MEAN GEOID-GENERATED GRAVITY DISTURBANCE ALONG PLUMBLINE

Beth-Anne Martin [1], Chris MacPhee [2], Robert Tenzer [1], Petr Vaníček [1] and Marcelo Santos [1]

[1] University of New Brunswick, Department of Geodesy and Geomatics Engineering, Fredericton, N.B., E3B 5A3, Canada

[2] Advanced Computational Research Laboratory, Faculty of Computer Science, University of New Brunswick, Fredericton, N.B., E3B 5A3, Canada

Abstract

In this contribution, some numerical aspects of computing the mean value of geoid-generated gravity disturbance along the plumbline within the topography are discussed.

Introduction

The mean value of gravity along the plumbline between the geoid and the Earth surface depends on the mass density distribution within the topography, the shape of the Earth surface and the vertical change of gravity generated by the geoid. The mean gravity is evaluated as the sum of the mean value of gravity generated by the Earth mass within the geoid and the mean value of topography-generated gravitational attraction. The geoid-generated gravity is further divided into the normal gravity and the geoid-generated gravity disturbance, i.e., the gravity disturbance in the No Topography space (Vaníček at al., 2003a). By removing the gravitational attraction of the topographical masses from the gravity generated by the whole Earth, the gravitational field becomes harmonic above the geoid. Therefore, the mean value of the geoid-generated gravity disturbance can be evaluated by solving Dirichlet's boundary value problem.

Mean gravity along plumbline

According the definition of "integral mean" the "*mean value of gravity along the plumbline between the geoid and the earth surface*" reads (e.g., Heiskanen and Moritz, 1967, Eq. 4-20)

$$\bar{g}(\Omega) \cong \frac{1}{H^{O}(\Omega)} \int_{r=r_g(\Omega)}^{r_g(\Omega)+H^{O}(\Omega)} g(r,\Omega)\,\mathrm{d}r \tag{1}$$

where $g(r,\Omega)$ is the gravity at a point of geocentric position $(r,\Omega) \equiv (r,\phi,\lambda)$, and the geocentric radius $r_t(\Omega)$ of the Earth surface is given (with an accuracy of a few millimeters) by the geocentric radius of the geoid $r_g(\Omega)$ plus the orthometric height $H^{O}(\Omega)$.

The gravity can be decomposed as

$$g(r,\Omega) \cong \gamma(r,\phi) + \boxed{\delta g^{\mathrm{NT}}(r,\Omega)} + g^{\mathrm{T}}(r,\Omega) \tag{2}$$

In this paper, we shall concentrate on the evaluation of the second term only. The evaluation of the first term is relatively standard in geodesy, while the evaluation of the third term is, among other things, discussed in the paper (Santos et al., 2003) presented at this conference.

Mean geoid-generated gravity disturbance along plumbline

The "*mean value of geoid-generated gravity disturbance along the plumbline between the geoid and the Earth surface*" is given by the second term on the right-hand-side of Eq. (3)

$$\overline{\delta g}^{\mathrm{NT}}(\Omega) = \frac{1}{H^{O}(\Omega)} \int_{r=r_g}^{r_g+H^{O}(\Omega)} \delta g^{\mathrm{NT}}(r,\Omega)\,\mathrm{d}r \cong \frac{1}{H^{O}(\Omega)} \int_{r=\mathrm{R}}^{\mathrm{R}+H^{O}(\Omega)} \delta g^{\mathrm{NT}}(r,\Omega)\,\mathrm{d}r \tag{3}$$

Since the geoid-generated gravity disturbance multiplied by *r* is harmonic above the geoid (really co-geoid, to be more precise), the mean gravity disturbance can be evaluated by **averaging the Poisson's integral** (e.g., Kellogg, 1929) .

Performing the radial integration of Poisson's integral kernel $\mathrm{K}[r,\psi(\Omega,\Omega'),\mathrm{R}]$ multiplied by 1/*r*, the following expression can be found for the averaging of Poisson's kernel (Vaníček at al., 2003c)

$$\int_{r=\mathrm{R}}^{\mathrm{R}+H^{O}(\Omega)} \frac{1}{r}\mathrm{K}[r,\psi(\Omega,\Omega'),\mathrm{R}]\,\mathrm{d}r = \mathrm{R}\int_{r=\mathrm{R}}^{\mathrm{R}+H^{O}(\Omega)} \frac{1}{r}\frac{r^2-\mathrm{R}^2}{l^3[r,\psi(\Omega,\Omega'),\mathrm{R}]}\,\mathrm{d}r = \left| -\frac{2\mathrm{R}}{l[r,\psi(\Omega,\Omega'),\mathrm{R}]} + \ln\left|\frac{\mathrm{R}-r\cos\psi(\Omega,\Omega')+l[r,\psi(\Omega,\Omega'),\mathrm{R}]}{r\sin\psi(\Omega,\Omega')}\right| \right|_{r=\mathrm{R}}^{\mathrm{R}+H^{O}(\Omega)} \tag{4}$$

