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# Assessment of GOCE Models Over Mexico and Canada and Impact of Omission Errors

M.C. Santos, D. Avalos, T. Peet, M. Sheng, D. Kim, and J. Huang

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## Abstract

This paper reports a series of comparisons of geoidal heights derived from several GOCE models with (1) geoidal heights derived from GPS on benchmarks (referred to as geometric geoidal heights) over Mexico and Canada, and with (2) geoidal heights derived from the latest geoidal maps of Mexico (GGM2010) and Canada (PCGG2013) (referred to as gravimetric geoidal heights). The paper also looks quantitatively into omission errors. Comparison (1) and (2) were carried out not including and including omission errors. The GOCE models used in comparison (1) disregarding omission errors are the direct solution model (first, second and third generations), the time-wise solution model (first, second and third generations), GOCO02S model and GIF48 model, all evaluated up to their maximum degree/order. Only GOCE direct third generation model was used in comparison (1) considering omission errors, and in comparison (2) disregarding omission errors, the latter with respect to GGM2010. The GOCE models used in comparison (2) including correction for omission errors are the GOCE direct third generation, GOCO01S, GOCO03S and DGM-1S models, evaluated up to degree/order 180. This makes GOCE direct third generation as the only model common in all comparisons. Omission errors were evaluated based on the extra-high degree harmonics of EGM2008. The omission errors in Mexico and in Canada show a similar behaviour, with a near zero mean and a standard deviation at the order of  $\pm 50$  cm in Mexico and  $\pm 45$  cm in Canada. In both cases, maximum differences reach more than 4 m. The effect of omission errors can be better appreciated by looking at performance of the only GOCE model used in all comparisons, the direct third generation model. Comparing it with Mexican geometric geoidal heights: without correcting for omission errors, mean and standard deviation of  $-5.1$  and  $\pm 45.7$  cm; including correction for omission errors,

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mean and standard deviation of  $-1.6 \pm 30.6$  cm. Comparing it with GGM2010: without correcting for omission errors, mean and standard deviation of  $-17.4$  and  $\pm 51.3$  cm; including correction for omission errors, mean and standard deviation of  $-2.8 \pm 34.8$  cm.

### Keywords

Earth gravity model • EGM2008 • GGM2010 • GOCE • PCGG2013

## 1 Introduction

GOCE (Visser et al. 2002), the Gravity Field and Steady-State Ocean Circulation Explorer, is a satellite gradiometry mission that maps the Earth's gravity field in a homogenous way over most of the globe. ESA (1999) states as the mission goals “the determination of the stationary gravity field – geoid and gravity anomalies – to high accuracy and spatial resolution.” The expectation is that it will provide a geoid model within one centimeter accuracy and gravity anomalies to an accuracy of a few mGal; all within a spatial resolution of 100 km. GOCE offers as its main product global gravity field models.

A major task is to validate these models by using external sources of information, of terrestrial and/or space origin, all of them with their own limitations. For example, terrestrial sources of information rely on GPS on benchmarks, and both techniques (GPS and geodetic leveling) have uncertainties associated with them. Regional gravimetric geoids, which can also be used to evaluate GOCE models, are based on both space (low degree terms of a geopotential model) and terrestrial data (gravity anomalies), all with uncertainties. Finally, GOCE models can be compared to other geopotential models built solely on space information. Several authors have discussed the limitations of the different data sources, such as Featherstone (2011).

There is already a huge family of GOCE models, computed using a variety of methods and either using just GOCE data or combining data from other satellite missions or terrestrial data with GOCE data. They also use data which cover different periods of time (ICGEM 2013).

Efforts in evaluating GOCE models are under way. For example, the IAG sponsored GGHS Meeting, held in Venice, in 2012, had a session dedicated to just that. By the time this paper is published, the proceedings of the GGHS Meeting will have been published already.

