Does a navigation algorithm
have to use Kalman filter?

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Abstract

Complications are known to arise when Kalman filter is used in the design of navigation algorithms. Many improvements have been introduced over the past thirty years. Still, problems seem to occur when navigation based on a single source data is attempted. Such navigation is then somewhat dependable upon the knowledge of the vehicle's dynamics model. This knowledge is almost never available at a level to satisfy the basic reliability requirements.

We seek a solution through an approach different from that of Kalman filtering in so far that no physical modelling is called for. We use a purely mathematical approach based on the laws of mechanics (e.g., Hamilton's principle), applied in a statistical space. The result is a new navigation algorithm (NNA) that relies solely upon the measurements of vehicle's positions (and possibly velocities), and their error statistics. The force field in which non-Newtonian forces are at work, is induced simply by confidence regions of the position fixes. It is assumed that instantaneous position fixes and their corresponding error statistics can be obtained by one or more navigation services, such as GPS or Loran C . One promising application of the NNA that will be investigated in the near future is the tracking of evasive targets based on position and/or velocity measurements. The results of the new algorithm are presented for a couple of real-life trajectories and a comparison with the best Kalman filter navigation is shown.

1   Introduction

Since the early 1960's, modern navigation has been making use of the hybrid (integrated) navigation systems, where various electronic sensing devices (sensors) are used side by side, to collect the information necessary to find the "continuous" position of the navigated vehicle. These systems are installed on-board of vehicles, such as ships, aircraft, or missiles. Sensors that are being used in such systems are the Inertial Navigation Systems (INS), radio-navigation aids (LORAN, GPS, etc.), Doppler Velocity Sensors (DVS), laser-ranging devices, barometric altitude-meters, etc.

The potential benefits from multi-sensor system implementation lead to a development of mathematical algorithms for merging the data from various sources (sensors). The automation of continuous positioning, and the decrease of direct human involvement in the positioning process became important considerations. These developments lead to a new general concept of navigation: the computational solutions were sought in algorithms that make use of filtering. At the beginning of the sixties, a Hungarian (electrical) engineer Rudolf Kalman introduced a new kind of filtering, where the observations relating to the motion of the vehicle, as well as a dynamic model derived from some physical laws, were required. Consequently, the success of

this filter to predict the correct position depends to a large extent on the validity of both kinds of information.

Although initially introduced as a solution to electrical engineering problems, the Kalman filter (KF) found early applications in navigation [Antoulas, 1991]. The reason for this stemmed from the fact that the method was compatible with the emerging digital computing technology. Monitoring of the vehicle movement presented no real problem, but the second requirement of Kalman filter, the physical modeling, was indeed the major reason for concern. Nevertheless, the benefits were thought to outweigh the drawbacks, the industry went spontaneously for the new concept, and the drawbacks were soon forgotten.

Almost forty years after it was first introduced, the most widely applied navigation algorithms are still based on Kalman filter. Here, we give several examples that highlight the desirability of replacing Kalman filter in the navigation systems used in marine vessels, aircraft and missiles, by a more appropriate algorithm. We will discuss these examples as they also serve the purpose of putting the new navigation algorithm in the proper context:

1) As a part of aerodynamics in military applications synthesis, it has been known for some time that the development of highly agile aircraft requires an understanding of the unsteady flow associated with these aircraft. The a priori aerodynamic force and moment input used for the design of these aircraft are studied by dynamic experiments in wind tunnels. Such experiments include both oscillatory and rotary techniques capable of testing at extreme attitudes and/or high angular rate motions. These motions represent a highly variable and **unpredictable dynamic environment**.

