Chapter 3

On Ambiguities in Definitions and Applications of Bouguer Gravity Anomaly

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Abstract Over decades diverse definitions and use of the Bouguer gravity anomaly found place in geodetic and geophysical applications. We discuss three distinct Bouguer anomalies. Their definitions vary due to the presence or absence of various effects (corrections), such as the geophysical indirect effect and the secondary indirect effects. Here we discuss the significance and magnitude of these effects. We point out the different understanding of the Bouguer anomaly in geophysics compared to geodesy. We also address the diverse demands on the gravity data in geophysical and geodetic applications, such as the issue of the topographic density and the lower boundary in the volume integral for the topographic correction, as well as the need for the bathymetric correction. Recommendations are made to bring the definitions and terminology into accord with the potential theory.

3.1 Anomalous Gravity – Gravity Anomaly and Disturbance

Having the pairs *actual potential* and *actual gravity* (g), *normal potential* and *normal gravity* (γ), we would anticipate to encounter the pair *disturbing potential* (T) and *disturbing (anomalous) gravity*. In fact, two such anomalous quantities have been used, the *gravity anomaly* (Δg) and the *gravity disturbance* (δg). Both the *disturbance*, see Eq. (1), and the *anomaly*, see Eq. (2), can be defined either using *actual gravity*, cf. the left-hand sides of Eqs. (1) and (2), respectively – we refer to such a definition as "point-wise definition" – or using the *disturbing potential*, cf. the right-hand sides of Eqs. (1) and (2), respectively

$$\delta g(h,\Omega) \overset{\text{def}}{=} g(h,\Omega) - \gamma(h,\Omega) \cong -\frac{\partial}{\partial h} T(h,\Omega), \tag{1}$$

$$\Delta g(h,\Omega) \overset{\text{def}}{=} g(h,\Omega) - \gamma(h-Z,\Omega) \cong \left\{ -\frac{\partial}{\partial h} - \frac{2}{R} \right\} T(h,\Omega). \tag{2}$$

The definition of the gravity anomaly using the right-hand side of Eq. (2) is known as the *fundamental gravimetric equation* (in spherical approximation, R being the mean earth radius). We refer the positions of points in geodetic (Gauss-ellipsoidal) coordinates that are respective to a geocentric properly oriented equipotential reference ellipsoid (RE), such as GRS'80, where h is height above the RE, and Ω denotes the pair of latitude and longitude. The same RE plays the role of the normal ellipsoid generating normal gravity. Above, Z is the *vertical displacement*, i.e., the separation between the actual and the equivalent normal equipotential surfaces at (h, Ω), given by the *generalized Bruns equation* (e.g., Heiskanen and Moritz, 1967)

$$Z(h,\Omega) = T(h,\Omega)/\gamma(h,\Omega). \tag{3}$$

The two sets of definitions in Eqs. (1) and (2), the left-hand vs. right-hand sides, are not rigorously compatible. They differ by the *effect of the deflection of the vertical* at the order of 10 μgal (e.g., Vaníček et al., 1999, 2004), which is typically negligible in both

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geodetic and geophysical applications. Both sets define the disturbance or anomaly at the observation point located anywhere, not only on the geoid or the topo-surface. The "point-wise definition" is used to compile anomalous gravity from observed gravity, while the definition using the disturbing potential is used when modeling the observed gravity data synthetically based on density, or when computing other parameters of the gravity field based on disturbing potential. The approximate compatibility of the two sets of definitions for both the anomaly and disturbance is vital for both geodetic and geophysical applications in inverting or interpreting observed gravity and in computing the from potential derived parameters of the gravity field.

3.2 Topographically Corrected Anomalous Gravity

Let V^T be the gravitational potential of topographic masses globally enclosed between the relief (topo-surface, earth's surface) and the geoid/quasigeoid ("sea level" as vertical datum), and let A^T be the attraction (the vertical component of the gravitational attraction vector) of those masses. By removing the effect (potential and attraction) of topographic masses from the gravity disturbance and the gravity anomaly, in both the sets of their definitions presented in the previous section, we obtain (e.g., Vajda et al., 2007) the topographically corrected disturbance and anomaly

$$\delta g^T(h,\Omega) \stackrel{\text{def}}{=} \delta g(h,\Omega) - A^T(h,\Omega) \cong -\frac{\partial}{\partial h}[T(h,\Omega) - V^T(h,\Omega)], \tag{4}$$