## 2 Comparisons Without Accounting for Omission Errors

### 2.1 Comparison of GOCE Geoidal Heights with Geometric Geoidal Heights

This paper builds on an earlier and unpublished work by Peet et al. (2012), which compared recently (at that time) developed GOCE gravity models and tested their applicability across the topography, in Canada and Mexico, by means of comparing Global Positioning System (GPS) observations taken on first-order orthometric benchmarks in both countries. Table 1 shows the models used in this evaluation.

This comparison used data provided by the Geodetic Survey Division (GSD) of Natural Resources Canada (NRCan): GPS-derived geodetic heights on first order benchmarks of the Canadian first-order levelling network – realization NOV07 (a total of 2,579 benchmarks – located mostly in the southern portion of the country); and, data from the Instituto Nacional de Estadística y Geografía (INEGI): GPS-derived geodetic heights on first order benchmarks of the North American Vertical Datum (NAVD88) (a total of 1,487 benchmarks – spread throughout Mexico). Differently from Canada, where the levelling lines go through rough terrain only in the West, the Mexican levelling lines go through rough terrain throughout the country, and it is expected to contain distortions reaching several decimeters in amplitude of medium wavelength.

A set of geoidal heights (from now on referred to as ‘geometric’) were derived from these data sets. GOCE geoidal heights were calculated over these benchmarks using the Fortran code developed by Rapp (1982) and expanded by others (Pavlis 1996). These were called as the ‘gravimetric’ undulations. Omission errors were not taken into account and geoidal heights were evaluated up to their maximum degrees.

**Table 1** List of gravity models (after ICGEM 2013)

Model	Year	Degree	Solution	Data used (months)
GOCE TIM Gen1	2010	224	Time-wise	GOCE (2)
GOCE DIR Gen1	2010	240	Direct	GOCE (2)
GOCE TIM Gen2	2011	250	Time-wise	GOCE (6)
GOCE DIR Gen2	2011	240	Direct	GOCE (6)
GOCE TIM Gen3	2011	250	Time-wise	GOCE (18)
GOCE DIR Gen3	2011	240	Direct	GOCE (18), GRACE (6)
GOCO02S	2011	250		GOCE (2), GRACE (7)
GIF48	2011	360		GRACE, terrestrial observation

**Table 2** Comparison of geometric geoidal height differences

Model	Mexico results (m)				Canada results (m)			
	Max	Min	Mean	Std. Dev.	Max	Min	Mean	Std. Dev.
TIM1	1.903	-2.286	-0.066	0.479	1.295	-1.529	-0.006	0.411
DIR1	1.985	-2.559	-0.063	0.440	1.228	-1.144	0.027	0.334
TIM2	1.788	-2.275	-0.069	0.448	1.302	-1.243	0.023	0.362
DIR2	1.740	-2.288	-0.075	0.457	1.290	-1.261	0.020	0.381
TIM3	1.816	-2.195	-0.057	0.441	1.276	-1.156	0.028	0.343
DIR3	1.844	-2.298	-0.051	0.457	1.377	-1.197	0.016	0.359
GOCO02S	1.852	-2.310	-0.077	0.480	1.271	-1.481	0.001	0.404
GIF48	1.844	-2.298	-0.051	0.457	1.366	-1.227	0.013	0.364

Table 2 presents statistics of comparison between GOCE and the geometric geoidal heights (in the sense of GOCE geoidal height minus geometric geoidal height). Mean differences are at the cm-level with 1-sigma standard deviation at the dm-level. The latter may be partially due to the commission and omission errors, which were not accounted for during the comparison. The third generation product yields the smallest mean values of geoidal heights differences except in Canada, where the smallest difference was obtained using the first generation time-wise.