2) In the context of geodesy and geomatics, a vehicle or missile, equipped with the INS/GPS navigation device(s) [Coffee & Maganty, 1996] requires a specific algorithm (filter) to perform the integration of an Inertial Navigation System (INS) with the Global Positioning System (GPS). A need for such integration is well documented by, e.g., Kleusberg [1988]: it should be performed so as to allow an at-all-times reliable navigation solution. For instance, the simple navigation filters built-in the GPS receivers are dependent on the vehicle dynamics model when GPS alone is used for navigation purposes—they require model set/reset input from the pilot/navigator as soon as a **significant state change** has occurred. Another source of problems lays in the unavoidable loss of the GPS signal, due to maneuvering (masking). A model-less algorithm, capable of self-following and guidance of the vehicle under all-conditions maneuvering is then preferable.

3) In marine navigation, the problems appear when trying to model tidal and other ocean currents, or the atmospheric variations. Here the difficulty in applying the Kalman filter lays in the fact that it requires an a-priori system model to be given; in marine navigation any such **model can never be realistic**. An algorithm that will do the self-following and guidance of the vessel under all-conditions maneuvering is thus required.

4) When employed in a combat situation, a fighter jet's navigation system based on Kalman filter does not perform very well if input data are based on position/velocity measurements. One of the reasons for this is that at such time the aircraft navigation system suffers the "memory loss", i.e., the model dependent filter **needs a new model** to cope with the fast developing situation. Such a model is, of course, not available and this results in a navigation failure. A model-less filter, capable of self-following and guidance under all-conditions maneuvering is needed.

5) In (military) industry, the **need to re-design** algorithms based on the Kalman filter, to run efficiently on every new platform (application) can be very costly. An adaptable, platform-independent algorithm could be highly desirable.

6) Tracking a target that employs evasive maneuvering could be very unreliable when the tracking algorithm is based on a Kalman filter. Particularly so, if the input for the algorithm consists only of position and/or velocity measurements.

2   Mathematical background of the new navigation algorithm

It is known from dynamics that the motion of a system in which all forces (except those derived from constraints) can be derived from a single generalized potential function that depends on coordinates, velocities and time. This concept is implied by, e.g., the Hamilton's principle [Boccaletti & Pucacco, 1996]. We define the generalized potential function in such a way that it can be constructed solely from the position fixes and their confidence regions, and call it the **position potential**. We assume that instantaneous position fixes and the corresponding error statistics for the navigated vehicle are obtained by one or more navigation services, such as GPS or Loran C.

Adopting the above concept, in its kinematic form, the acceleration is given by:

$$a_i = \ddot{\mathbf{r}}_i = \frac{\partial U}{\partial \mathbf{r}_i}, \tag{1}$$

where U is the potential of the force field we want the navigated vehicle to obey. The navigated vehicle is then represented simply by a mass particle. The notion of "position potential" enables us to define that force field which drives the free particle along its trajectory.

Conceptually, the solution uses the force field in which non-Newtonian forces (i.e., forces that are **not** inversely proportionate to squared distances) are induced by the confidence regions – called here also force sources – that pull the (undetermined) trajectory of the particle (*Fig*.1) to pass through them. In fact, the solution is built on a time-varying position potential field U, defined as a summation of n individual position potential fields defined by the quadratic forms of the confidence regions. Thus

$$U(\mathbf{r}, t; \alpha, G) = Ge^{-\alpha t} \sum_{i=1}^{n} (\mathbf{r} - \mathbf{r}_i^0)^{\mathrm{T}} \mathbf{C}_i^{-1} (\mathbf{r} - \mathbf{r}_i^0) \cdot e^{\alpha t_i}, \quad t \geq t_n \tag{2}$$

with

$$\mathbf{r} = \left\| \begin{matrix} x \\ y \end{matrix} \right\| \quad \text{and} \quad \mathbf{r}_i^0 = \left\| \begin{matrix} x_0 \\ y_0 \end{matrix} \right\|, \tag{3}$$

being the position vectors (here two-dimensional, but can be extended into three dimensions) of the particle and of the i-th position fix, respectively, both considered at time t_i. Here G is a positive scale factor, analogous to the gravitational constant in the Newtonian mechanics, and α is the temporal attenuation coefficient. Both numbers, G and α, can be either selected beforehand, or evaluated during the navigation process from the fit of the predicted trajectory to new position fixes, following one of many possible schemes. Different applications may require different approaches to the selection and updating of these two parameters to ensure the optimum performance of the algorithm. So far only a few simple approaches have been tried on the trajectories we had at our disposal [Xu, 1996]. Further investigation is clearly indicated here.

