UNB North American geoid 2000 model: theory, intermediate and final results

J. Janák (Department of Geodesy and Geomatics Engineering, University of New Brunswick, P.O. Box 4400, Fredericton, New Brunswick, E3B 5A3, Canada; e-mail: jjanak@unb.ca; fax: 506-453-4943)
P. Vaníček (Same address and fax; e-mail: vanicek@unb.ca;)

Introduction

This contribution is a progress report of the University of New Brunswick (UNB) research group about the GEOIDE project #10, "Precise Geoid Determination for Geo-referencing and Oceanography". The gravity field of the earth contains important information useful not only to scientists but also to people from practice. One of these surfaces, called the geoid, is especially important as it is the most natural representation of the earth shape and also because it serves as the reference surface for the heights above the sea level.

There are presently four institutions in Canada seriously focused on the precise geoid determination problem. These are:

1. Geodetic Survey Division, National Resources Canada, whose mandate is the data collection and preprocessing;
2. Department of Geomatics at the University of Calgary, whose prime interest is the data analysis;
3. Department of Oceanography at the Dalhousie University, whose prime interest is the sea surface topography modeling and the geoid modeling at sea.
4. Department of Geodesy and Geomatics Engineering at the University of New Brunswick, whose prime interest is the investigation of the basic theoretical principles and the physical correctness of the geoid solution.

Each of these research groups is now compiling a precise geoid for North America following more or less different computation scheme (as there are many approaches on how to solve this problem).

The most recent geoid model computed at UNB for the whole of Canada was compiled in 1986. Since that time the research of the UNB group has mostly focused on the theoretical improvements of the geodetic boundary value problem formulation resulting in theoretical refinements of the gravimetric geoid compilation method. Moreover, some new input data sets have become available, e.g., the digital topo-density model. Consequently, the new Geoid Model 2000 is closer to reality than the previous one.

Problem Statement

The problem is to determine the geoid for North America as accurately as possible, using the available gravity and other auxiliary data, such as the digital elevation model, the digital topo-density model, the global elevation model and the global potential model.

Methodology

The theory we use to solve this problem is based on the Stokes-Helmert approach. The original Stokes-Helmert theory was revised and significantly improved at UNB during the last decade, so it is accurate enough to give the geoid undulations with 1 cm -accuracy provided that the input data are errorless and available at any point on the earth surface. The biggest obstacle to obtaining this accuracy is in the input data errors and area coverage.

The improved Stokes-Helmert approach can be characterized by the following features:

1) The transformation of gravity values g_t observed at the earth surface to g_t^h in Helmert space, where the Helmert space differs from the real space by the removal of topographical masses and the addition of topographical masses condensed on the geoid (Direct Topographical Effect).
2) The construction of mean Helmert's gravity anomalies, mean (Δg_t^h), on the earth surface in the Helmert space.
3) The downward continuation of mean Helmert's gravity anomalies from the earth surface to the Helmert co-geoid, i.e., the geoid in Helmert space. As the disturbing potential T^h in Helmert's space is harmonic (it satisfies the Laplace partial differential equation of second order) everywhere above the geoid, we use the Poisson solution formulated for harmonic functions.
4) The acceptance of satellite derived long wavelength global field of degree and order 20 as a reference field in both the real and Helmert spaces, with the different attendant spheroids N_{20} and $(N^h)_{20}$ in both spaces.
5) A numerical Stokes integration of the mean Helmert's anomalies, mean$(\Delta g_t^h)^{20}$, reduced by the 20 by 20 reference field, on the Helmert reference spheroid. For this integration, the Stokes integration kernel is modified in the Molodenskij fashion to minimize the far zone contribution and the far zone contribution is then evaluated analytically from a global gravity field model (of a higher degree and order than the reference satellite derived field).
6) The addition of the Stokes integration results $(N^h)^{20}$ to the reference spheroid $(N^h)_{20}$ in Helmert's space to obtain the complete Helmert co-geoid N^h.
7) The transformation of Helmert's co-geoid N^h to the geoid N by subtracting from it the effect of condensed topography and adding to it the effect of real topography (this difference is called Primary Indirect Topographical Effect).

For the evaluation of the topographical effects the numerical integration using a spherical model was adopted. The far zone contribution was evaluated analytically. The effect of lateral variation of topographical density on the geoid through the Direct and Primary Indirect Topographical Effect was also computed.

Results

The result is a digital geoid on a 5 by 5 arc-minute grid. This grid covers the whole of Canada, except the northernmost islands, and the United States, except the western part of Alaska. The accuracy of the model is not known at the time of preparation of this synopsis. This digital model will find many applications in geodesy, geophysics, civil engineering and navigation. As byproducts, many auxiliary quantities were computed, e.g., various topographical effects, the downward continuation, which can be used for further studying of the earth.

Conclusions

The model computed at UNB is going to be compared with the solutions of the others research groups mentioned above, as well as against the independent GPS/leveling testing points. The evaluation will be done at the Canadian Geophysical Union (CGU) 2000 meeting in Banff. The ultimate goal then is to produce as accurate a geoid for North America as the existing data allow, by combining the best traits of the individual approaches.

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