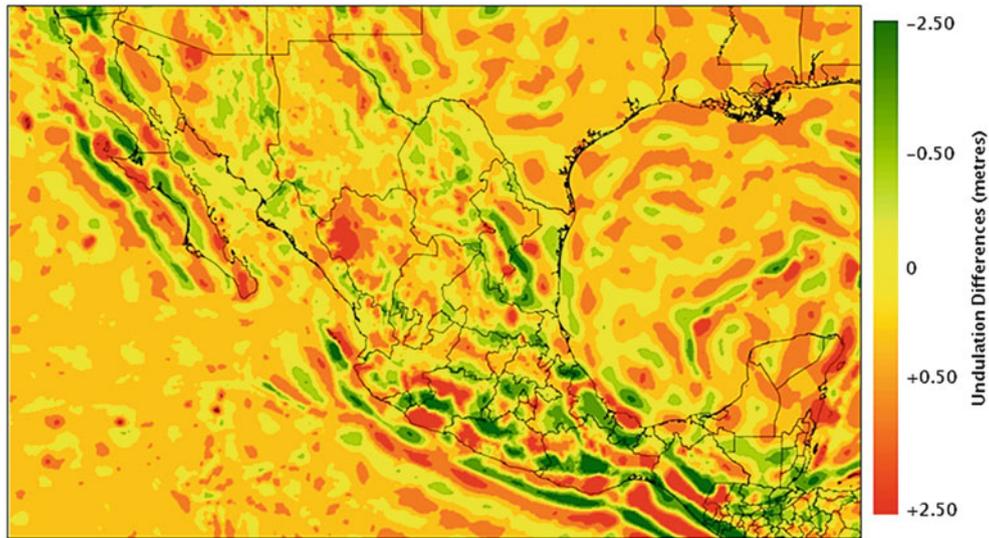
## 2.2 Comparison of GOCE Geoidal Heights with GGM2010 Gravimetric Geoidal Heights

Still in Peet et al. (2012) there was a comparison between geoidal heights derived from the Geoide Gravimétrico Mexicano GGM2010 (Muoz-Abundes 2011) and from GOCE direct third generation, which yielded the best results over Mexico. Again, omission errors were disregarded. Figure 1 shows the differences computed along a grid (2.5' by 2.5'). Statistics for only mainland Mexico include a mean of

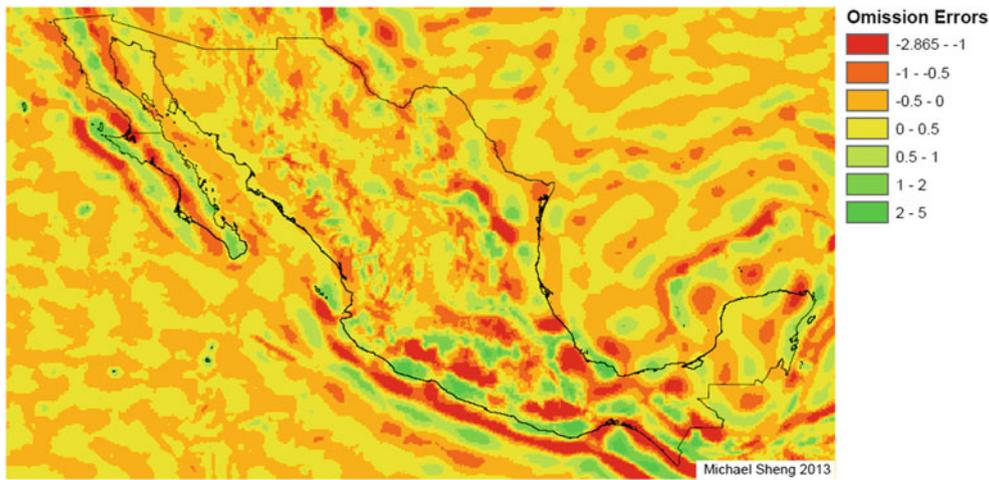
-0.174 m with a spread of 0.513 m and maximum and minimum values of 3.215 and -2.354 m respectively. There are larger variations in the regions of Baja California and southern Mexico. These regions show large amplitudes with a short wavelength (around 120 km, beyond degree 60). This feature may be due to disagreement between the GOCE dir 3 and the terrestrial data used to feed the GGM2010: GGM2010 uses a reference field up to degree 40 from the model EIGEN-GRACE\_03S (Reigber et al. 2005) and terrestrial data for all higher degrees. Pavlis (1996) routine was also used here.