The equation of motion is then obtained from *eqn*.(2) by differentiation with respect to $\mathbf{r}_i$:

$$\frac{\partial U}{\partial \mathbf{r}_i} = \ddot{\mathbf{r}}_i = -e^{-\alpha t}(A\,\mathbf{r}_i - B) \tag{4}$$

where

$$\left.\begin{aligned}
A &= 2G \cdot \sum_{i=1}^{n} e^{\alpha(t_i - t_n)} \mathbf{C}_i^{-1} \\
B &= 2G \cdot \sum_{i=1}^{n} e^{\alpha(t_i - t_n)} \mathbf{C}_i^{-1} \mathbf{r}_i^{\,0}
\end{aligned}\right\}. \tag{5}$$

For $t \geq 0$, this happens to be the second order Bessel differential equation of order 0 [Xu, 1996], the solution ($\mathbf{r}_i$) of which is given by Bessel functions of first kind. For more information on the mathematical background and previous work on the new algorithm, the interested reader is referred to Xu (1996).

3   Summary of test results

Tests made with the NNA consisted of examining its performance on both synthetic and real two dimensional trajectories, while considering various schemes for determining the parameters G and α. The best performing scheme turned out to be a pre-selection of a specific value of α, suitable for the kind of tested motion accompanied by continuous updating of the G value based on the "past experience" of the algorithm. The results were then compared with the best Kalman filter (KF) navigation.

As shown in *Fig*.3, the NNA can indeed be regarded as a filtering technique. Clearly, the most accurate results are obtained when the algorithm is used in its "smoothing mode", while the "estimation mode" gives worse results, and the "prediction mode" performs the worst. The NNA represents also a capable blunder detector - it picked up two artificially implanted blunders, at the 0.001 confidence level.

A comparison of the performance of the NNA with that of the KF was done over two real ship trajectories. The NNA performed, on average, three to four times better than the KF, i.e., with an improvement of 60 to 80%. The results are shown in *Fig*.2 and summed up in *Table* 1. We note that the speed of processing by the NNA is slower than that by the KF, by a factor 5 to 10. This drawback of the NNA could be alleviated by using parallel processing or some other advanced computing methodology which were not at our disposal during the testing.

Table 1: comparative results of KF and NNA tests – speed vs. accuracy

Test results from	Time needed for processing	Max. position error	Average position error
Kalman filter	0.02 sec	8 m	3 m
New Navigation Algorithm	0.11, 0.23 sec	2 m	1 m

To summarize, the NNA has the following advantages. It does not require any model of vehicle's dynamic environment, and, consequently, it does not depend on the platform it is mounted on; it suffers from the phenomenon of "overshooting" much less than the KF-based algorithm does; as Kalman filter, NNA can also be used for detecting blunders in position fixes; and can be easily modified to accept also velocity information. Its disadvantage is in its slower processing speed.

4 Discussion

At its present form, the New Navigation Algorithm (NNA) is a 2-D self-learning algorithm (G, and possibly α, are being continuously updated), flexible and responsive to a changing dynamic environment, with minimal overshooting (*Fig*.2). It represents a unique, platform-independent navigation tool, capable of multi-sensor information integration. Tested with both simulated and real navigation data, it proved to be a valid estimator, predictor, smoother and blunder detector (*Fig*.3). Compared with the Kalman filter, the new algorithm requires the uncertainties of observations to be known only relatively, as cofactor matrices. Further, the NNA allows a direct input and real time integration of position and/or velocity measurements (taken at the vehicle, i.e., it is independent of the ground control or navigator's input) to speed up the response of the algorithm to changing dynamics.

