results. Unfortunately, it is impossible to find such a profile before we construct the maximum curvature surface. Figure 6 shows the difference of the foot lines determined by two-dimensional profiles (dashed line) and through the maximum curvature surface (solid line).

Local Detection of Ridge Points

As mentioned above, the foot line of a continental slope should correspond to one of the ridges on the maximum curvature surface; thus, the problem of location of the foot line is transformed into the problem of the location of ridges. In order to detect ridge points, we first analyze the matrix of points that defines the maximum curvature surface. We begin by classifying each point on the basis of its neighborhood. As we know, the defining characteristic of a ridge is that the points located on the ridge have larger values than the points located on either side of the ridge. We can thus formulate the following, rather self-evident, criterion:

> Criterion #1: On the maximum curvature surface, any point located on the ridge has a larger value than the points located on either side of the ridge in the neighborhood of that point.

Then the following algorithm can be implemented: Starting with an arbitrary point, e.g., point A in Figure 7, we search for a point with the largest maximum curvature value among its neighboring points (e.g., point 3). Then we test whether the points located on both sides of the line connecting A and 3, i.e., points 2 and 4, have smaller maximum curvature values than points A and 3. The value of point 2 is compared with the value at the foot of the normal from point 2 to the connecting line A–3, obtained by linear interpolation between A and 3. If points 2 and 4 have smaller maximum curvature values, points A and 3 are both considered to be ridge points. Then we examine the next point until all points have been examined.

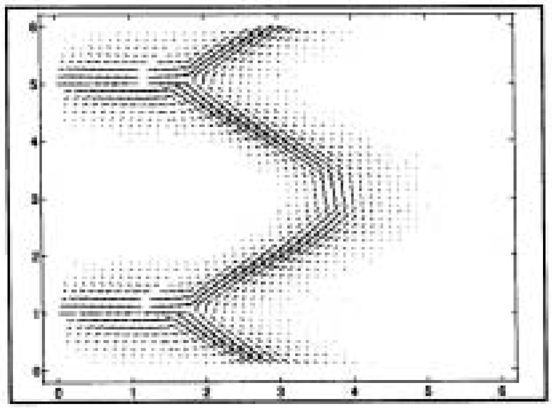

Figure 5. The direction lines of minimum curvature. The length of the arrows denotes the magnitude of the maximum curvature.

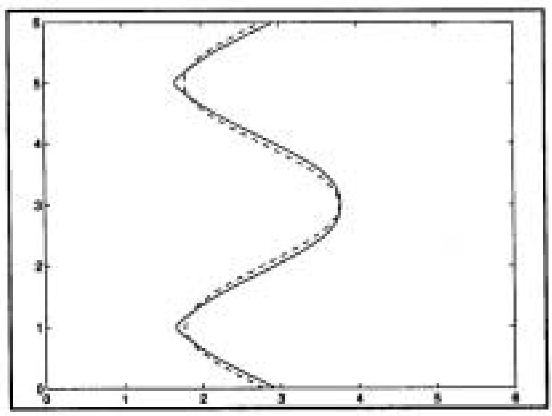

Figure 6. Comparison of two foot lines, determined by the two-dimensional profiles (dashed) and by the maximum curvature surface (solid).

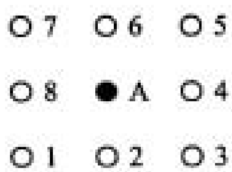

Figure 7. Point classification.

In order to eliminate the influence of errors (in the computation as well as in the observed bathymetric data) on this detection procedure, we may introduce a specific threshold. Only those points whose maximum curvature is larger than that of the side points by more than the prescribed threshold are identified as ridge points.

In addition, we are only interested in the foot-line of the continental slope. Therefore, we may wish to discard right away some ridges of maximum curvature that are caused by other local features. This can be done by considering the maximum curvature surface and the bathymetric surface simultaneously in another self-evident criterion:

> Criterion #2: On the bathymetric surface, the depth of a point on the coastward side of the foot line should be smaller than the depth of the corresponding point on the seaward side of the foot line.

Figure 8 shows a profile of the sea floor, where A is the point with the maximum positive curvature and B and C are two neighboring points of A located on the seaward side and the coastward side, respectively. Hence, on the maximum curvature surface, A is a ridge point and B and C should have smaller values than A. But on the bathymetric surface, the absolute value of the depth difference between C and B should be equal to the sum of the absolute values of the depth differences between C and A, and A and B.

Figure 9 shows all the ridge points detected by applying criterion #1, and Figure 10 shows the ridge points remaining after discarding those which fail criterion #2.