## 3 Omission Errors

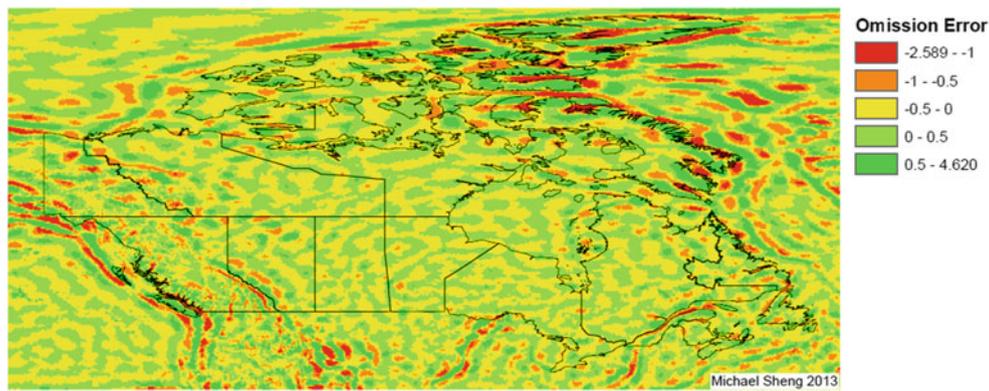
Omission errors were evaluated by computing the contribution of EGM2008 (Pavlis et al. 2012, 2013) coefficients from degree 181 up to 2190. Figures 2 and 3 show the distribution and variation of omission errors over Mexico and Canada. Table 3 summarizes the statistics. Program Harmonic\_Synth\_v.02 was used for this evaluation. This program still uses the same approach as described in Pavlis (1996).



**Fig. 1** GGM2010 vs. GOCE direct third generation



**Fig. 2** Omission errors over Mexico (in metres)



**Fig. 3** Omission errors over Canada (in metres)

**Table 3** Statistics about omission errors (values in m)

	Mexico	Canada
Mean	-0.001	0.001
Standard deviation	0.498	0.447
Maximum value	4.772	4.260
Minimum value	-2.880	-2.589

**Table 4** List of Gravity models (after ICGEM 2013)

Model	Year	Degree	Solution
DGM-1S	2012	250	GOCE, Grace
GOCO03S	2012	250	GOCE, Grace
GOCE dir3	2011	240	GOCE, Grace, Lageos
GOCO01S	2010	224	GOCE, Grace

## 4 Comparisons Including Omission Errors

Model assessment considering omission errors was carried out by simply comparing geometric and gravimetric geoidal heights with those computed from the GOCE models, according to the following relationship:

$$\Delta N = (N_G - \delta N_{\text{EGM2008}}) - N_{\text{GOCE}}, \quad (1)$$

where  $N_G$  is the geoidal height as given by the geometric or gravimetric model,  $N_{\text{GOCE}}$  is the geoidal height as computed using a GOCE model, and  $\delta N_{\text{EGM2008}}$  represents the omission errors.

### 4.1 Comparison Against Mexican Geometric Height

The comparisons considering omission errors using GOCE direct third generation model were carried out with respect to Mexican geometric geoidals heights. Statistics related to this comparison resulted in a mean of  $-0.016$  m with a spread of  $0.306$  m; maximum and minimum values at the range of  $2$  m.

### 4.2 Comparison Against Gravimetric Height

The comparisons considering omission errors were also done with respect to the gravimetric geoid GGM2010 of Mexico and PCGG2013 (in a  $2'$  by  $2'$  grid) of Canada (Huang and Véronneau 2013). The four GOCE models used in this comparison are shown in Table 4, being evaluated by the Harmonic\_Synth\_v.02 program. Note that only GOCE direct third generation is common to all comparisons. The other models were used because they are more recent. All models were evaluated up to degree 180. The use of degree 180 as a limit was a recommendation by Rummel (2012).