$$\Delta g^T(h,\Omega) \stackrel{\text{def}}{=} \Delta g(h,\Omega) - A^T(h,\Omega) + SITE(h,\Omega) \cong \cong \left\{-\frac{\partial}{\partial h} - \frac{2}{R}\right\}[T(h,\Omega) - V^T(h,\Omega)], \tag{5}$$

where

$$A^T(h,\Omega) = -\frac{\partial}{\partial h}V^T(h,\Omega) \tag{6}$$

is the attraction of topography, and

$$SITE(h,\Omega) = \frac{2}{R}V^T(h,\Omega) \tag{7}$$

is the *secondary indirect topographic effect* on gravity anomaly (e.g., Vaníček et al., 2004). The two sets (Eqs. 4 and 5) define the topographically corrected gravity disturbances and anomalies, respectively, either based on actual gravity, or based on disturbing potential. Again the approximate compatibility of these two sets is vital for geodetic and geophysical applications, as it mediates the modeling or interpretation of observed topo-corrected anomalous gravity, or the computation of gravity field parameters derived from topo-corrected anomalous gravity data. Notice that the computation of the topo-corrected gravity anomaly (the so-called Bouguer anomaly) from observed gravity involves not only the removal of the attraction of the topography, but also the secondary indirect topographic effect (SITE). The SITE is a long-wavelength (trend-like) signal of the magnitude at the order of 100 mgal in mountainous regions. To illustrate it we show

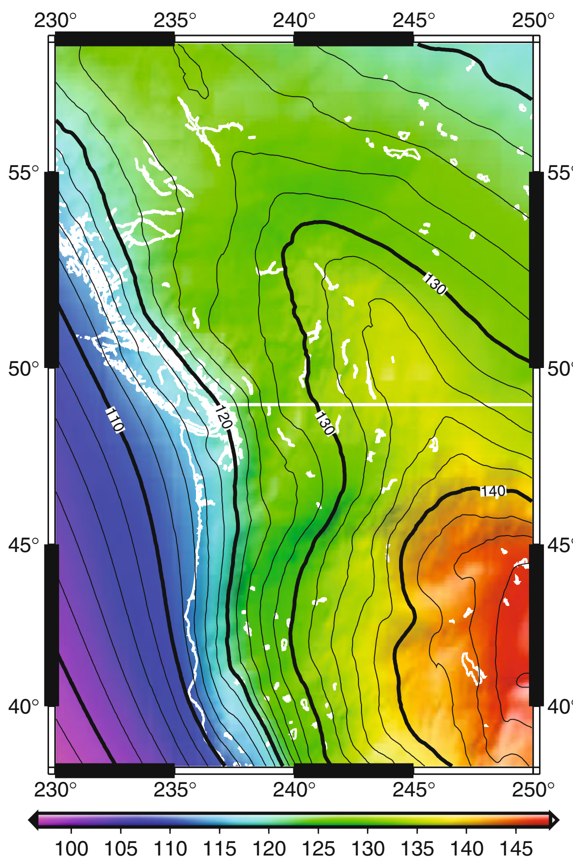

Fig. 3.1 The SITE in the Canadian rocky mountains (mgal). The contour interval is 2 mgal. The magnitude of SITE reaches 150 mgal

SITE computed for the region of Canadian Rockies in Fig. 3.1.

When SITE is not included in the topographic correction in the definition of the topo-corrected gravity anomaly based on actual gravity, then this definition will not be compatible with the definition of the topo-corrected gravity anomaly based on the topo-corrected disturbing potential (based on the fundamental gravimetric equation). In geophysical and geodetic applications the incompatibility will introduce a systematic error equal to the SITE.

To sum it up, when we remove the gravitational potential of topographic masses from disturbing potential, we have to remove from the gravity anomaly the effect of topographic masses, which consists of the attraction and secondary indirect effect of topographic masses. Similarly, if we were to remove from disturbing potential the gravitational potential of the bathymetric density contrast, or of the ice or sediments density contrast, we would have to remove from the gravity anomaly the complete effect of these density contrasts – not only the attraction of these density contrasts, but also the secondary indirect effects of these contrasts.

3.3 Bouguer Anomaly – Ambiguous Definitions

Under the name "Bouguer anomaly" there are currently at least three distinctly different quantities used in the geophysical and geodetic practice. We review and compare them below. For simplicity, we define them at stations $P \equiv [h_T(\Omega),\Omega]$ on the topo-surface. All the topo-corrections (evaluated at P) will be written in a general form, involving numerical integration of the Newtonian volume integrals over the entire globe. The numerical aspects of evaluating them, such as the truncation to a spherical cap (e.g., of the Hayford-Bowie radius, about 1.5 arcdeg), splitting into shell/cap and terrain ("roughness") terms, various approximations to the integral kernel, planar approximation, etc. are of no concern to us here.

