A forward modelling approach for estimation of 3D density effects on geoidal heights

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Abstract

1. Introduction

The Stokes-Helmert scheme for geoid computation requires calculation of effects of topographical masses both as they exist in reality, and as masses condensed onto the geoid. The most comprehensive current evaluations of topographical effects in the Stokes-Helmert scheme account for terrain effects, use a spherical (rather than planar) approximation of topography, and incorporate 2-dimensional laterally varying models of topographical mass-density (e.g. Vaníček et al., 1999). A 2-dimensional mass-density model is used to approximate the Earth's real 3-dimensional density distribution because it is difficult and expensive to accurately determine the actual distribution of density within topography. We set aside the problem of determining exactly the real topographical mass-density distribution and instead seek an indication of how much accuracy is lost by using a 2-dimensional rather than 3-dimensional model of topodensity.

2. Method

We consider numerous hypothetical but realistic 3D mass-density distributions whose effect would not be accounted for in a 2D model. We have written a software program (Rad_Eff_Geoid) to apply forward modeling in each case to evaluate numerically the direct topographical effect (DTE), and the primary and secondary indirect topographical effects (PITE and SITE) of these masses. Our approach is a generalization of the work of Martinec (1998) dealing with topographical effects on geoid. Rather than repeat the mundane details of our approach, here we will provide an overview of the method emphasizing point of departure from approaches dealing with 2D models.

We consider any density distribution as a series of interfaces, whether these interfaces are drawn from

We apply analytical radial integration and numerical horizontal integration of the Newton kernel and its various derivatives to find the contribution of a 3D density distribution considered to represent the anomalous topographical density unaccounted for in a 2D model. Integration is performed over a limited spherical cap, with size determined by testing (normally no more than 3 degrees), to determine effects on gravity and gravity potential of the anomalous masses over a grid of points. The DTE on gravity, the PITE on gravity potential and geoidal height, and the SITE on geoidal potential are calculated in this way. Stokes integration of the DTE and SITE is then performed over a spherical cap of maximum size 6 degrees to determine the DTE and SITE on geoidal height.

3. Results and Discussion

To evaluate the accuracy of our software, we first consider simple shapes whose effects on gravity may be calculated analytically at specific points. Here we present one such example, a disc of anomalous density of 660 kg/m^3, 30 km wide and 250 m thick, embedded at a depth of 1500 m in topography 2000 m thick. In a real world context, this might represent a disc of basalt embedded in a sandstone.

The SITE resulting from this shape is negligible. The DTE on gravity, and the PITE on potential, are given in Figures 1a and 1b below. These and later figures show an arbitrary 1 degree by 1 degree region, centered on the anomalous mass whose effect they depict.

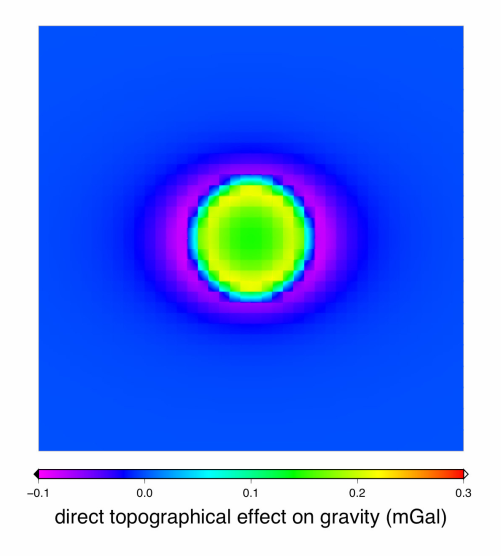

Figure 1a

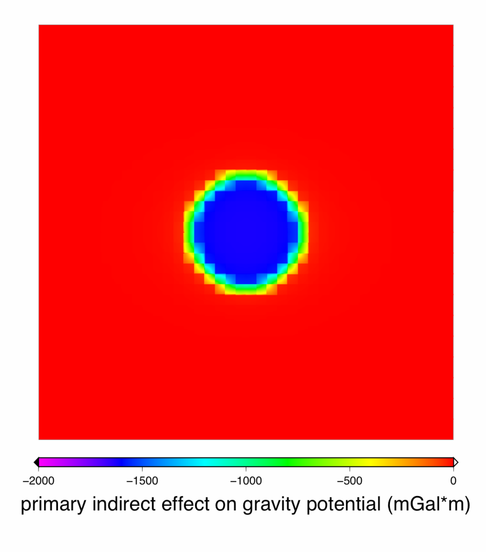

Figure 1b

For a computation point over the center of the disc, both of these effects may be calculated entirely analytically, without numerical integration. Figure 1a shows that this is not the maximum of the DTE, but has similar magnitude. Comparison with the analytical calculation at this point shows the error in our numerical approach. In this particular case our software calculates the DTE as 0.142 mGal, while an analytical determination yields 0.120 mGal (14.3 % error). Our software calculates the PITE as -1623 mGal*m, while the analytical result is -1716 mGal*m (5.4 % error). By performing the calculation with discs of various sizes and densities, we find no percent error greater than 15%, which we consider suitable for merely estimating whether the differences between 2D and 3D models are significant.

We further calculate effects on geoidal height corresponding to the DTE, PITE and SITE we have calculated. Again, the SITE is negligible. Results for the DTE and PITE are given in Figures 2a and 2b.