In this evaluation, we applied a zero degree term (sum of the mass and potential terms so that the geoidal heights are referred to the GRS80 ellipsoid) of  $-0.53$  m in the evaluation involving the GGM2010 and  $-0.44$  m for the

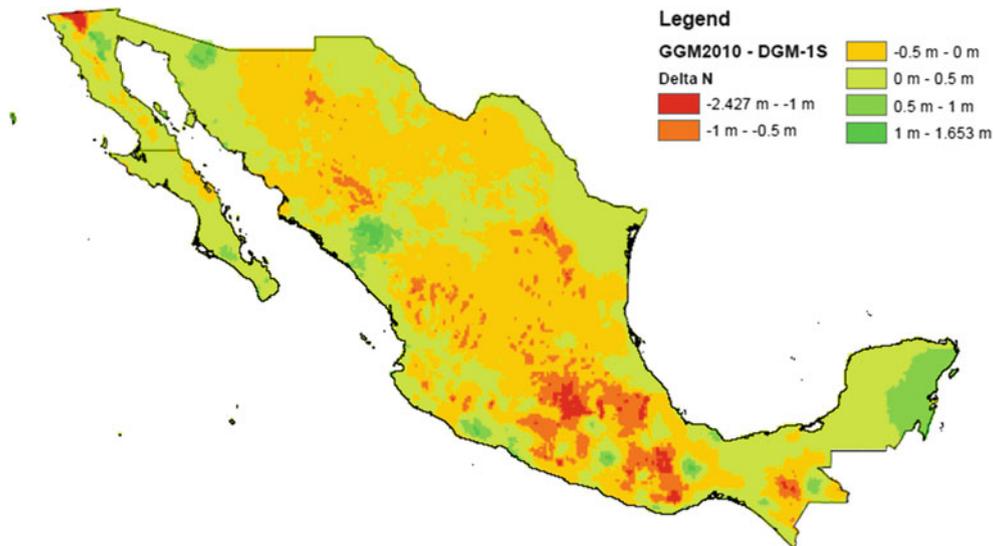
evaluation involving the PCGG2013. Figures 4 and 5 show the results for the comparison done with respect to DGM-1S, for Mexico and Canada, respectively. Statistics summarized in Tables 5 and 6.

## 5 Concluding Remarks

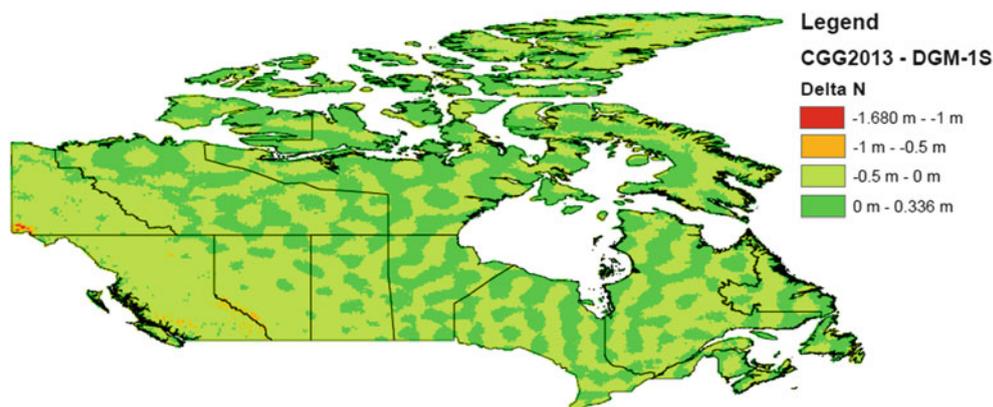
We have performed a series of comparisons of geoidal heights derived from several GOCE models with the ones derived from GPS on benchmarks over Mexico and Canada and with the latest geoidal maps of Mexico (GGM2010) and Canada (PCGG2013). Some of the comparisons did not take omission errors into account, whereas some others did. The results indicate unequivocally the benefits of taking omission errors into account, as in doing so it results in a reduction in the spread of the differences. The limitation in the approach of using EGM2008 to evaluate omission error is that EGM2008 truncates at degree 2190. Therefore, it does not model all omission error (i.e., those beyond degree 2190). The omission errors that remain unaccounted for may be contributing to the remaining spread.

