Continental slope foot-line determination: geometrical aspects

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ABSTRACT

This paper is based on our earlier theoretical work that had culminated in the formulation of an algorithm for computing the surface of maximum curvature corresponding to a bathymetric surface. Here we descibe the results of our first experiments with real bathymetric data. We show that bathymetric data density and distribution as well as the morphology of the sea-bed affect the results in a subtantial way and as such must be given a serious consideration when deploying the algorithm.

INTRODUCTION

The United Nations Conference on the Law of the Sea III [United Nations, 1983] Article 76, ¶ 4(b) defines the <u>foot of continental slope</u> as "...the point of the maximum change in the gradient at its base." In mathematics, the change in the gradient is called the <u>curvature</u>. The UNCLOS III document thus speaks of the foot of the continental slope as the point (at its base) in which the curvature is the maximum - see **Figure 1** . In our discussion here, we define additionally the <u>foot-line</u> as the line that connects all the foot-points. We will thus understand the continental slope foot-line to consist of all the points, where the curvature of the slope, at its base, is the maximum.

We will focus our attention exclusively at this purely geometrical aspect as described above, leaving alone the complications of geological and geophysical nature arising when the foot-line is covered with sediments of certain thickness. These complications are treated by other contributors to this volume.

How can the foot-line be traced? To the best of our knowledge, all the attempts up to now used the bathymetric surface (the image of sea bottom constructed from bathymetric data) directly. In some cases, human eye can trace the points of maximum curvature rather well, in other cases, it is next to impossible to do it. In all the cases, such tracing is subjective and no two people will come up with the same result. If an objective determination of the foot-line is desired, a different approach must be used.

We have investigated one such approach, based directly on the legal definition of the foot in the UNCLOS III, whereby we construct the surface of maximum curvature (SMC for brevity) and trace the ridges (i.e., lines of maximum curvature!) on this surface, if they exist. The tracing of ridges can be done automatically, i.e., objectively; this constitutes the reason why we became interested in this technique. We note that such automated tracing has not yet been implemented in our software and here we report only on the first stage of our investigation, the construction of the SMC.

CONSTRUCTION OF THE SURFACE OF MAXIMUM CURVATURE

The SMC has to be constructed from available bathymetric soundings, such as shown in **Figure 2** and **Figure 3**. These soundings were collected during different seasons, using different techniques and different measuring systems and the two areas represent two rather extreme cases.

The mathematical formulae for the maximum normal curvature (the two-dimensional equivalent of the uni-dimensional curvature used in the UNCLOS III definition is the normal curvature of the surface) use the first and second partial derivatives of the bathymetric surface. These are evaluated from bathymetric soundings; for the actual mathematical formulation see Vaníč ek et al. [1994].

The maximum curvature values can be evaluated either at the locations of the available soundings, or on some selected grid. The former option is preferred because it best honours the collected data. The latter option implies the use of some approximation or interpolation and artifacts caused by the used approximation or interpolation technique may be expected to occur. Unfortunately, the preferred option may not give the optimal results either if the bathymetric data density and configuration do not match the sea bottom morphology. We shall address these problems in the next paragraph.

Before we do that, let us discuss another aspect of the foot-line determination: What happens if the change in continental slope curvature is very gradual? This situation will appear as a very gentle ridge in the SMC. In the extreme case, when the curvature is constant, the ridge will disappear all together, the foot-line cannot be located following the wording of Article 76, ¶4(b) and an alternative approach must be used. An example of such an alternative is given in [United Nations, 1993].

The uncertainties in collected soundings can be propagated into the uncertainties of SMC and ultimately, into the uncertainties of the ridge tracing. Statistically derived standard deviations of maximum curvature values based on standard deviations of collected soundings are routinely determined by our software.

SOME PROBLEMS WITH AVAILABLE SOUNDING DISTRIBUTIONS

When the location configuration of collected soundings is highly irregular (cf. Figure 2), the SMC computed at these locations may be affected by artifacts caused by the computational technique: the partial derivatives evaluated at and from the actual locations would not be representative of the bathymetric surface as such. An example of such artifacts is shown in **Figure 4.**

When the density of collected soundings is too high for the sea bottom morphology, small bottom features cause the values of maximum curvature to become too large. These large values then dominate the SMC, masking the gentler ridge associated with the continental slope foot-line - cf. **Figure 5** .

The collected soundings may also be too sparse for a meaningful determination of the foot-line. This can be easily understood and we do not offer any example of this case here.

Clearly, the answer to all these problems is to match the data density with the sea bottom morphology and use as regular a data distribution as possible. When the available data coverage is too thin and/or too irregular, there exists no mathematical technique to remedy the situation - more data have to be collected. When a denser than necessary data coverage is available, this can be happily achieved.

DATA REGULARIZATION

The technique that has worked well with the 3 data sets we investigated is the <u>areal data averaging</u>. The idea of this technique is to divide the region of interest into regular (small) cell, evaluate the centroid of all the data locations in each cell and associate a representative depth with the centroid. The representative depth is best chosen as a weighted average of all the soundings that fall within the cell, where the weight is selected to be inversely proportionate to the distance between the sounding to be weighted and the centroid. We note that this process is not equivalent to equidistant data gridding. Rather, it results in a thinner, still irregular coverage as seen in **Figure 6**, that shows the areal averaging of the actual data set from **Figure 2** using areas of 0.4 by 0.4 degrees.

The only artifact created by this areal averaging is a smoothing of the bathymetric surface. If the size of the averaging cell is selected properly, then the areal averaging aleviates both problems described in the previous paragraph. Just what constitutes a proper size of a cell, depends on the sea bottom morphology. More numerical experimentation is needed before definite prescription can be formulated. For an illuatration, **Figure 7** shows the SMC constructed from the regularized data set in **Figure 6**, while **Figure 4** shows the SMC for the same data set but averaged for 0.1 by 0.1 degree cells.

One may, of course, now ask the obvious question: Having abandonned the principle of honouring the collected data, may other approximation, interpolation or griddig schemes work just as well? The answer at this point must be yes, they may; based on our somewhat limited experience, we have to leave this question open to further investigation.

CONCLUSIONS

Based on our first systematic experience with contructing continental slope foot-line from real bathymetric data, there is, of course, nothing really definitive we can conclude as yet. Perhaps one provisory conclusion can be stated as: the densest sounding data coverage is not necessarily the best for this purpose. Even though, it is desirable to use an algorithm which does not depend on any mathematical technique (model) and relies on the collected point data alone, it may not be possible to by-pass the necessity to a mathematical model into the process.

REFERENCES

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