Tracing, Connecting, and Confirmation of the Ridge

By applying the algorithms discussed above, we get a set of discrete ridge points. How these ridge points should be connected to form ridge lines (one of which will be the sought foot line) is the problem we have to discuss next. The ridge points can be connected either by hand or by computer. We are interested in the latter and have thus formulated an algorithm that can automatically connect the ridge points. The approach we follow is similar to the "surface specific line tracing" proposed by Bevacqua and Floris (1987),

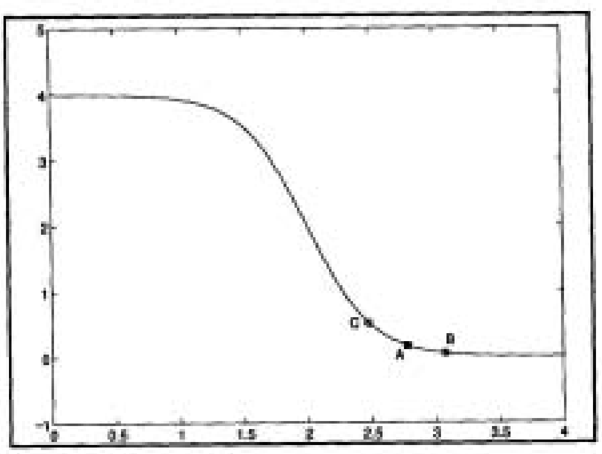

Figure 8. A profile of a simulated sea floor.

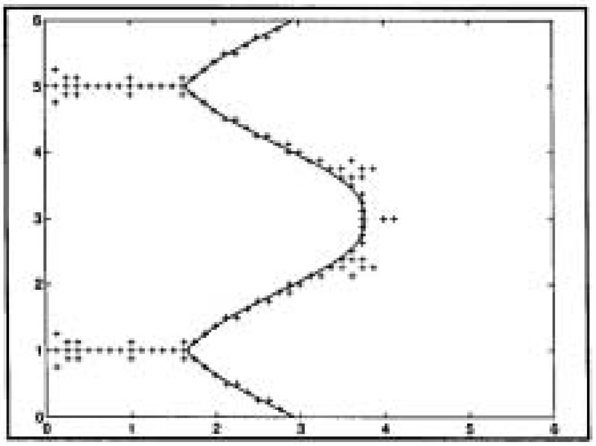

Figure 9. All detected ridge points at the maximum curvature surface depicted in Figure 3. The solid line is the theoretical foot line.

who used it to detect the ridge and valley lines in a gridded digital elevation model. In order to suit an irregular data set, our algorithm is divided into three separate stages: tracing, connecting, and confirmation. In this order, the three stages are elaborated below.

Tracing

Step A. We search for the highest ridge point *P* which has not been examined yet. If there is such a point, we take it as the starting point and then take step B. Otherwise, there are

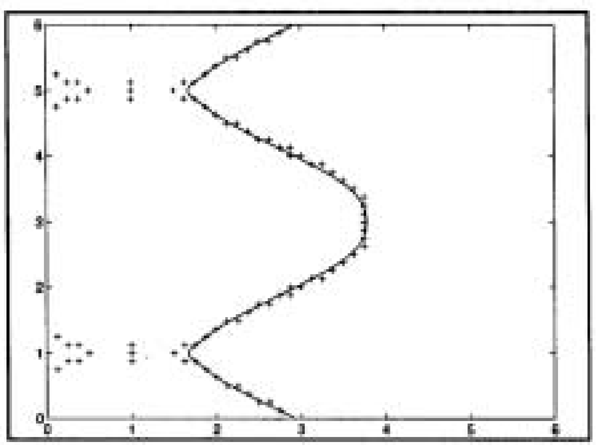

Figure 10. Ridge points after discarding points caused by local features. The solid line is the theoretical foot line.

no more ridge points that have not been examined and we can start connecting the identified ridges directly.

Step B. At the starting point, we search for the nearest neighboring point in the direction of the minimum principal curvature which simultaneously meets criteria #1 and #2. If such a neighbor ridge point cannot be found, we discard point P and go back to step A. Otherwise, we go to step C.

Step C. The new point is noted and taken as a starting point P for subsequent search and we go back to step B. We repeat this procedure until no further ridge points can be found.

Applying the described sequence of steps, we obtain generally several separate ridge lines of varying lengths. In the next stage we connect them together to form a foot line.

Connecting

Because of the infuences of the observation errors and the roughness of sea bottom, the ridges on the maximum curvature surface, which correspond to the foot of the continental slope, may be discontinuous. We wish to connect them together, if possible. To carry out the connection we have to be able to measure, one way or another, the length of the curves and distances between points. This can be done by computing the distances directly from the coordinates of ridge points for an irregular data set or by counting the intervals between adjacent grid points for a regular data set.

Step A. We select the longest ridge line as the hypothesized image (HI) of the foot line we are after.

