# Atmospheric Density Admittance Function for Gravity Reduction

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## 1. Introduction

Recent rapid improvements in technology has created extremely precise measuring systems that are affected by the smallest effects that were once much too small to be detected. The superconducting gravimeter is no exception; it is a sensitive device, which can detect minute changes in surface gravity at the nanogal level. Gravity variations are caused by many physical phenomena e.g. lunar and solar tides, Earth rotation, atmospheric and ocean loading, and others (e.g., Crossley and Xu, 1998; Goodkind, 1999; Hinderer and Crossley, 2000). It is, with no doubt, a challenge to identify and/or separate minute signal(s) of interest in a specific band of interest.

Atmospheric mass change is one of the most significant environmental phenomena that affects Earth surface gravity. There are two approaches that are usually followed to model the atmospheric pressure effect on gravity signals: physical and empirical. The latter is also called "*the admittance function*" or "*the transfer function*" method and represents the response of gravity to atmospheric pressure variation. In this paper we take a new approach that allows us to model the response of gravity to air density variations rather than to the air pressure as it has traditionally been done. The atmospheric correction to gravity is achieved by using air density time series that are synthetically produced from temperature, pressure and humidity time series recorded simultaneously with gravity at Canada's fundamental gravity station, in Cantley PQ.

A constant admittance is not adequate to describe the air pressure or density effect, which is admittedly frequency dependent. This frequency-dependent admittance that was first introduced by Warburton and Goodkind, (1977) and later by Crossley et al., (1995), Neumeyer, (1995) and others, shows that it increases smoothly and monotonically from 0.2 μgal/mbar at low frequencies (<0.3 cpd) to about 0.35 μgal/mbar at high frequencies (>1 cpd). However, Sun et al., (2002) found that the admittance is 0.378 μgal/mbar at low frequencies and decreases to 0.147 μgal/mbar at high frequencies.

In this paper, we adopt an alternative approach for the determination of the admittance that is based on the least-squares (LS) product spectrum of the air density and gravity time series. The air density is synthetically produced from pressure, temperature and humidity records using the equation of state of the atmosphere. The air density admittance is then estimated from common spectral peaks identified in the gravity and air density series, using the product spectrum and rigorous statistical analysis tools. The common spectral peaks in both gravity and air density series are suppressed to estimate their amplitudes and phases and subsequently the yearly admittance amplitude and phase in the band 700h to 2h. Finally, the weighted LS regression is used to estimate the admittance as a function of frequency.

## 2. Atmospheric Density

The atmospheric density can be calculated using the equation of state of the atmosphere, which after some lengthy derivation gives the total air density as a function air pressure *P*, temperature *T:*

$$\rho = \frac{P}{R_d T_v}, \qquad T_v = \left( \frac{1 + \frac{r}{\varepsilon}}{1 + r} \right) T, \tag{1}$$

where $R_d$ is the specific gas constant for dry air, $T_v$ is the virtual temperature, $r$ is the mixing ratio and $\varepsilon$ is the ratio between specific dry and wet gas constants.

## 3. Methodology

We use the Least Squares Spectral Analysis (LSSA) to estimate the spectra of the gravity and air density series and subsequently produce their product LS spectrum. More details on the LSSA and related statistical properties can be found in Vanicek (1969; 1971) and Pagiatakis, (1999).

The LSSA spectrum is described by the percentage variance $s(\omega_i)$ of the spectral content of a time series $f(t)$ with variance-covariance $C_f$ at a specific frequency $\omega_i$ :

$$s(\omega_i) = \frac{f^T C_f^{-1} \hat{p}(\omega_i)}{f^T C_f^{-1} f}, \tag{2}$$

where $\hat{p}(\omega_i)$ is the projection of $f(t)$ on a manifold characterized by a specific base functions (trigonometric function). It has been shown (Pagiatakis, 1999) that the probability density function (*pdf*) of the LS spectrum is the *beta* distribution, which can be used to define the *pdf* of the product of two LS spectra using standard statistical approaches (e.g. Hogg and Craig, 1995). After lengthy derivations, the *pdf* of the product LS spectrum *z* for two random variables $x_1$ and $x_2$ is:

$$f(z) = \int_z^0 \beta_1 \beta_2 e^z \left(1 - e^{z - x_2}\right)^{\beta_1 - 1} \left(1 - e^{x_2}\right)^{\beta_2 - 1} dx_2, \tag{3}$$

where $\beta_i = 0.5(m_i - u_i - 2)$, $m_i$ is the length of time series and $u_i$ is the number of unknown parameters estimated by the LS procedures. The above *pdf* that underlines the product LS spectrum can be used to identify statistically significant common peaks in both gravity and air density series via their product spectrum.

