On the Estimation of Variance Components Using GPS, Geoid and Levelling Data

G. Fotopoulos and M.G. Sideris

Department of Geomatics Engineering, University of Calgary, 2500 University Dr. NW, Calgary, Alberta, T2N 1N4, Canada
Tel: 403-220-4984, Fax: 403-284-1980, email: gfotopou@ucalgary.ca

OVERVIEW

In this paper a well-known approach for estimating the variance components of heterogeneous groups of observations is used in the combined adjustment of GPS, geoid and levelling data. Specifically, the minimum norm quadratic unbiased estimation (MINQUE) algorithm [Rao and Kleffe, 1988] is employed to determine the individual variance components for each of the three height types. This method is explored in order to address the question of *'What accuracy level can be achieved using the GPS-levelling method?'* Over the past decade, numerous advances have been made which have placed us in a position where we can begin to address this issue with more confidence, namely (i) improved data processing capabilities, (ii) increased data availability/quality for computing gravimetric geoid models, and (iii) refined mathematical models/techniques for dealing with GPS data.

The general model applied for the 1D multi-data vertical network adjustment is:

$$h_i - H_i - N_i = f_i + v_i \tag{1}$$

where h_i, H_i and N_i refer to the known ellipsoidal, orthometric and geoid height values at each of the network points, f_i describes the correction term for the systematic errors and biases inherent among the different types of height data (usually modeled by a parametric surface) and v_i describes the zero-mean random errors in the GPS, levelling and geoid data [see Fotopoulos et al., 2003 for more details]. The combined second-order statistical properties of the observational errors, v_i, are described by the covariance matrix:

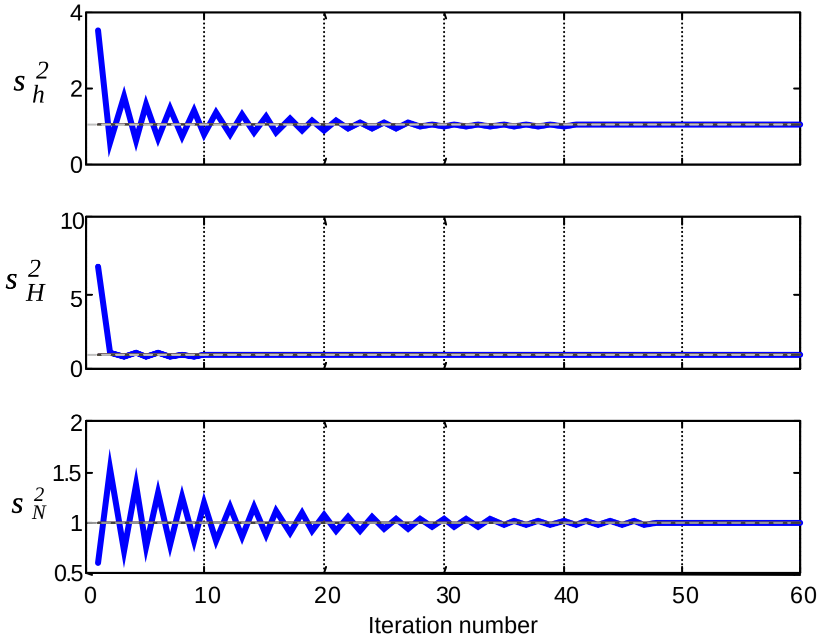
$$\mathbf{C} = \sum_k s_k^2 \mathbf{Q}_k \quad \text{where } k = [h \quad H \quad N] \tag{2}$$

Fully populated empirical covariance matrices ($\mathbf{Q}_h, \mathbf{Q}_H$ and $\mathbf{Q}_N$) are computed from a-priori information about the accuracy of the three height types and used as initial input into the MINQUE algorithm. These a-priori covariance matrices were successively 'updated' by the corresponding estimated variance components $s^2 = [s_h^2 \quad s_H^2 \quad s_N^2]$ in an iterative procedure. The iterations stop and the final estimated factors are computed once a pre-specified convergence criterion is met.

In order to apply this algorithm in practice through a combined adjustment of GPS, geoid and levelling data, there are a number of issues that must be addressed, namely (i) the effect of the a-priori covariance matrices on the final estimated variance components, (ii) the provision for estimating only *non-negative* variances, (iii) the effect of using fully populated versus diagonal covariance matrices and (iv) the role of the parametric model type on the estimated variance components. These challenges were encountered when implementing the MINQUE algorithm in practice using balanced data for a GPS/levelling network in Switzerland [Marti, 2002]. A brief sample of some of the results is discussed in the following section.

RESULTS

A network of 111 GPS/levelling benchmarks covering all of Switzerland was used for the numerical tests. The following figure shows the estimated variance factors at each iteration as they ultimately converge to unity. From top to bottom, the three subplots represent the estimated variance factors for GPS, levelling and geoid heights, respectively. Upon inspection of the graphs it is evident that the estimated variance factor for the levelled heights converges at a much faster rate, approximately 40% (27 iterations) as compared to 42 and 47 iterations for GPS and geoid heights. This can be attributed to the fact that the initial accuracy estimates for the levelled heights were more realistic than the available covariance matrices for the other types of height data. In fact, for this case, it was found that the GPS covariance matrix resulting from an independent GPS network adjustment was too optimistic, while the a-priori covariance matrix for the geoid proved to be too pessimistic overall.



CONCLUSIONS

The analysis conducted for this paper provides some indication into the practicality and usefulness of estimating variance components in mixed vertical networks. Notably, the estimation of realistic variance components provides us with important insight with regards to the GPS-levelling problem in addition to other uses of combined GPS, geoid and levelling data, such as assessing the accuracy of a gravimetric geoid model.

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