Calculation of the spherical terrain correction to Helmert’s orthometric height

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Basic concepts, principles and definitions:

Definition of Orthometric Height: \( H^O = C(T) - \frac{1}{\gamma(T)} \)

Helm’s Mean Gravity: \( \gamma(T) = \sum G \left( \int dV + \gamma'(\Omega) \right) \)

Rigorous Mean Gravity: \( \gamma(T) = \frac{1}{G} \int \left( \frac{\partial^2 \Phi}{\partial r^2} \right) dV + \gamma'(\Omega) \)

Effect of terrain roughness on gravity at a point:

Integration of terrain mass around computation point:

\[ g^W(r) = G \oint_{\partial V} \frac{\partial^2 \Phi}{\partial r^2} dV \] (Martinec 1998)

Effect of terrain roughness on mean gravity:

This effect may be computed using three methods: Mader (1954), Niethammer (1932) and U.N.B. The Mader method approximates the mean value using an average of gravity effect values calculated from parts of the integration area below the spherical Bouguer shell. The Niethammer method computes the surface and geoid values of the effect on gravitational potential and divides by orthometric height, yielding an exact solution.

Niethammer (1932) Method:

\[ \gamma(T) = \gamma(T) + \sum G \left( \int dV + \gamma'(\Omega) \right) \]

Mader (1954) Method:

\[ g^W(r) = G \oint_{\partial V} \frac{\partial^2 \Phi}{\partial r^2} dV \]

U.N.B. Method:

\[ g^W(r) = G \oint_{\partial V} \frac{\partial^2 \Phi}{\partial r^2} dV \]

Correction to Helmert orthometric height:

\[ \text{Correction to Helmert mean gravity: } \text{corr}_{T} = \frac{\text{corr}_{W} \times \gamma(T)}{G} \]

\[ \text{Correction to Helmert orthometric height: } \text{corr}_{H} = \frac{\text{corr}_{W} \times H^O}{G} \]

Necessary DTM Data:

\[ \begin{array}{c|c|c|c|c|c} \text{Method} & \text{Calculation Time (Approximate)} \\ \hline \text{Neithammer (100m)} & \approx 20h \\ \text{Mader} & \approx 2h \\ \text{U.N.B.} & \approx 25h \\ \hline \end{array} \]

Conclusions:

1. The correction to Helmert orthometric height resulting from terrain roughness may be as great as 26cm in areas of very rough terrain. Generally, however, it is small – around 1cm in the test area. Though the correction may sometimes be negative, negative values are quite small.

2. The correction is best computed using the U.N.B. method, which yields an exact result, and is much faster than the Niethammer method, which yields comparably good results.

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