Step B. Then we search for the ridge line nearest to the HI ridge line. If such a ridge line is found, we identify the ridge points on the two ridge lines which are the closest to each other and proceed with step C. If there is no ridge line that has not yet been considered on the investigated surface, the connection search is over and we can start the confirmation procedure.

Step C. Next, we decide whether or not the two near ridge lines should be connected together at the two closest ridge points. These closest points do not have to be the ridge-line end points. For example, let us assume that the length of the HI ridge line is L_1 and the length of its nearest ridge line is $L_2 < L_1$. Assume further that the nearest points between them are in the ith position on the HI ridge line and in the jth position on the second ridge line. Each line is thus divided into two segments by the ith and jth points, respectively. The segment lengths are equal to L_{11}, L_{12} for the HI ridge line and L_{21}, L_{22} for the second ridge line, respectively. If the distance between the closest points is smaller than some given threshold, which is selected manually in order to get "reasonable" connections, and at the same time the two longer segments connected together result in a ridge line longer than the HI ridge line (i.e., longer than L_1), we connect them into a new HI ridge line and go back to step B. Since there are no ridge points between the closest points of the two ridge-lines, the new HI ridge line bridges this gap in one step. If these conditions are not satisfied, we discard this ridge line, keep the current HI ridge line unchanged, and go back to step B.

After having examined all ridge lines, we finally obtain the longest ridge line (the latest HI ridge-line) that can be constructed within the data window. Under normal circumstances, the longest ridge line should represent the foot line to be determined. We recommend keeping discarded parts of the ridge lines in step C in a file and using visual inspection to safeguard against possible pathological cases.

Confirmation

In the final stage the resulting HI ridge line is checked. We check all the points along this ridge line to see if criteria #1 and #2 are met. If a point does not meet these two criteria—this case usually appears only between the connecting points of two adjacent ridge lines—we check whether there is any closely neighboring ridge point that does. If such a point is found, we use it to replace the troublesome point. If not, the troublesome point is simply discarded. The foot line then strides over this gap with a straight line.

We have applied this algorithm to the simulated bathymetric model shown in Figure 3. Figure 11 shows a comparison of the foot line identified by this algorithm and the correct foot line.

Effect of Bathymetric Data Density

Now, we are in a position to measure the effect of bathymetric data density on the accuracy of the foot-line determination. To measure this effect, we first have to adopt a norm with which to measure it. We have decided to use two norms, N_1 and N_2, side by side. N_1 is the area enclosed by the two lines whose closeness we are trying to measure (a geometric criterion), and it can be written as

$$N_1 = \int_L |f(x) - g(x)|\, dL \tag{22}$$

where $f(x)$ and $g(x)$ are the functional prescriptions for the two curves and L is the foot line. Because the recovered foot line is a polygonal line, the above integral is evaluated piecewise. The N_2 is the discrete quadratic norm, or the root-mean-square deviation (a statistical criterion), and it is given by

$$N_2 = \sqrt{\frac{\sum_{i=1}^{m} d_i^2}{m}} \tag{23}$$

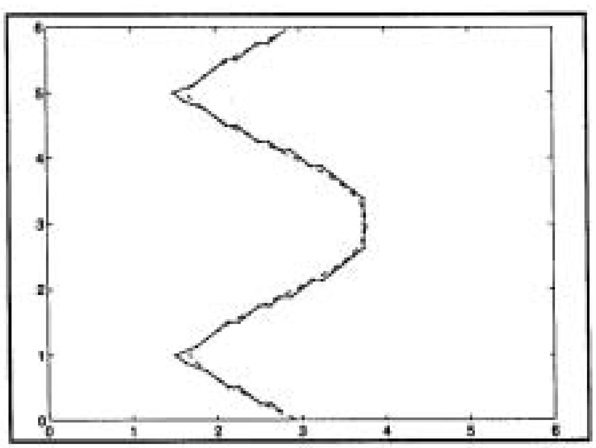

Figure 11. Comparison of the correct foot line (dashed) and the foot line recovered from a simulated sample of 49 × 49 bathymetric data points (solid).

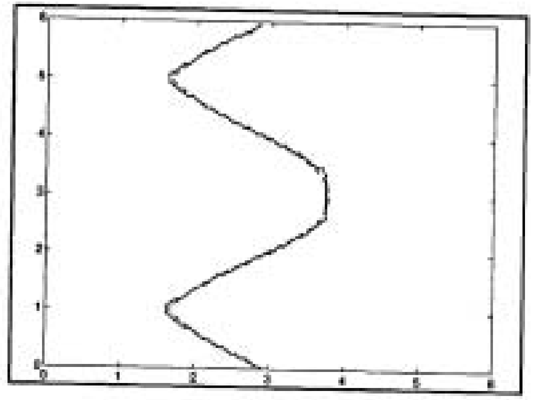

Figure 12. Comparison of the theoretically correct foot line (dashed) and the recovered foot line (solid) from 97 × 97 sampling data points.