The new filter "learns" (i.e. updates the model parameters regularly) all the time, hence it can keep pace with the kinematic change of the vehicle and thus achieve better results. It appears that the use of this algorithm would be more appropriate than that of the Kalman filter for navigating vehicles that are expected to maneuver under their own power. The inclusion of velocity observations in this algorithm results in a more accurate navigation than that using either observed positions alone, or velocities alone (dead reckoning). The performance of the new algorithm appears to be sturdy enough to detect possible blunders in position fixes. Compared with the Kalman filter for the vehicle under maneuvers, the new algorithm achieves better results, but its processing speed is slower.

5 Conclusions

The introduction of the new navigation algorithm would have positive consequences for navigation. These consequences are:

- increased safety, as the new algorithm would result in a navigation that is stable, safe and reliable under all conditions, and

- increased economy, as the algorithm is platform independent.

So far, the algorithm is formulated only for two-dimensional navigation, but an expansion into three dimensions can be achieved relatively easily so that its application in aviation can be realistically contemplated. Further improvement of the new algorithm can be expected from a selection of an optimal "position potential function" (to replace the generic "position potential

function" used in our investigations so far), which requires more experimental testing. Because of algorithm's high flexibility, it is possible to envision an improvement to its self-learning capabilities.

References

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Figures

Figure 1 -   History of trajectory evolution.
Figure 2 -   Comparison between the NNA and the KF test results.
Figure 3 -   Time history of actual position errors for one of the real ship trajectories.

t₁

t₂

t₃

t₄

- - - trajectory

confidence
region of a
position fix (center)

t_o time instant
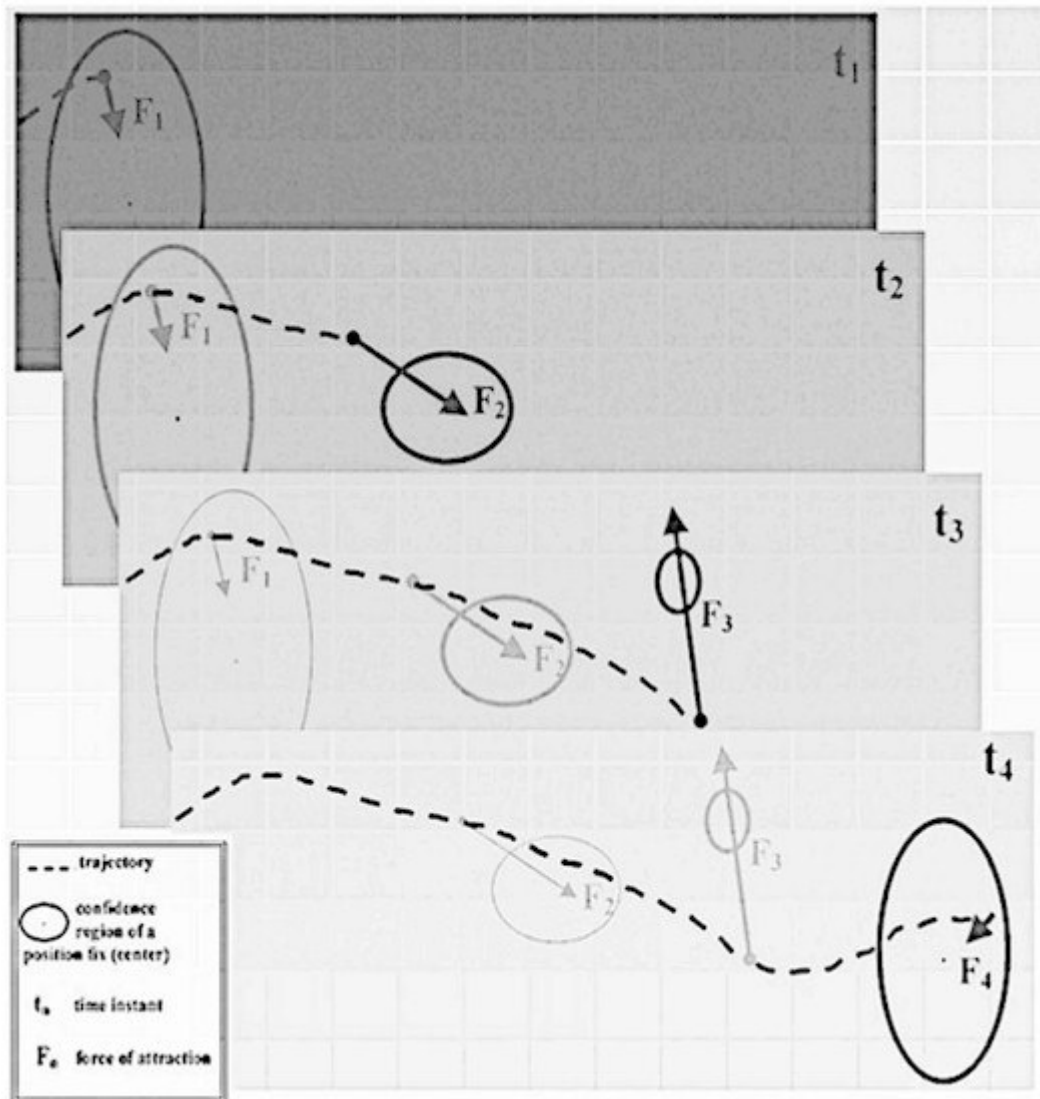
F_o force of attraction

