An Introduction To The Stokes-Helmert's Method For Precise Geoid Determination

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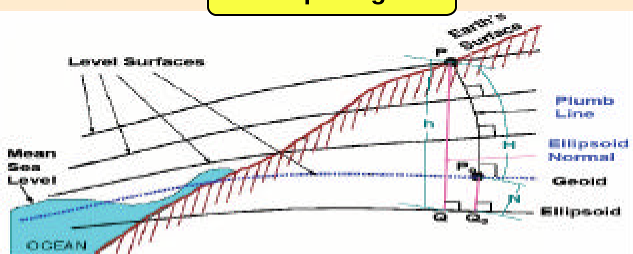
Introduction

The geoid plays a very important role in geodesy. It can not only be seen as the most natural shape of the earth, but it also serves as the reference surface for most of the height system. Geoid is the equipotential surface of the Earth gravity field that best approximates the mean sea level. Such a reference surface is needed for a number of modern mapping, oceanographic and geophysical applications.

Concept of geoid



Level Surface = Equipotential Surface,
N (geoid height) = Distance along ellipsoid normal (P_0 to Q_0),
H (orthometric height) = Distance along plumb line (P to P_0),
h (gedetic height) = Distance along ellipsoid normal (P to Q),
These quantities are related by the expression H = h-N.

Methods to compute the geoid

- Geometric method (GPS/leveling)

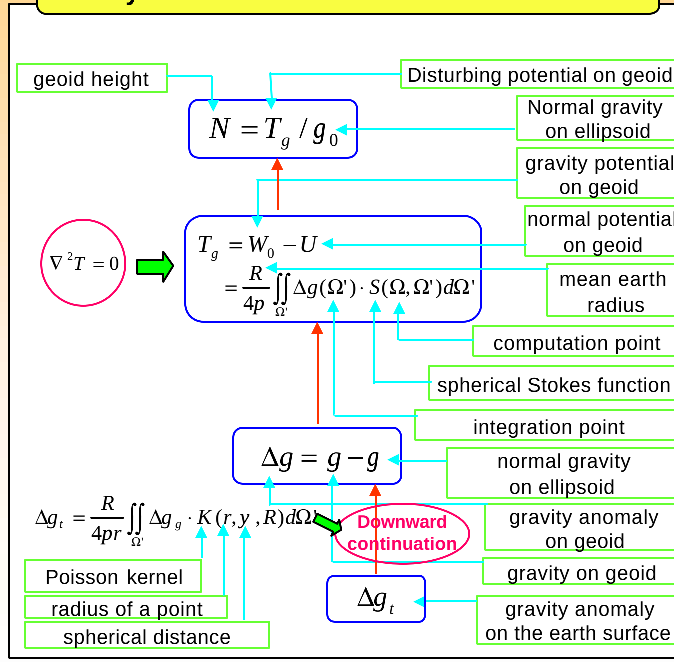
The simplest method is to use GPS/leveling points, where both the geodetic and orthometric heights are given. From these data the point geoid height can be calculated with a simple subtraction. Orthometric heights can be derived from a surveying technique called "leveling".

Unfortunately this solution can not provide high-resolution geoid, due to the distribution of the GPS/leveling points.

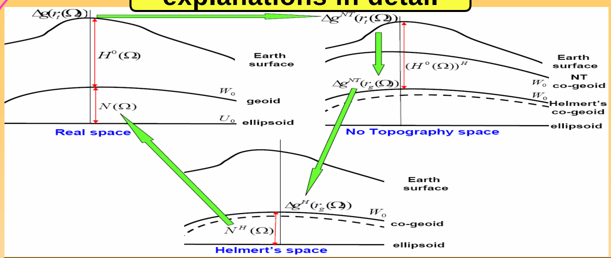
- Gravimetric solution

Stokes-Helmert's method, one of gravimetric solutions, is adopted and developed in University of New Brunswick.

The way to understand Stokes-Helmert's method



explanations in detail



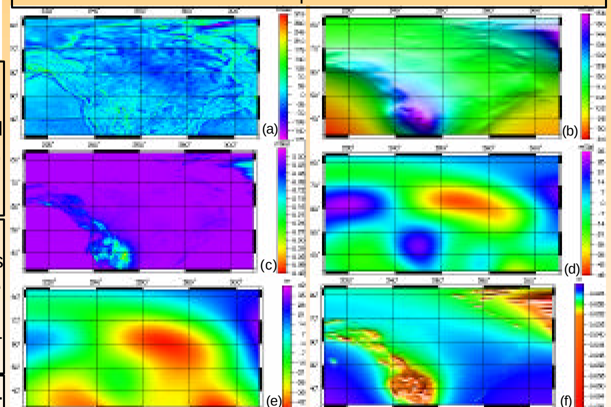
$r_t(\Omega)$: a point at the earth surface, $r_g(\Omega)$: a point at the geoid, $N^H(\Omega)$: co-geoid height in Helmert space, $(H^0(\Omega))^H$: orthometric height in Helmert space, $\Delta g(r_t(\Omega))$: gravity anomaly on earth surface in real space, $\Delta g^{NT}(r_t(\Omega))$: gravity anomaly on earth surface in NT space, and $\Delta g^{NT}(r_g(\Omega))$: gravity anomaly on geoid surface in NT space, and $\Delta g^H(r_g(\Omega))$: gravity anomaly on geoid surface in Helmert space.

The space characterized by the mass distribution obtained after Helmert's condensation is called Helmert's space. The quantities given in Helmert's space are denoted by superscript h. Compare with this, the space is called real space, and the space after being removed all topographical masses is called No-Topography space (NT space) or Bouguer space.

At first we can get $\Delta g(r_t(\Omega))$ Then $\Delta g(r_t(\Omega))$ is transformed to $\Delta g^{NT}(r_t(\Omega))$. This step is numerically realized by subtracting the effect on the gravitational attraction of the topographical and atmospheric masses. Thus the gravitational field in NT space becomes harmonic. To obtain $\Delta g^{NT}(r_g(\Omega))$, $\Delta g^{NT}(r_t(\Omega))$ is downward continued from the earth's surface onto the geoid surface. $\Delta g^H(r_g(\Omega))$ are evaluated by adding the effect of the condensed topographical and atmospheric masses on the gravitational attraction to the geoid-generated gravity anomalies. $N^H(\Omega)$ is calculated by solving the Stokes formula in the Helmert gravity space. To obtain the final geoid in the real space, the primary indirect topographical effect on the geoidal heights is subtracted from $N^H(\Omega)$.

parts of preliminary results

Base on this theory the UNB geoid software package (SHGeo software) for precise geoid determination was developed covering all aspects of the gravimetric geoid computations. This software uses standard input data.



(a) Free-air gravity anomaly on the earth surface,
(b) Secondary indirect effect of topographical masses attraction,
(c) Geoid-quasigeoid correction,
(d) Helmert's reference gravity anomaly,
(e) Reference co-geoid height,
(f) Primary indirect topographical effect.

Conclusions

The geoid could be regarded as a reference surface for geo-referencing, positioning and navigation, and also be used in conversion of GPS heights to orthometric height.

The theory and the software package of Stokes-Helmert's method for precise geoid determination have been developed to enable the geoid computation to an accuracy of one centimeter in grid spacing 5' by 5'. The actual accuracy will, of course, depend on the available data, their accuracy and their spatial distribution.

Because the computation of several programs is done in 1 by 1 arcdegree regions it is rather time consuming. It would take a few months to finish the geoid computation covering the whole Canada in full-time work.

Reference

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