where m denotes the number of points on the recovered foot line and d_i is the normal distance of the ith recovered foot-line point from the theoretically correct foot line. The accuracy of the recovered foot line as shown in Figure 11 is characterized by the following values: $N_1 = 0.2573$ and $N_2 = 0.04915$. For comparison, Figure 12 shows another foot-line determination attempt, this time using 97 × 97 sampling data points. Not surprisingly, for our smooth surface, the denser sample gives more accurate results, characterized by $N_1 = 0.1753$ and $N_2 = 0.02727$. This may not be the case for a rougher bathymetry, as demonstrated by Vaníček et al. (1994b).

It is worth pointing out that for a rough sea bottom it may not always be the best strategy for the purpose of foot-line determination to collect indiscriminately as much bathymetric data as possible. When the density of collected soundings is too high for the sea bottom morphology, small bottom features cause the values of maximum curvature to become too large. These large values then dominate the maximum curvature surface, masking the gentler ridge associated with the continental-slope foot line. Thus, how to match the data density with the sea bottom morphology is still a problem which needs to be investigated further.

Geometric Uncertainty

There are two sources of foot-line location uncertainty that have to be considered. The first is the geometric uncertainty caused by the real geometric shape of the region where the continental slope changes into the abyssal plane. The second source of uncertainty is, of course, the errors in the bathymetric data. Here we shall deal only with the first cause.

What happens if the change in continental-slope curvature is very gradual or none at all? If the slope-to-plane transition of the continental slope follows a circular cylinder (i.e., if the profiles of the slope-to-plane transition have the shape of a circular arc), the "maximum curvature" or "maximum change in the gradient of the slope" will be mathematically indeterminate within the region where the cylindrical shape applies. Three possible solutions for imposing a juridical determination have been proposed in (Vaníček et al., 1994a):

1. To depict the foot line as a "fuzzy" line (strip), with a width that varies according to the width of the region for which the circular approximation is valid
2. To select the central point in the circular arcs as the foot line
3. To emphasize the phrase "at its base" in the foot-line definition, and select the most seaward point on the circular arcs

No matter what solution is chosen, the key problem is to find the boundary of the region in which the circular approximation applies. Once this boundary is determined, any of the above three options can be easily applied. On the maximum curvature surface, this situation appears as a strip of constant maximum curvature, i.e., a ridge with a flat top. When the curvature is not constant but changes very gradually, the ridge on the maximum curvature surface will be very gentle and thus difficult to detect. Thus, perhaps, we should be generally seeking not a foot line but a "foot strip," which will become a line along the stretches where the maximum curvature does indeed change abruptly.

As we have shown in Figure 5, the extension of the maximum curvature ridge is along the direction of minimum curvature. Hence, the direction of the maximum curvature is perpendicular to the run of the ridge. This suggests that we can use the direction of the maximum curvature for locating the boundaries of the "foot strip." A boundary point is found (along the profile running in the principal direction of maximum curvature) where the value of the maximum curvature starts or stops being constant. Mathematically, these are the points where the first derivative of the profile becomes equal to zero, or stops being equal to zero. Since, as we pointed out above, in the real world there will also be uncertainties coming from observation errors, we should consider using "fuzzier" criteria for locating these boundary points.

As an illustrative example, Figure 13 plots the foot strip for the foot line shown in Figure 11. Since the foot line is well defined, the threshold is arbitrarily selected as the percentage (85%) of the largest maximum curvature; at the boundary we have 85% of the

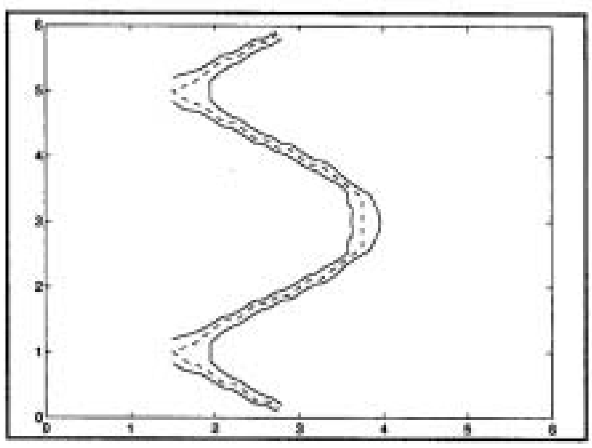

Figure 13. The 85% foot strip that contains the foot line in Figure 11.

largest maximum curvature on the corresponding profile, i.e., the value at the ridge point itself. It is seen that the strip becomes wider where the ridge of the maximum curvature is more gentle.

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