F_1

F_2

F_3

F_4

Figure 1: History of trajectory evolution: confidence regions (error ellipses) of position fixes tend to attract the undetermined trajectory to pass through them. The impact of the force sources diminishes with time (as the trajectory evolves). The position fix with smaller confidence region (the more accurate fix) attracts more than a fix with larger confidence region. The closer the position fix to particle position (at the time of the fix) the weaker the attracting force.
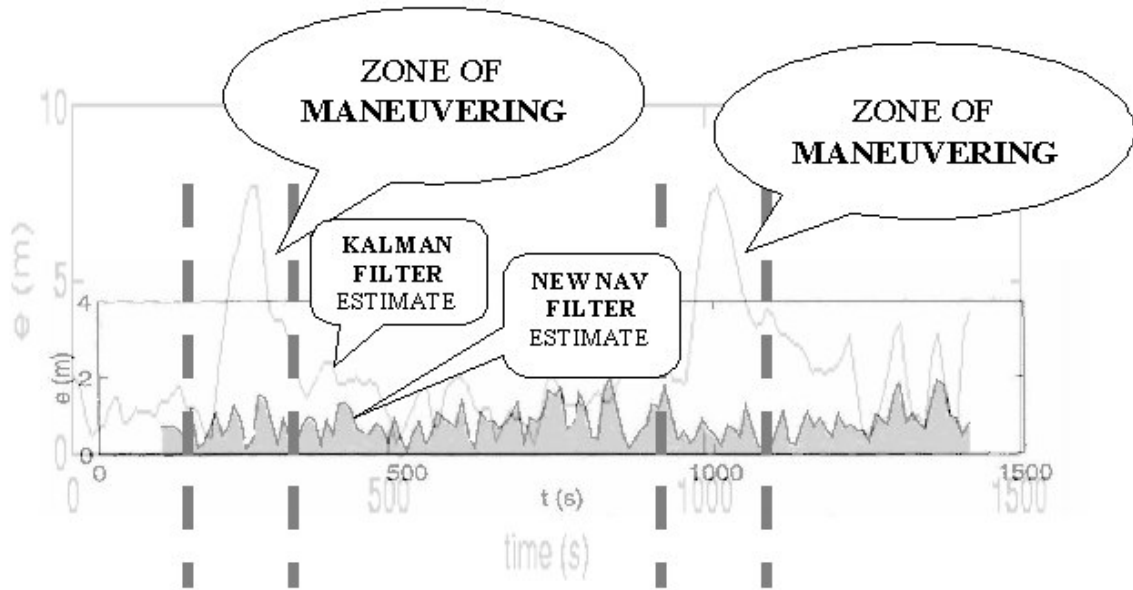