The mean gravity disturbance along the plumbline takes the following form (Vaníček at al., 2003c)

$$\overline{\delta g}^{\mathrm{NT}}(\Omega) = \frac{1}{4\pi}\frac{\mathrm{R}}{H^{O}(\Omega)} \iint_{\Omega'\in\Omega_0} \left[\frac{2\mathrm{R}}{l[\mathrm{R},\psi(\Omega,\Omega'),\mathrm{R}]} - \frac{2\mathrm{R}}{l[r_t(\Omega),\psi(\Omega,\Omega'),\mathrm{R}]} + \ln\left| \frac{\mathrm{R}}{r_t(\Omega)} \left(\frac{\mathrm{R}-r_t(\Omega)\cos\psi(\Omega,\Omega')+l[r_t(\Omega),\psi(\Omega,\Omega'),\mathrm{R}]}{\mathrm{R}(1-\cos\psi(\Omega,\Omega'))+l[\mathrm{R},\psi(\Omega,\Omega'),\mathrm{R}]} \right) \right| \right] \delta g^{\mathrm{NT}}(\mathrm{R},\Omega')\,\mathrm{d}\Omega' \tag{5}$$

Computational considerations and numerical results

The surface integration in Eq. (5) has to be carried over the entire Earth. It has been divided into the near-zone domain and the far-zone domain integration. The near-zone contribution to the mean geoid-generated gravity disturbance is computed by the numerical integration of an integral shown in Eq. (5), while the far-zone contribution can be evaluated from the global geopotential model. In this contribution we discuss only the evaluation of the near-zone contribution. Brief tests have shown that the far-zone contribution to the mean values of the geoid-generated gravity disturbance is 3 orders of magnitude smaller than the near-zone contribution.

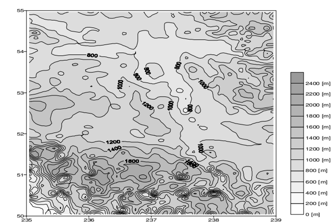

Fig. 1 Topography in the testing area of the Canadian Rocky Mountains

These gravity disturbances were then convolved numerically with the **averaging Poisson's integration kernel** over the near zone, which was selected to be a spherical cap of radius of 5 arc-degrees. The numerical results of the computed near-zone contribution to the mean geoid-generated gravity disturbances in the part of the Canadian Rocky Mountains $\left(\phi\in\langle 50^{\circ},55^{\circ}\rangle, \lambda\in\langle 235^{\circ},239^{\circ}\rangle\right)$ that we have used for our test, are shown in Fig. 2. They range between –169.2 mGal and 128.2 mGal (average –14.6 mGal).

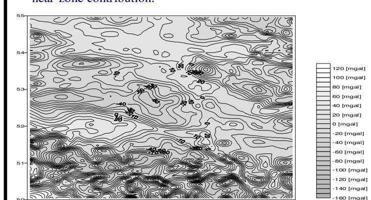

Fig. 2. Mean geoid-generated gravity disturbances

The mean values of the geoid-generated gravity disturbances are interesting from both the geodetic as well as geophysical point of view. Their main geodetic application is as part of the corrections to Helmert's orthometric heights (Santos et al., 2003), where the actual value of mean gravity along the plumbline is needed. It can be seen from Eq. (2) that the geoid-generated field is one part of the real gravity field needed in that correction. The correction to Helmert's orthometric height due to the near-zone contribution to the mean geoid-generated gravity disturbance along the plumbline is shown in Fig. 3; it varies between –3.4 cm and 7.9 cm with an average of 0.1 cm. The correction due to the far-zone contribution is totally negligible. Fig. 3 shows one part of the correction to Helmert's orthometric height. The other part is the Terrain correction which is discussed in Santos et al. 2003.

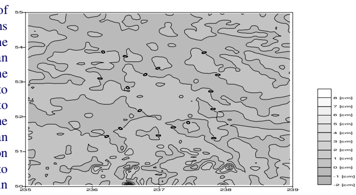

Fig. 3: Correction to Helmert's orthometric height due to the mean geoid-generated gravity disturbance along the plumbline.

Clearly, from the numerical investigation shown in Fig. 3, applying the correction due to mean geoid-generated gravity disturbance to Helmert's orthometric heights improves the accuracy of these orthometric heights significantly.

Interpretations

The mean gravity disturbances reflect the mass distribution below the geoid level (i.e., sea level) and also the topography because they are referred to the approximate mid-point between the geoid and the Earth surface. It has been shown that the gravity values at the geoid are not correlated with the topography, while the mean value of gravity is correlated with the topography.

In this contribution, we have attempted to show that it is not only possible but also feasible to compute that part of the real gravity field whose origin is in the masses below the sea level. When the part of the field whose origin is within topography is added to the here-discussed part, it is possible to reconstruct the actual field within topography with accuracy adequate for many applications.

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