The definition of the *Bouguer anomaly* in its *classical sense*, e.g., according to Heiskanen and Moritz (1967), denoted here by superscript "CB" (standing for "Classical Bouguer"), reads

$$\Delta g^{CB}(P) \overset{\text{def}}{=} g(P) - \gamma(P^*) - A^T(P), \qquad (8)$$

where the normal gravity is evaluated at the point (P^*) which is the vertical projection of the observation point P onto the telluroid, and where A^T is the attraction of the global topography, i.e., of masses contained between the sea level (geoid/quasigeoid) and the topo-surface. The *Bouguer anomaly* in its classical sense lacks the SITE (Fig. 3.1). Therefore, the Δg^{CB} is not a "topo-corrected gravity anomaly" in a rigorous sense. This anomaly is not well suited for geophysical interpretation of gravity data either, because normal gravity is not evaluated at the station P, and because the topo-masses in the topo-correction are defined with the geoid rather than RE as their lower boundary (cf. Vajda et al., 2006). When this gravity anomaly is used in geodetic applications, the SITE must be added to have a complete effect of the topo-masses (cf. Vaníček et al., 2004). The definition of the *Bouguer anomaly* in accord with the rigor, denoted here simply by superscript "B" (standing for "Bouguer"), reads (e.g., Vaníček et al., 2004, Eq. [53])

$$\Delta g^{B}(P) \overset{\text{def}}{=} g(P) - \gamma(P^*) - A^T(P) + SITE(P). \qquad (9)$$

Compared to the classical definition, it contains also the required SITE,

$$\Delta g^{B}(P) = \Delta g^{CB}(P) + SITE(P).$$

In literature the adjective "Bouguer" is sometimes replaced by synonyms "No Topography" ("NT") or "geoid-generated" and the superscript "B" by superscripts "NT" or "g", respectively. This "Bouguer anomaly" finds its practical use in geodesy, when solving the *boundary value problem* defined with sea level as the boundary, in computing the geoid, either via the "NT space" (e.g., Vaníček et al., 2004) or via the "Helmert space" (e.g., Vaníček et al., 1999), using the Stokes-Helmert scheme, where Bouguer anomalies are transformed into Helmert anomalies (ibid). The Bouguer anomaly defined by Eq. (9) is not suitable for geophysical applications for the same reasons as that defined by Eq. (8). It has been recognized (Hinze et al., 2005; Vajda et al., 2006; and references therein) that geophysical interpretation of gravity requires the normal gravity to be evaluated at the observation point (station) and the topographic masses to be defined (for the sake of the topo-correction) with the RE (replacing sea level) as their bottom boundary.

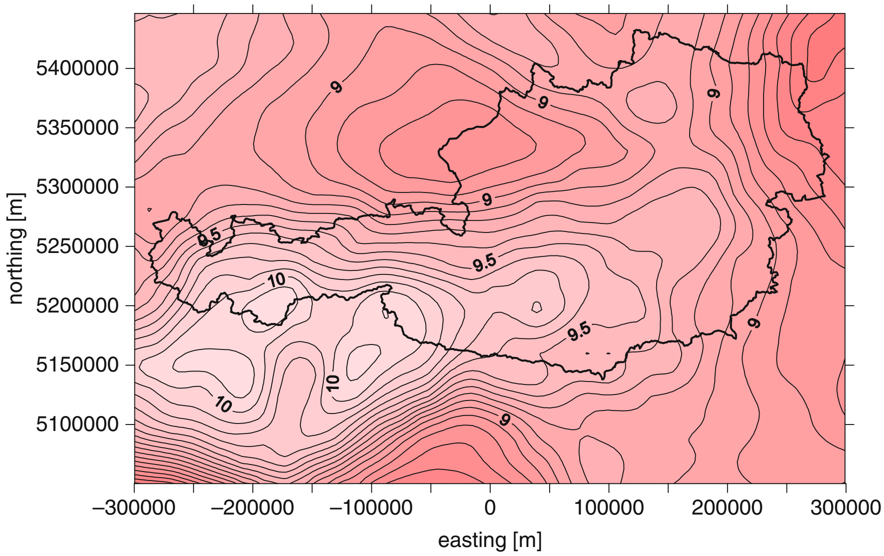

Fig. 3.2 The geophysical indirect effect (mgal) for the area of the Eastern Alps (Austria). It reaches the magnitude of 10 mgal. It is of long-wavelength ("regional trend") nature