The statistically significant common peaks (periods) in the product spectrum are suppressed separately in the gravity and air density series to estimate their amplitude and phase:

$$g_i = a_{iG}(\cos(\omega_i t) - \varphi_{iG}), \tag{4}$$

$$\rho_i = a_{i\rho}(\cos(\omega_i t) - \varphi_{i\rho}), \tag{5}$$

where $a_{iG}$, $a_{i\rho}$ are the amplitudes of gravity and air density constituents, respectively and $\varphi_{iG}$, $\varphi_{i\rho}$ are their phases. The magnitude and phase of the air density admittance is then estimated from:

$$\alpha(\omega_i) = \frac{a_{iG}}{a_{i\rho}}, \tag{6}$$

$$\Delta\varphi_i = \varphi_{iG} - \varphi_{i\rho} \tag{7}$$

## 4. Data processing and Analysis

Three year long time series of gravity, air pressure, temperature and humidity starting 1st January 2000 are used to estimate the air density admittance. First, the solid Earth tide effect is removed from the 1s gravity records using GWave (Merriam, 1992). The ocean loading effect is also removed by least-squares fitting of eight most significant periods; this is done simultaneously with the estimation of the gravity spectrum using the LSSA software. The 1s gravity residual series is then filtered using a Parzen weighting scheme that produces unequally spaced series along with their standard deviation at a sampling interval ranging from 2 to 5 minutes. The air density and its associated standard deviation are produced every 30 minutes using Eqs. (1).

The product LS spectrum of gravity and air density is then calculated from the two individual LS spectra respectively to show common peaks. Subsequently, the product spectrum and its statistical properties are used to estimate the Earth gravity response to air density. Yearly data are processed to estimate the spectrum in the band 700h to 2h (0.0014cph to 0.5cph). The statistically significant common peaks (95 percent confidence level) in the product spectrum are identified using the *pdf* given by Eq. (3). The periods of these peaks are then suppressed in each of the gravity and air pressure series provided that they are also statistically significant (at 95 percent) in these series. The suppression of the significant periods gives an estimate of their amplitudes and phases. The magnitude of the air density admittance along with its standard deviation is estimated for the period of three years (Fig. 1). The weighted LS regression is used to define the best fit to three years admittances (μgal/g $m^{-3}$):

$$\alpha(f) = 0.13261 - 0.08965 e^{\left(-\frac{2\pi}{240\omega}\right)} \tag{8}$$

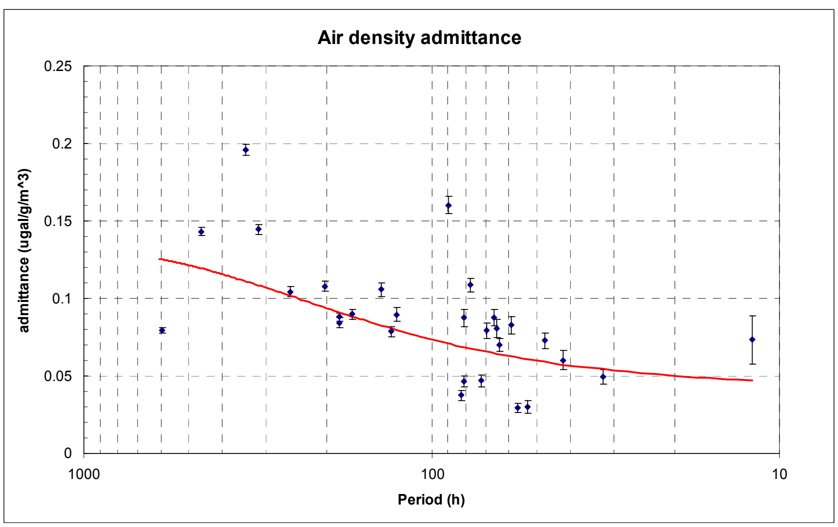


Fig. 1. Air density admittance as a function of period (h)

## 5. Discussion and conclusion

The new air density admittance is frequency dependent. It is relatively constant in the high frequency band, and it increases exponentially starting from 24h. The combination of the three physical environmental effects namely, pressure, temperature, and humidity through the equation of state of the atmosphere (physical law) is expected to reduce the signal-to-noise ratio in the gravity spectrum, more so than the pressure admittance alone. Research is continuing to determine the seasonal variations of the air density admittance.

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