Isolating and Estimating Undifferenced GPS Integer Ambiguities

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Undifferenced Ambiguity Resolution has been an elusive goal in GPS processing.

Recent techniques have been introduced that appear to "show the way". (All aspects addressed?)

Concurrently, there have been on-going investigations into the so-called "code-biases".

The goal of the presentation is to show how:

- the "standard model" of undifferenced ionosphere-free observables is sub-optimal; and
- rigorous modelling of code biases facilitates estimation of integer ambiguities from undifferenced observables.
Standard Observation Equations

\[ C_1 = \rho + T + I + c(dt^r - dt^s) + b_{C1}^r - b_{C1}^s + \varepsilon_{C1} \]
\[ P_1 = \rho + T + I + c(dt^r - dt^s) + b_{P1}^r - b_{P1}^s + \varepsilon_{P1} \]
\[ P_2 = \rho + T + q^2 I + c(dt^r - dt^s) + b_{P2}^r - b_{P2}^s + \varepsilon_{P2} \]
\[ \lambda_1 (\Phi_1 + N_1) = L_1 = \rho + T - I + c(dt^r - dt^s) + b_{L1}^r - b_{L1}^s + \varepsilon_{L1} \]
\[ \lambda_2 (\Phi_2 + N_2) = L_2 = \rho + T - q^2 I + c(dt^r - dt^s) + b_{L2}^r - b_{L2}^s + \varepsilon_{L2} \]

- distinguish between geometric and non-geometric (timing) parameters
- \( b_* \) represent synchronisation errors between measurements – codes and phases measured separately
- understanding their role is crucial to isolating integer ambiguities from undifferenced carrier phases
Standard Observable Model
Re-assessed

\[ P_3 = \rho + T + c(dt^r - dt^s) + b_{rP3}^r - b_{sP3}^s + \epsilon_{P3} \]
\[ L_3 = \rho + T + c(dt^r - dt^s) + b_{L3}^r - b_{L3}^s - \lambda_3 N_3 + \epsilon_{L3} \]

- singular due to functionally identical clocks & biases
- by combining code clock and bias parameters and retaining common oscillators:

\[ P_3 \equiv \rho + T + c(dt_{P3}^r - dt_{P3}^s) + \epsilon_{P3} \]
\[ L_3 \equiv \rho + T + c(dt_{P3}^r - dt_{P3}^s) + A_{P3} + \epsilon_{L3} \]

where \[ A_{P3} = b_{L3}^r - b_{P3}^r - b_{L3}^s + b_{P3}^s - \lambda_3 N_3 \]

- hence, even if \( b_{L3}^* \) known, ambiguities are not isolated
Because each carrier phase is uniquely ambiguous, the pseudoranges provide the datum for the clock solutions.

Implication:
- A change in dual-frequency pseudoranges manifests itself in estimated clocks and ambiguities.

Example:
- Compute P1-C1 bias from 2 standard model solutions:
  \[ P_3 = f(P_1, P_2) \]
  \[ P_3' = f(C_1, P_2') \text{ where } P_2' = C_1 + (P_2 - P_1) \]
  \[ dt_{P3}^s - dt_{P3'}^s = b_{P1-C1}^s = A_{P3} - A_{P3'} - b_{P1-C1}^r \]
Deriving satellite P1-C1 biases

- standard model still optimally parameterised if $b_1^*$ are constant, but…

RMS of fit = 0.3ns/0.1m
YELL-AMC2 clock error
(Jan07-Jan13, 2007; common linear fit removed)
Problem:
- Apparent code and phase oscillator measures are significantly different.

Solution:
- Decouple the code and phase clocks:
  \[ P_3 = \rho + T + c(d_{P3}^r - d_{P3}^s) + \varepsilon_{P3} \]
  \[ L_3 = \rho + T + c(d_{L3}^r - d_{L3}^s) - \lambda_3 N_3 + \varepsilon_{L3} \]

  No assumptions about bias ‘stability’ required.