Hinze et al. (2005) proposed a new definition of the "Bouguer anomaly" to meet the geophysical requirements. The definition of the *Bouguer anomaly* according to the newly proposed standards for gravity databases (Hinze et al., 2005), denoted here by superscript "NB" (standing for "New Bouguer"), while the new standards propose a term "Bouguer gravity anomaly", optionally to be preceded by the adjective "ellipsoidal", reads

$$\Delta g^{NB}(P) \stackrel{\text{def}}{=} g(P) - \gamma(P) - A^{ET}(P). \qquad (10)$$

This definition of "Bouguer anomaly" differs from the classical definition (Eq. (8)) in two aspects: (1) normal gravity is evaluated at the observation point (station), (2) the *reference ellipsoid* replaces the *sea level* as the lower boundary of topo-masses in the topo-correction, hence the superscript "ET" replacing "T". There is only one problem with this definition – that of terminology. The quantity defined by Eq. (10) is by the standards of the theory of the gravity field, which applies equally well to both geodesy and geophysics, the "topo-corrected gravity disturbance" (Vajda et al., 2006, 2007).

The difference between the "classical" and the "new" Bouguer anomalies amounts to the so-called *geophysical indirect effect* (GIE),

$$\Delta g^{NB}(P) = \Delta g^{CB}(P) + GIE(P),$$

which consists of two terms. One accounts for the difference between normal gravity at the topo-surface and that at the telluroid (between P and P^*). The other accounts for the attraction of masses globally enclosed between the RE and the geoid/quasigeoid (Vajda et al., 2006; and references therein). To illustrate the *geophysical indirect effect* we show it computed for the region of the Eastern Alps (Austria) in Fig. 3.2.

3.4 Gravity Data – Geophysical Versus Geodetic Applications

Just to illustrate that the demands on defining, compiling and using gravity data may differ between geophysical and geodetic applications, we compare (cf. Table 3.1) two examples: A geodetic application being represented by the boundary value problem (BVP) for geoid computation using the Stokes approach and the geoid as the boundary, and a geophysical application represented by interpretation or inversion of gravity.

The Stokes approach requires the use of a gravity anomaly, as opposed to the Hotine approach, which requires the gravity disturbance. The BVP requires harmonicity above the boundary, hence no topo-masses above the geoid. That implies the geoid as the lower boundary (bottom interface) of the topo-masses in the topo-correction and no need of a bathymetric

Table 3.1 Comparison between a geodetic and a geophysical application

Task (an example)	Geodetic application geoid computation, BVP based on geoid and Stokes approach	Geophysical application gravity interpretation or inversion (Vajda et al., 2006, 2008)
Anomalous gravity	Gravity *anomaly*	Gravity *disturbance*
Topographic correction	Attraction of topo-masses and SITE	Attraction of topo-masses
Lower boundary of topo-masses	Geoid/quasigeoid ("sea level")	Reference ellipsoid (RE)
Topographic density in topo-correction	Real	Reference (e.g., constant, average crustal) for solid topography onshore, that of water for liquid topography offshore
	NO need	YES, based on the RE

correction. The use of gravity anomaly implies that the topo-correction consist of two terms, the attraction of topo-masses and the SITE (cf. Sect. 3.2).

The geophysical interpretation or inversion of gravity data, by means of direct inversion, forward modeling, or pattern recognition, etc. (e.g., Blakely, 1995), requires that the gravity data exactly equal (match) the attraction of the anomalous masses being sought. It has been proved (Vajda et al., 2006, 2008) that such a requirement is satisfied by the bathymetrically and topographically corrected gravity disturbance (the so-called "BT disturbance"). In geophysical practice the "BT disturbance" has been and unfortunately remains to be called the "Bouguer anomaly" (Hinze et al., 2005).

3.5 Conclusion

The ambiguous terminology regarding the "Bouguer anomaly" could cause confusion when using the Bouguer gravity anomaly databases by geodesists and geophysicists, as the two groups would anticipate different quantities under the same name, as shown in Sects. 3.3 and 3.4. There is but one gravity field of the earth with its theory, and it is an inseparable part of the research and applications in both the geophysics and geodesy. These two disciplines overlap and therefore there is a great need not only for common standards but also for common terminology. We propose to refer to the "Bouguer anomaly" needed in geophysics, defined by Eq. (10), correctly as the *topographically corrected gravity disturbance*. When a bathymetric correction is applied in addition to the topographic correction, we propose to call it the *BT disturbance* (cf. Vajda et al., 2008).

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