Figure 2: Comparison between the new navigation algorithm and the Kalman filter test results (approximately scaled)
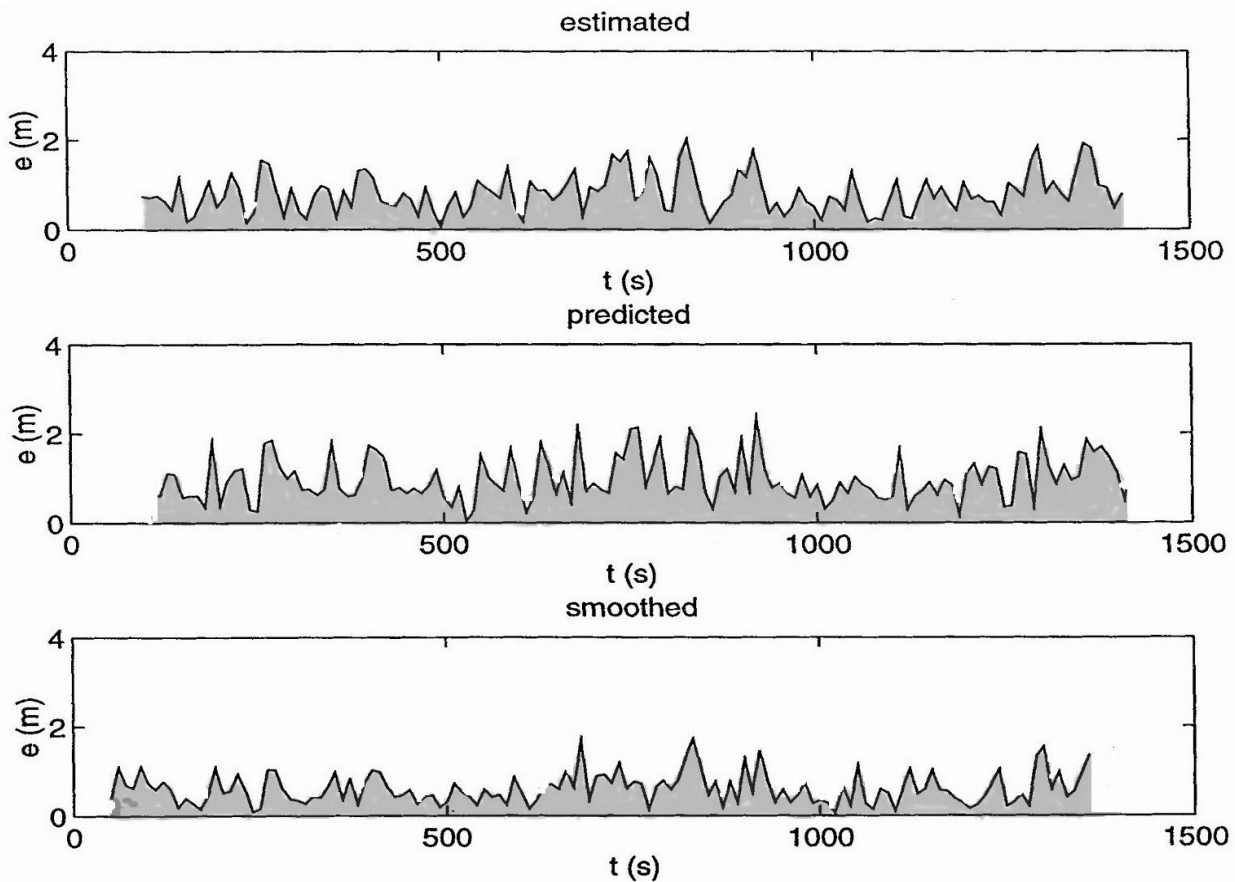
Figure 3: Time history of actual position errors for one of the real ship trajectories