Implication:
- Pseudorange datum removed from carrier phase.
- Replace with Ambiguity Datums.
Ambiguity Datum Fixing

- One ambiguity per phase clock fixed, less one
- One phase clock fixed as the network datum
  - Identical concept to fixing the ‘reference clock’ in standard model network processing
  - One code clock fixed also
- Ambiguity can be fixed to arbitrary integer value
  - acts as Partial Integer Constraint
    - remaining ambiguities are integer!
- Phase clock estimates are integer ambiguous with respect to the code clock estimates.
Relationship to Goad Model


\[
\begin{align*}
\Phi_1^1(t) &= G_1^1(t)/\lambda + B_1^1(t) \\
\Phi_1^2(t) &= G_1^2(t)/\lambda + B_1^2(t) \\
\Phi_2^1(t) &= G_2^1(t)/\lambda + B_2^1(t) \\
\Phi_2^2(t) &= G_2^2(t)/\lambda + N_{12}^{12} + B_2^1(t) + B_1^2(t) - B_1^1(t)
\end{align*}
\]

4 observations: 4 clk/amb unknowns when \( G(t) \) known

- \( G(t) \): a-priori values or pseudorange estimates

\[
\Phi_1^1(t) = G_1^1(t)/\lambda + B_1^1(t) \\
\Phi_2^1(t) = G_2^1(t)/\lambda + B_2^1(t) + B_1^1(t)
\]

network datum

\[
\Phi_1^2(t) = G_1^2(t)/\lambda + B_1^2(t) \\
\Phi_2^2(t) = G_2^2(t)/\lambda + B_2^1(t) + N_{12}^{12} + B_1^2(t)
\]

base-station–base-satellite \(\Rightarrow\) datum fixing
Problem:
\[ \lambda_{IF}(L1,L2) \approx 6 \text{mm}. \ (\text{Note: } \lambda_{IF}(L2,L5) \approx 12 \text{cm}) \]

Implication:
Intermediate step required for L1,L2 processing

Solution:
Melbourne-Wübbena combination for WL
\[ A_4 = L_4 - P_5 = b_{A4}^* - b_{A4}^s - \lambda_4 N_4 + \varepsilon_{A4} \]
\[ b_{A4}^* \text{ are not constant} – 'delta-clocks' \]
Ambiguity Datum fixing
Processed simultaneously with \( P_3 \) and \( L_3 \)
With WL fixed, \( \lambda_{IF}(L1,L2) = \lambda_{NL} \approx 11 \text{cm} \)
YELL P3 & L3 Average Residuals

Elapsed Time (hour)

std model P3 avg res (m)
-2.0E+00
-1.5E+00
-1.0E+00
-5.0E-01
0.0E+00
5.0E-01
1.0E+00
1.5E+00
2.0E+00
0 12 24 36 48 60 72
ext model P3 avg res (m)
-2.0E+00
-1.5E+00
-1.0E+00
-5.0E-01
0.0E+00
5.0E-01
1.0E+00
1.5E+00
2.0E+00
0 12 24 36 48 60 72

Elapsed Time (hour)

std model L3 avg res (m)
-2.0E-06
-1.5E-06
-1.0E-06
-5.0E-07
0.0E+00
5.0E-07
1.0E-06
1.5E-06
2.0E-06
0 12 24 36 48 60 72
ext model L3 avg res (m)
-2.0E-06
-1.5E-06
-1.0E-06
-5.0E-07
0.0E+00
5.0E-07
1.0E-06
1.5E-06
2.0E-06
0 12 24 36 48 60 72
Station Clock Parameter Estimates

Rapid/phase clock de-trended RMS = 0.08ns/0.02m
Satellite Clock Parameter Estimates

Rapid/phase clock de-trended RMS = 0.17ns/0.05m
Widelane Ambiguities – Float & Fixed

WL Ambiguities for YELL (cy)

Ambiguities (cy)

Hour-of-Day (006, 2007)

Ambiguities (cy)

Hour-of-Day (006, 2007)
L3 Residuals – Float & Fixed

Phase Residuals for YELL (m)

Phase Residuals (m)

Elevation Angle (deg)

Phase Residuals for YELL (m)

Phase Residuals (m)

Elevation Angle (deg)
Implications for PPP — Summary

- Each observable requires a satellite ‘clock’ parameter:
  - \( dt_{P3}, dt_{L3}, b_{A4} \)
  - as well as satellite X, Y, Z coordinates.
- In practice \( (dt_{P3} - dt_{L3}) \) and \( b_{A4} \) variations may allow transmission as ‘slow’ corrections.
- Standard Ambiguity Resolution Techniques (e.g. LAMBDA) become applicable to PPP.
- PPP-AR becomes possible in principle.
- ALL predicated on good orbits! (IGS Rapid here)
Conclusions

- Synchronisation of code and phase measurements is significantly different.
- The Standard Model allows the pseudorange biases to directly interfere with the carrier phase biases.
- The Decoupled Clock Model provides:
  - unambiguous, but imprecise code clock estimates
  - precise, but ambiguous phase clock estimates
  - integer ambiguities.
- Extended Model required for L1, L2 processing.
- Provides a path for PPP-AR in a very generic way
  - extension of generic LS, no a-priori bias assumptions.