There are other important features in the study that need to be stressed in order to properly understand the results. In this study, in the comparison of GOCE models against geometric geoidal heights, the GOCE models were evaluated up to their maximum degree; whereas when omission errors were accounted for the GOCE models were evaluated up to degree 180. Other point is that not all models were the same in both comparisons: only GOCE direct 3<sup>rd</sup> generation was common to all comparisons. Nevertheless, the conclusion (about the benefits of taking omission errors into account) holds exactly due to the solution obtained with the latter model.

Results can be summarized as follows. Let us first call the comparisons of geoidal heights derived from several GOCE models with geoidal heights derived from GPS on benchmarks (referred to as geometric geoidal heights) over Mexico and Canada as comparison (1), and call the comparisons of geoidal heights derived from several GOCE models with geoidal heights derived from the latest geoidal maps of Mexico and Canada (referred to as gravimetric geoidal heights) as comparison (2). Comparison (1) disregarding omission errors, presented in Sect. 2.1, resulted in a mean difference in the order of  $-6 \pm 46$  cm in Mexico (best result



**Fig. 4** Comparing GGM2010 with DGM-1S



**Fig. 5** Comparing PCGG2013 (same as CGG2013 in the legend) with DGM-1S

GOCE direct third generation), whereas in Canada most comparisons resulted in  $-2 \pm 37$  cm, being that the ones involving GOCE time-wise first generation and GOCO02S gave millimetre-level mean difference (which seems difficult to justify); maximum differences are slightly larger than 2 m. Comparison (1) taking omission errors into account, presented in Sect. 4.1, resulted in  $-1.6 \pm 30.6$  cm. Comparison (2) without taking into account omission errors, presented in Sect. 2.2, resulted in  $-17.4$  and  $\pm 51.3$  mm. Comparison (2) taking the omission errors into account, presented in Sect. 4.2, seem to have resulted in a reduction in all parameters, being the mean difference in the order of  $-3 \pm 35$  cm in Mexico and  $-3 \pm 9$  cm in Canada. Maximum values are 2.5 m and 1.7 m, respectively.

The omission errors in Mexico and in Canada show a similar behaviour, with a near zero mean and a standard

**Table 5** Summary of comparisons, only landmass considered, values in m

Mexico	DGM-1S	GOCO03S	GOCE dir3	GOCO01S
Mean	-0.030	-0.030	-0.028	-0.031
Standard deviation	0.348	0.348	0.348	0.349
Maximum	1.657	1.686	1.674	1.694
Minimum	-2.427	-2.489	-2.443	-2.515

**Table 6** Summary of comparisons, only landmass considered, values in m

Canada	DGM-1S	GOCO03S	GOCE dir3	GOCO01S
Mean	-0.030	-0.030	-0.028	-0.030
Standard deviation	0.093	0.091	0.093	0.101
Maximum	0.336	0.334	0.342	0.338
Minimum	-1.680	-1.704	-1.691	-1.708

deviation at the order of  $\pm 50$  cm in Mexico and  $\pm 45$  cm in Canada. In both cases, maximum differences reach more than 4 m.

The effect of omission errors can be better appreciated by looking at performance of the only GOCE model used in all comparisons, the direct third generation model. Comparing it with Mexican geometric geoidal heights: without omission errors, mean and standard deviation of  $-5.1$  and  $\pm 45.7$  cm; including correction for omission errors, mean and standard deviation of  $-1.6 \pm 30.6$  cm. Comparing it with GGM2010: without omission errors, mean and standard deviation of  $-17.4$  and  $\pm 51.3$  cm; including correction for omission errors, mean and standard deviation of  $-2.8 \pm 34.8$  cm.

**Acknowledgements** Thanks to Natural Resources Canada and to Instituto Nacional de Estadística y Geografía for the provision of the geoid models PCGG2013 and GGM2010. GOCE models were obtained through ICGEM's website, also cordially acknowledged.

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