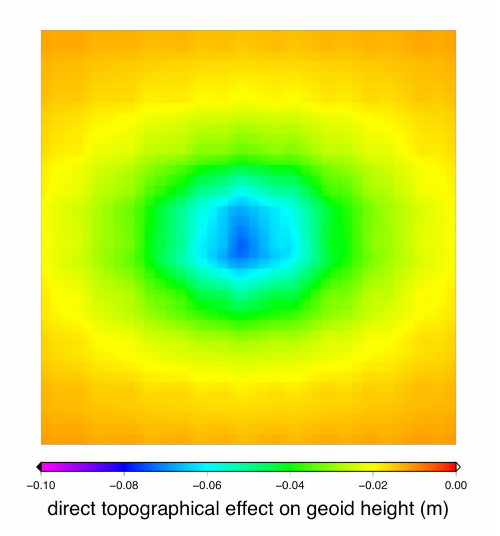

Figure 2a

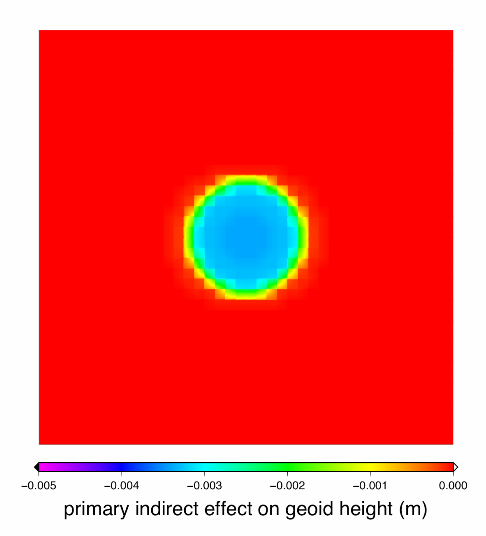

Figure 2b

Figure 2a indicates a minimum DTE of -7.1 cm, and Figure 2b a minimum PITE of -0.3 cm. As expected, the PITE is an order of magnitude smaller than the DTE. The magnitudes indicate however that for a realistic density distribution the contributions not accounted for in a 2D topodensity model may reach centimeters in magnitude.

We have further extended our software to deal with more complex density distributions, to estimate effects in more realistic situations. As an example application, we consider radial density effects over Lake Superior. We assume that a laterally varying model has already been applied based on the extent of the lake surface. Thus radial density effects will mainly be significant near to shore, since in areas where the lake bottom is deeper than the geoid there is no radial density variation within topography. Our results for the DTE on gravity and the PITE on gravity potential are given in Figures 3a and 3b.

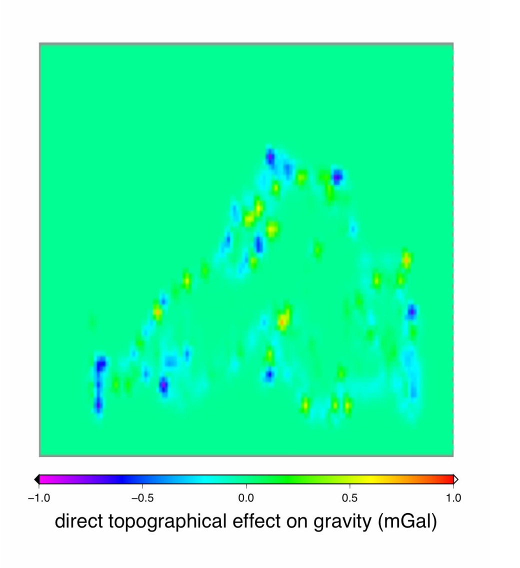

Figure 3a

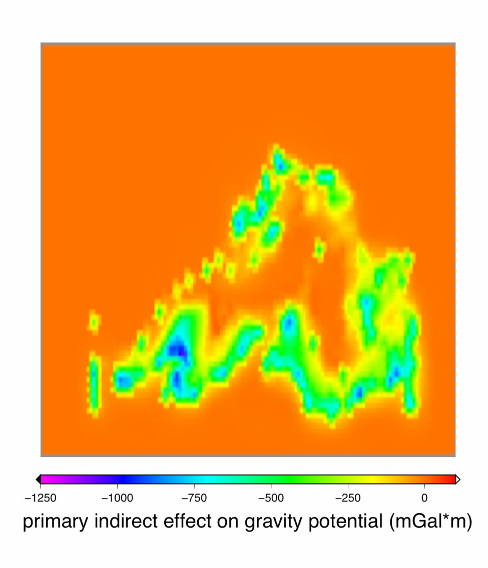

Figure 3b

We see that while effects of lake waters are relatively small, they are still significant. The DTE reaches -0.99 mGal and the PITE -1190 mGal*m in the most extreme areas.

4. Conclusions

We have created a method for calculating the DTE, PITE and SITE resulting from masses neglected in 2-dimensional models of topographical density. Our results are within roughly 15% error of values determined through an entirely analytical calculation. We find that the differences between geoidal heights calculated using 2-dimensional and 3-dimensional models might be on the order of centimeters, based on realistic simulations. While the software we have developed would allow implementation of a 3-dimensional model, we lack sufficient information on the radial topographical density distribution to apply it in such a way. In the future we will attempt through various simulations to determine how often 2-dimensional models are insufficient and to identify characteristics of such situations.

References

Martinec, Z. (1998). *Boundary Value Problems for Gravimetric Determination of a Precise Geoid (Lecture Notes in Earth Sciences).* Springer-Verlag Telos.

Vaníček, P., J. Huang, P. Novak, S. Pagiatakis, M. Veronneau, Z. Martinec, W. E. Featherstone (1999). Determination of the boundary values for the Stokes-Helmert problem, *Journal of Geodesy,* 73: 180-192.